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29-NOV-RICHARD L. GARWIN

DEC 3 2 39 PM 1981

Richard L. Garwin
IBM Fellow
Thomas J. Watson Research Center
P.O. Box 218
Yorktown Heights, New York 10598

Dear Dick;

I first heard about this boat when I was in high school from my friend David Smullin who credited it to his brother Joe. Joe explained roughly how it worked to me as I started college. I told a teacher of a course called "Yaght Design" about it when I was a freshman. He didn't believe it. Thats when I worked out the analogies and went through the energy arguments in some more detail. In graduate school someone pointed out the enclosed Blackford article to me. Since the article had three conclusions, and all three wrong, and was published I thought I could publish something correct.

Now if I resubmit this I will make some changes. I will leave all the discussion of the variable "a" to an appendix and center the argument on the basic fact that Power = Force x Velocity x constant. The constant being allways bigger or less than one depending on whether you are trying to generate thrust or power. This result follows from the argument with the parameter "a" or from other general arguments for certain models of fluid behavior (1) The particle model of a fluid in the Sci. Am. article or 2) the assumpion that the fluid interacting with the propeller eventually returns to the speed of the surrounding fluid and that the work is the integral of pressure x velocity over a remote surface).

Another idea I would like to add is this. A tacking sailboat is really a reciprocating windmill powering a reciprocating propeller. An iceboat tacking downwind faster than the wind is a reciprocating "mechanism" collecting power for a reciprocating fan.

Any comments?

Enclosed; 1) Article
2) Reviews
3) Two Earlier articles on same subject.

Sincerely,

A. L. Ruina

Andy L. Ruina
Theoretical and Applied Mechanics
Thurston Hall
Cornell University
Ithaca, New York 14853
(607) 256-7108
5062 (msg)
272-8377

112981.ALR

P.S. You were an entertaining, inspiring & hospitable host.

THE "PUSH-ME PULL-YOU" BOAT IS BETTER STILL

Andy Ruina

Division of Engineering
Brown University
Providence, R.I. 02912

In a recent article[1] (Oct. 1978) B.L. Blackford discusses the physics of the push me/pull you boat and reaches conclusions about the optimum performance of such a boat. By using an inappropriate definition of efficiency, however, he deduces overly-harsh restrictions on the boat's "ideal" performance. In fact, the boat can do the following: 1) it can sail into the wind at any speed, not just twice the wind speed as the article concludes; 2) contrary to intuition, the boat can sail in the same direction as the wind at <u>any</u> speed (even faster than the wind). In fact, if one can think of air and water merely as two fluids with some relative velocity, it can be seen that both of these features are the same. That is, a boat sailing into the wind can be thought of, as seen from an observer who moves with the wind, as a "current vehicle" currenting in the same direction as the water, faster than the water.

The push me/pull you boat sits at the interface of water and air with a propeller in each fluid and has some kind of transmission to transmit power from one propeller to the other. A force acts at each propeller and a drag force acts on the boat directly (whether the drag is from wind or water does not matter since both drag forces will act in the same direction in the interesting cases). Before embarking on a discussion of the consequences of force and energy balance for this boat, first consider a simpler mechanical analogy:

Two coaxial rollers of different radii are rigidly attached to each other and are each in contact with a rail (see figure 1). The rollers represent the boat, and the rails represent water and air. Assume the rollers roll without slip on the rails. If the rail (representing wind) in contact with the larger roller is moved, then the rollers will move in the opposite direction. This is like a boat sailing into the wind. Similarly, if the rail in contact with the smaller roller is taken as air and is moved, keeping the other rail fixed, then the rollers will move in the same direction as the rail faster than the rail. This is like a boat traveling in the same direction as the wind faster than the wind. Neither of these demonstrations is affected by the introduction of a drag force. If the rollers have radii that are nearly equal, very fast speeds are obtained. (A less than full spool of thread demonstrates this. Place the spool on a smooth surface, and slowly pull the loose end from the underside of the spool). Rollers which do not slip are, in fact, strictly analogous to maximally efficient windmills and propellers if efficiency is measured correctly.

A windmill (or watermill) on a push me/pull you boat is operating best when it offers the most power for the least drag force. In eq. (7), Blackford's article tells us that

$$P = FV(1-a) \qquad (1)$$

where P is the power provided to the transmission, V is the <u>relative</u> speed of the inviscid incompressible fluid relative to the boat, F is the force of the fluid on the propeller (measured in the <u>same</u> direction as V) and a is the fractional reduction in relative fluid velocity at the propeller relative to the far field. As the article points out, the derivation is valid for both mills (propellers that provide power to the transmission) and thrusters (propellers that use power from the transmission). In the latter case a is negative and F and V must have opposite signs. Thus, a , F and V may take on any values subject to the constraints

$$a < 1 \quad ; \quad \mathrm{sign}(a) = \mathrm{sign}(F)\mathrm{sign}(V) \tag{2}$$

where $\mathrm{sign}(x)$ is plus or minus one, depending on whether x is greater or less than zero.

For whatever values of a , F and V are chosen, a propeller of the proper size, A , for fluid with density ρ can be designed by Blackford's formula (valid for mills and thrusters)

$$F = 2\rho AV^2 a(1-a) \ . \tag{3}$$

Now a boat can be designed using (1) and (2) and then the propellers designed by (3). Assume the wind moves at velocity W . The wind causes a force F_1 on the air propeller which provides power P_1 to the transmission. The water exerts a force F_2 on the water propeller which provides power P_2 to the transmission. A drag force D acts directly on the boat which is moving at speed u . All forces and velocities are taken to be positive if in the same direction as the wind. Our design constraints are that D is imposed (possibly a function of u and possibly negative), propellers obey (2) and (3), and the energy and momentum of the boat must be constant. These last two laws, which are the essential physics, can be written from the point of view of an observer who sits on the boat, as

$$F_1 + F_2 + D = 0 \tag{4a}$$

$$P_1 + P_2 = 0 \tag{4b}$$

where a perfectly efficient transmission is assumed.

The relative velocity that the wind-propeller feels is $W-u$ and the relative velocity the water propeller feels is $-u$. Applying these relative velocities to (1) and then to (4b), equations (4) can be rewritten:

$$F_1 + F_2 = -D \qquad (5a)$$

$$F_1(W-u)(1-a_1) + F_2(-u)(1-a_2) = 0 \quad . \qquad (5b)$$

Equations (5) can be solved (almost always) to give

$$\begin{aligned} F_1 &= -D[1 + (W-u)(1-a_1)/u(1-a_2)]^{-1} \\ F_2 &= -D[1 + u(1-a_2)/(W-u)(1-a_1)]^{-1} \end{aligned} \qquad (6)$$

where the solution is sensible only if (2) is satisfied.

Consider first the case of sailing into the wind $(u<0)$. In this case, the drag D will be positive and the constraint (2) applied to (6) with the appropriate relative velocities, can be re-expressed as

$$\begin{aligned} (1 + |a_2|)/(1 - |a_1|) &< 1 + W/|u| \\ \text{sign}(a_1) = -\text{sign}(a_2) &= 1 \ . \end{aligned} \qquad (7) \quad \Rightarrow \frac{|u|}{W} < \frac{1-|a_1|}{|a_1|+|a_2|}$$

In Dr. Blackford's article, a_1 is taken as 1/3 to maximize "efficiency" and hence the maximum possible $|u|$ of $2W$. In fact, both a_1 and a_2 may approach zero and thus $|u|$ may be arbitrarily large. A boat designed to work at a speed $|u|$ into the wind is planned by successively applying equations (6) and (3) using values of a_1, a_2 that satisfy (7) and whatever value of D is imposed. For a fast boat, large propellers are needed. There is no fundamental limit on the ratio $|a_1|/|a_2|$, though they both may need to be small for a boat to travel at a given speed, and thus formulae (6) and (3) show no basic limitations on the ratio of the two propeller sizes.

Next, consider the boat to be traveling downwind faster than the wind, $u > W$, when one expects D to be negative. In this case the constraint (2) applied to (6) can be expressed by

$$(1 - |a_2|)/(1 + |a_1|) > 1 - W/u \qquad \text{sign}(a_1) = -\text{sign}(a_2) = -1 \; . \tag{8}$$

$\Rightarrow \frac{u}{w} < \frac{1+|a_1|}{|a_1|+|a_2|}$

These relations can be satisfied for any u if $|a_1|$ and $|a_2|$ approach zero. One should note the signs of a_1 and a_2, for they imply that the water propeller is acting as a mill, and the air propeller as a thruster. Again, there is no fundamental limit on how much faster than the wind the boat will sail.

For both upwind and downwind sailing the fastest boats are those with a_1 and a_2 as close to zero as possible. Thus, by (1), the most efficient windmill is the one that provides power equal to the product of retarding force and relative velocity, and the best thruster is one that provides thrust equal to the raio of power to relative velocity. The analogy to rollers without slip, where the above-named efficiencies are met exactly, should now be apparent. A roller that does not slip transmits power equal to the work rate on the rail (measured relative to the roller). A maximally efficient propeller transmits power equal to the very-far-field[2] work rate of the air.

The feasibility of the boat is based, then, on the fact that the power obtainable from one moving fluid causing a certain drag, may be used to provide a greater thrust in a second fluid with a slower velocity relative to the boat.

The push me/pull you sailboat can be made more intuitively acceptable by pointing out that it does nothing new. Connect two sailboats that are tacking upwind (always on opposite tacks) with a string and call them one boat. Here is a boat sailing directly into the wind. Iceboats are known to be able to tack downwind so that the component of velocity in the wind direction is greater than the wind speed (from the boat's point of view, it is tacking upwind!) Connect two oppositely tacking iceboats with a piece of string and you have a boat sailing directly downwind faster than the wind. It is interesting and not contradictory that, from the point of view of the boat sailing downwind faster than the wind, the still water is doing work on the boat (likewise with the iceboat pair).

Finally, I would like to say that the boat was first introduced to me by Joseph Smullin (of Bolt, ~~Berraneck and Neumann~~ Camb. Collaborative, Cambridge, Mass.) about ten years ago. The boat is indeed instructive to analyze as it provides a chance to use the laws of mechanics where intuition is not at first correct.

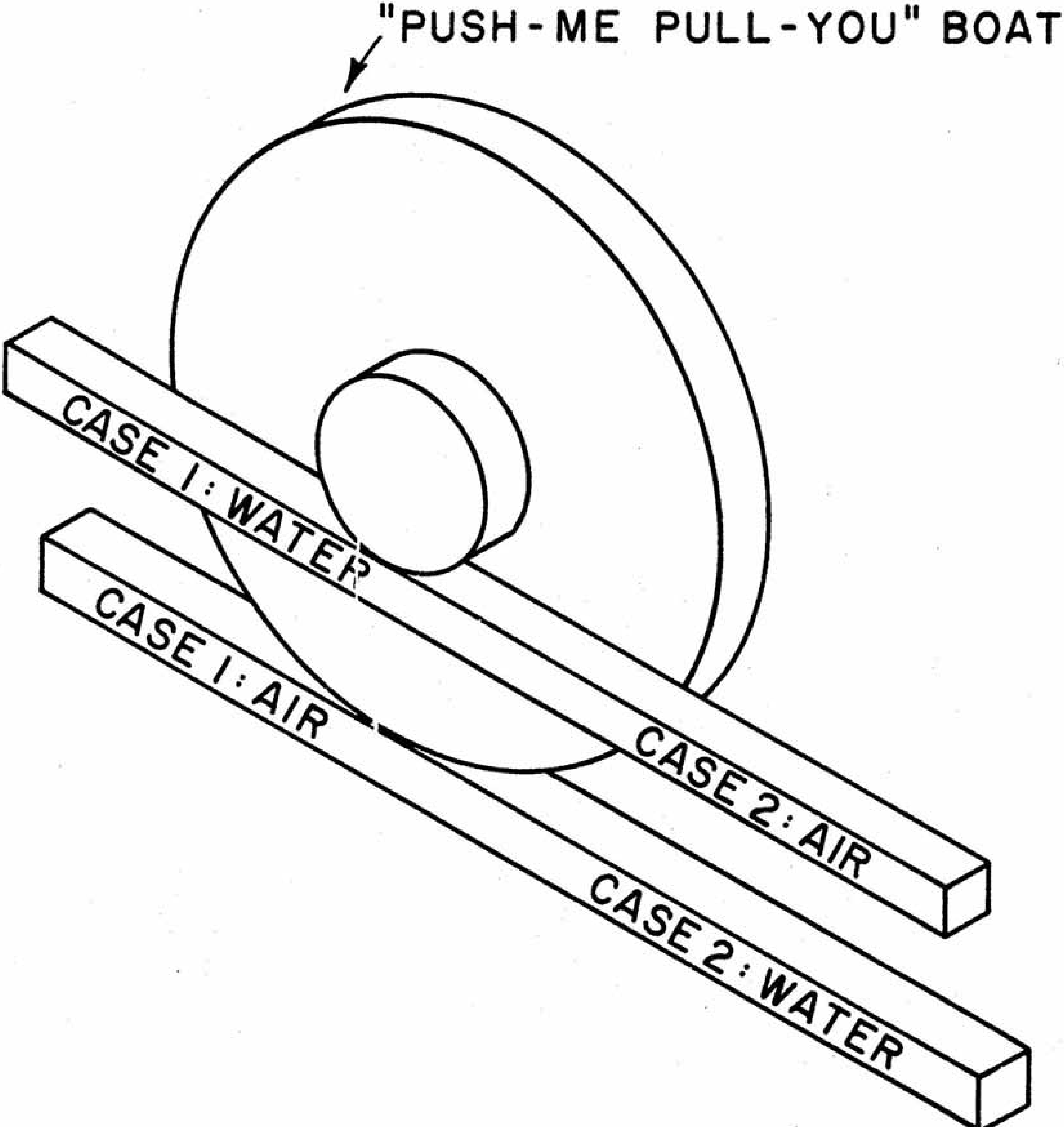
---

1 B. L. Blackford, Am. J. Phy. 46, 1004, 1978.

2 "Very-far-field" means so far away that all inhomogeneities in fluid velocity have been smoothed out by viscosity so that the work rate is again FV . This viscous smoothing is dissipative and is what is minimized by an optimal propeller.

FIGURE CAPTION

Figure 1. Rigidly connected coaxial rollers are analogous to a "push-me pull-you" boat. Two rigid rails represent water and air. In case #1 the upper rail is fixed, the lower rail is moved and the 'boat' moves up "wind." in case #2 the lower rail is fixed, the upper rail is moved and the 'boat' moves down "wind" faster than the "wind."


"PUSH-ME PULL-YOU" BOAT
CASE 1: WATER
CASE 1: AIR
CASE 2: AIR
CASE 2: WATER

AMERICAN
JOURNAL
of PHYSICS

A Journal of the American Association of Physics Teachers

John S. Rigden, Editor
Philip B. James, Assistant Editor

Room 240 Benton Hall
University of Missouri - St. Louis
St. Louis, Missouri 63121 U.S.A.

March 12, 1979

Professor Andy Ruina
Division of Engineering
Brown University
Providence, R. I. 02912

Dear Professor Ruina:

The two reviews of your manuscript "The 'Push-Me Pull-You' Boat is Better Still" are enclosed. On the basis of these reviews, it would appear that rather substantial revision must be made before the paper is acceptable. In particular, the comments of one reviewer concerning the constancy of the speed would seem to be particularly difficult in view of the Scientific American article (by Martin?) which you neglected to cite.

Since both referees feel that, properly revised, the paper is acceptable we shall keep it in the active file pending your reply. Further advice of outside reviewers will be sought at that time.

Sincerely yours,

Philip James

Philip B. James
Assistant Editor

CC: Referees

I couldn't disagree with anything more than this review!! (I get angry reading it.) (5)

RECEIVED
JAN 18 1979

This paper suffers from two major shortcomings which render it unsuitable for publication in the American Journal of Physics. First, the author considers only the case of a boat moving at a <u>constant</u> speed u , even though it states in the accompanying article from Scientific American that, "In the laboratory test, where the boat was in a large tank of water and a room fan was the source of wind, we placed the boat in front of the fan with the air propeller motionless. The breeze blew the boat away from the fan and started the propeller turning. As the propeller came up to speed, the boat stopped moving away from the fan and began accelerating toward it until the boat reached the fan." In view of this, there is no reason to believe that any conclusion reached by the author is applicable to the actual "push-me pull-you" boat, which clearly exhibits a <u>non-constant</u> speed u .

The second and more serious flaw in the manuscript is the wordy way in which the author deals with the problem at hand. His attempt to demonstrate intuitively, by means of discussions and analogies, that the boat under consideration can sail both into the wind and with the wind at any speed is simply not the way to solve a problem in mechanics. It is unsatisfactory as far as good science is concerned, and is disastrous with regard to the teaching of physics to students, for upon reading a paper such as this, students will come away with the mistaken notion that mechanics deals with qualitative verbal "explanations" for observed motions of dynamical systems, rather than quantitative predictions of these motions.

The author would surmount both of the above mentioned deficiencies in his paper by formulating a <u>first-order</u> ordinary differential equation of motion governing u (neglecting the gyroscopic effects of the propellers) and producing plots of u versus the time t for a series of values of the wind speed W . This would enable him to account satisfactorily for the non-constant behavior of u described in the Scientific American article, support (or reject) his contention concerning the boat's possible speeds, and eliminate the need to resort to "explanations" of any kind.

Comments on: "The push-me pull-you boat is better still"

by Andy Ruina.

I agree with the result that the boat can, in principle, sail upwind at speeds greater than 2W. This also follows from the Blackford paper when the factor "a" is allowed to vary and takes on values less than 1/3.

However, I disagree with the statement made by Ruina at the bottom of page 4 and the top of page 5 of his article. There he implies that the ratio $\rho_2 A_2/\rho_1 A_1$ is not a critical parameter for the operation of the boat. In a letter to Dr. Ruina (copy enclosed) I have pointed out why I believe that the ratio is still important, based on the results of the Blackford paper using a variable "a".

put in an inefficient propeller?

A similar conclusion is also reached from the Ruina paper. Suppose, for example, that we want to design a boat to sail upwind at speed $|u| = W$ using an air propeller with $a_1 = \frac{1}{3}$. Then, by combining Eqns. (6) and (3) one finds that a_2 is determined by

$$[1 - \frac{4}{3(1-a_2)}] \div [1 - \frac{3(1-a_2)}{4}] = \frac{9}{8}(\frac{\rho_2 A_2}{\rho_1 A_1}) a_2 (1-a_2) \quad .$$

That is to say, we are no longer free to choose a_2. The value of a_2 is determined by $(\frac{\rho_2 A_2}{\rho_1 A_1})$, which must exceed a critical value before the resulting a_2 will satisfy Eqn. (7).

Apart from this criticism, I am in general agreement with the paper and feel that it is suitable for publication in the AJP. I would, however, like my citicism addressed and the appropriate changes made.

B. L. Blackford
Department of Physics
Dalhousie University

The physics of a push-me pull-you boat

B. L. Blackford
Department of Physics, Dalhousie University, Halifax, Canada
(Received 20 March 1978; accepted 17 May 1978)

The basic laws of mass, energy, and momentum conservation are applied to a novel wind-driven water craft. The resulting analysis is an instructive application of these laws, suitable as an undergraduate physics exercise. We find a critical condition which must be met before the boat will accelerate against the wind, and we also obtain expressions for calculating the final speed.

I. INTRODUCTION

A novel wind-driven water craft was recently reported in the literature.[1] In this boat a windmill-type propeller is connected via a straight shaft to an underwater propeller, as shown in Fig. 1. Kinetic energy extracted from the wind by the air propeller is used by the water propeller to move the boat directly *against* the wind. We have demonstrated such a boat at recent public displays in our junior level physics laboratory. A typical nonphysicist is suitably impressed by the novelty of the boat, whereas the initial response of people with some physics background is often one of disbelief. Being familiar with the laws of energy and momentum conservation, they feel that somehow these laws are being violated. Indeed, a detailed analysis of the device is an instructive application of these laws, suitable as an undergraduate physics exercise. (The topic also aroused considerable interest when given to a graduate physics problems class.) We present such an analysis here. It is found that there is a critical condition on the relative sizes of the two propellers which must be met before the boat will move against the wind, even for *ideal* propellers. It is also found that the maximum speed attainable under ideal conditions is twice the wind speed. The performance of an actual boat is expected to be considerably less than this.

II. THEORY

A. Energy from the wind (efficiency of an ideal windmill) (Ref. 2)

Figure 2 shows a propeller of swept out area A_1 immersed in a fluid flow field (density ρ_1) whose undisturbed speed and pressure are W and P_∞, respectively. The air speed at the propeller is $W(1-a)$ where $0 < a < 1$, and the downstream speed is W_∞. If we assume incompressible flow along the streamlines, shown in Fig. 2, then mass conservation requires that

$$AW = A_1W(1-a) = A_\infty W_\infty, \tag{1}$$

where A and A_∞ are the cross-sectional areas of the streamtube far upstream and far downstream, respectively. The factor $1-a$ is simply a convenient way of expressing the decrease in wind speed at the propeller. Now apply Bernoulli's theorem, in the form $P + (1/2)\,\rho\, v^2 = \text{const}$, to the regions upstream and downstream of the propeller. Upstream of the propeller we have

$$P_\infty + (1/2)\,\rho_1 W^2 = P_u + (1/2)\,\rho[W(1-a)]^2, \tag{2}$$

where P_u is the pressure immediately upstream of the propeller. On the downstream side

$$P_\infty + (1/2)\,\rho_1 W_\infty^2 = P_d + (1/2)\,\rho_1[W(1-a)]^2, \tag{3}$$

where P_d is the pressure immediately downstream of the propeller.

Combining Eqs. (2) and (3) one finds a pressure discontinuity at the propeller, given by $P_u - P_d \equiv \Delta P = (1/2) \times \rho_1(W^2 - W_\infty^2)$. The force exerted on the propeller by the stream is therefore given by

$$F_W \equiv A_1\Delta P = (1/2)\,\rho_1 A_1(W^2 - W_\infty^2). \tag{4}$$

From momentum conservation, the force on the propeller can also be written

$$F_W = \rho_1 AW^2 - \rho_1 A_\infty W_\infty^2, \tag{5}$$

where the two terms represent the rates at which momentum enters and leaves the streamtube, respectively. Combining Eqs. (1), (4), and (5) one obtains $W_\infty = W(1-2a)$, which gives

$$F_W = 2\rho_1 A_1 W^2 a(1-a) \tag{6}$$

when substituted in Eq. (4). The power extracted from the wind is given by

$$P_W = F_W W(1-a) = 2\rho_1 A_1 a(1-a)^2 W^3. \tag{7}$$

Maximizing the power P_W with respect to the parameter a, gives $a_{max} = 1/3$. Using this optimum value of a, one finds that the efficiency of the propeller, as defined by $P_W/(1/2)\rho_1 A_1 W^3$, is equal to 16/27. This is the maximum efficiency of an ideal windmill. Real windmills of modern design[2,3] are capable of reaching 60%–70% of this value.

B. Net forward thrust

Assume that the boat is held fixed with respect to the undisturbed water and focus attention on the water propeller A_2, Fig. 1. By using arguments identical to those used above in obtaining Eqs. (1)–(6), one finds that the thrust delivered by the propeller A_2 is given by

$$F_2 = 2\rho_2 A_2 V^2. \tag{8}$$

In this case, it is helpful to note that the speed of the water in the streamtube far upstream of A_2 is zero while that far downstream is $V_\infty = 2V$, where V is the water speed at the propeller.

The rate of increase of the kinetic energy of the water is given by $F_2 V$ and if this is equated to P_W, as given by Eq. (7), one finds that

$$F_2 = 0.56\,\rho_2 A_2(\rho_1 A_1/\rho_2 A_2)^{2/3}\,W^2, \tag{9}$$

where $a = 1/3$ was used. In obtaining Eq. (9), the propeller A_2 was assumed to be capable of converting the shaft energy

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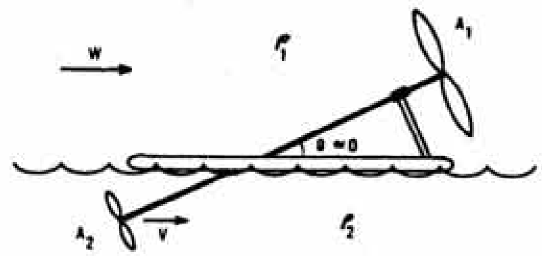

Fig. 1. Sketch of a boat which supports a shaft with a propeller on each end. One propeller, of swept-out area A_1, is located in the air and the other propeller, of area A_2, is located below the water surface. Both propellers are assumed to be far enough from the water surface so that its effect can be ignored. The speed of the air far upstream of the boat is W. The speed of the water immediately downstream of A_2 is V, where V is due to the effects of A_2. ρ_1 and ρ_2 are the densities of air and water, respectively.

kinetic energy of the water with 100% efficiency. This is *not* inconsistent with the 59% efficiency of an ideal windmill, as it refers to a different problem.

The net forward thrust $F_2 - F_W$ is given by subtracting Eq. (6) from Eq. (9) and rearranging the resulting expression, that is

$$F_{\text{net}} = \rho_1 A_1 W^2[0.56(\rho_2 A_2/\rho_1 A_1)^{1/3} - 4/9]. \quad (10)$$

This expression for F_{net} assumes that both propellers operate at their maximum possible efficiencies and that there are no energy losses in the coupling shaft. It also neglects wind-drag forces on the hull and superstructure of the boat. Even for these ideal conditions we see from Eq. (10) that the net force will be positive only if $\rho_2 A_2/\rho_1 A_1 \geq 1/2$.

The essential qualitative explanation of why the boat moves forward can be seen from Eq. (10). It clearly depends on the way in which the kinetic energy extracted from the air is added to the water. If the streamtube is narrow ($\rho_2 A_2 \ll \rho_1 A_1$) the momentum in the water jet will be less than that extracted form the air and the boat would move backwards. On the other hand, if $\rho_2 A_2 \gg \rho_1 A_1$ the momentum in the water jet is larger than that extracted from the air and a net forward force results. This is analogous to the situation of two masses having the same kinetic energy, whereas the heavier mass has the largest momentum.

C. Ultimate speed

We now consider the situation where the boat is allowed to move forward, with the aim of calculating the ultimate speed u. It is convenient to use a reference frame attached to the boat. The power extracted from the wind by the propeller A_1 will then be given by

$$P'_W = (8/27)\rho_1 A_1 (W + u)^3, \quad (11)$$

where $a = 1/3$ was used in Eq. (7) and the effective wind felt by the boat is $W + u$ rather than W. Similarly, the force on the propeller A_1 is given by

$$F'_W = (4/9)\rho_1 A_1 (W + u)^2. \quad (12)$$

The water propeller A_2 now sees an effective upstream speed equal to u, a relative water speed at the propeller of $u + V$ and a downstream speed equal to $u + V_\infty$. Applying arguments similar to those in Sec. II A, one still finds that $V_\infty = 2V$. By analogy with Eq. (4), the thrust produced by A_2 can then be written

$$F'_2 = (1/2)\rho_2 A_2[(V_\infty + u)^2 - u^2] = 2\rho_2 A_2 (u + V)V. \quad (13)$$

The force F'_2 adds kinetic energy to the water at the rate $F'_2(u + V)$ and if we again assume that the propeller A_2 is 100% efficient then we have, from Eqs. (11) and (13)

$$F'_2(u + V) = 2\rho_2 A_2 (u + V)^2 V = (8/27)\rho_1 A_1 (W + u)^3. \quad (14)$$

At the ultimate speed u all forces on the boat must balance, i.e.,

$$F'_2 - F'_W - F_D = 0, \quad (15)$$

where

$$F_D = (1/2)\rho_2 A_D C_D u^2 \quad (16)$$

is the drag force[4] of the water against the hull. A_D is the effective frontal area of the hull and C_D is the drag coefficient. Substituting Eqs. (12), (14), and (16) into Eq. (15) then gives

$$\frac{(8/27)\rho_1 A_1 (W + u)^3}{(V + u)} - (4/9)\rho_1 A_1 (W + u)^2 - (1/2)\rho_2 A_D C_D u^2 = 0. \quad (17)$$

Equations (14) and (17) represent two equations for the unknowns u and V, which must be solved simultaneously. The general solution is very messy algebraically and it is perhaps more instructive to look at some particular cases.

First, consider the case where the drag coefficient C_D is zero, as this will yield the maximum possible value of u. With this assumption, Eqs. (14) and (17) yield

$$u = \left(\frac{2\rho_2 A_2/\rho_1 A_1 - 1}{\rho_2 A_2/\rho_1 A_1 + 1}\right) W \quad (18)$$

and

$$V = u/[2(\rho_2 A_2/\rho_1 A_1) - 1]. \quad (19)$$

In the limit $\rho_2 A_2/\rho_1 A_1 \gg 1$, $u = 2W$ and $V \ll u$. That is, the maximum possible upwind speed of the device is equal to twice the wind speed. On the other hand, the maximum downwind speed under the same conditions would be equal to the wind speed, as one can easily show from Eqs. (14) and (17) with appropriate sign changes.

The performance of an actual device will be reduced below the maximum value, $u = 2W$, by (a) nonzero drag coefficients, (b) smaller values of the parameter $\rho_2 A_2/\rho_1 A_1$, and (c) propeller inefficiency. For example, if one chooses $A_1 = \pi$ m^2, $A_2 = 0.2$ m^2, $A_D = 0.1$ m^2, $C_D = 0.2$, $\rho_2 = 1000$ kg/m^3, and $\rho_1 = 1.20$ kg/m^3, then one finds from Eqs. (14) and (17), that $u = 0.56W$ and $V = 0.034W$. If propeller

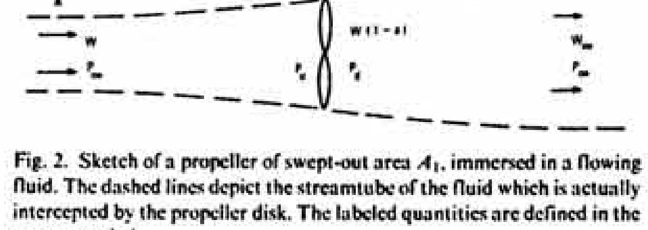

Fig. 2. Sketch of a propeller of swept-out area A_1, immersed in a flowing fluid. The dashed lines depict the streamtube of the fluid which is actually intercepted by the propeller disk. The labeled quantities are defined in the text as needed.

inefficiency is taken into account the value of u will be further reduced, by 50% or more.

D. Energy considerations

In the above derivation of Eq. (14) it was assumed that all of the kinetic energy extracted from the wind is added to the water by the propeller A_2. But, what about the work done by the drag force F_D? Have we violated energy conservation? To answer this question it is best to work in a reference frame at rest with respect to the undisturbed water. In this frame the force F_D moves at speed u and therefore dissipates energy in the water at a rate $F_D u$. If we are to conserve energy then the power extracted from the wind must supply this dissipation as well as the kinetic energy increase in the water passing through the propeller A_2.

First we note that, in the rest frame, the kinetic energy extracted from the wind by A_1 and that added to the water by A_2 will not be the same as in the boat frame. Recall that when a mass m has a speed change $v_f - v_i \equiv \Delta v$ in the rest frame, then the change in kinetic energy is

$$\Delta \mathrm{KE} = (\tfrac{1}{2})m(v_f^2 - v_i^2). \qquad (19')$$

The corresponding kinetic energy change in a reference frame moving with speed u is given by

$$\Delta \mathrm{KE}' = \Delta \mathrm{KE} + m\Delta v u. \qquad (20)$$

Applying this relation to the propellers, in the rest frame, we find that the power extracted from the wind is

$$P_W = P'_W - (\tfrac{4}{9})\rho_1 A_1(u+W)^2 u = P'_W - F_W u. \quad (21)$$

where P'_W is the power from the wind in the moving frame, as given by Eq. (11). F_W is the force of the wind against propeller A_1 and is the same in both frames. Similarly, the power added to the water by A_2 is given by

$$P_2 = P'_2 - 2\rho_2 A_2(u+V)Vu = P'_2 - F_2 u. \qquad (22)$$

where P'_2 is the power added to the water by A_2 in the moving frame and F_2 is the force produced by A_2. The energy balance in the rest frame therefore takes the form

$$P_W - P_2 = (P'_W - P'_2) + (F_2 - F_W)u. \qquad (23)$$

Recalling that $P'_W = P'_2$ in the moving frame and using Eq. (15), Eq. (23) becomes

$$P_W - P_2 = F_D u. \qquad (24)$$

Therefore a satisfactory energy balance does exist in the rest frame.

III. CONCLUDING REMARKS

A successful demonstration boat used a 36-cm-diam model airplane propeller for A_1 and a 10-cm one for A_2. These were connected by a 65 cm × 3.2 mm stainless-steel shaft mounted (shaft inclined at 22° to the horizontal) on a catamaran-type hull made from two thin-walled aluminum tubes (41 × 2.5 cm) separated by 22 cm. Performance can be optimized by trying propellers of various pitch. This unit produced a net forward thrust of nearly 1 N in a windspeed of 5 m/s provided by a household fan.

We note that a vertical-axis windmill[3] could easily be utilized with the above type of boat. The omnidirectional nature of such windmills should give them a practical advantage in this application. We also note that a wheeled vehicle could also be driven against the wind by coupling the propeller shaft to the wheels. The maximum attainable speed, under ideal conditions, is expected to be twice the wind speed in this case also.

ACKNOWLEDGMENT

This work benefited from numerous discussions with students and colleagues.

[1] S. Martin, Sci. Am. **233** (No. 12), 125(1975).

[2] *Standard Handbook for Mechanical Engineers*, edited by T. Baumeister and L. S. Marks (McGraw-Hill, New York, 1967), p. 9-8. *Handbook of the Engineering Sciences*, edited by J. H. Potter (Van Nostrand, Princeton, NJ, 1967), Vol. 2, p. 298.

[3] P. South and R. S. Rangi, NAE-LTR-LA-74, National Research Council of Canada, Ottawa, 1972.

[4] *Handbook of Tables for Applied Engineering Science*, edited by R. E. Bolz and G. L. Tuve (Chemical Rubber, Cleveland, 1973), p. 516.

ting to a brass plate that I brazed to the outer end of the pipe.

"In practice the antenna appears to have a signal gain of between 10 and 13 decibels through an angle of approximately 25 degrees on each side of the target toward which it is pointed. In other words, the directivity of the antenna falls to half power at the edges of a beam 50 degrees wide. It is not necessary to continuously track the satellite with the antenna. I determine in advance the azimuth at which the satellite has been predicted to rise and aim the antenna in that direction at an elevation of approximately 20 degrees. Usually the signal strength is adequate for making a good picture of cloud cover throughout the pass."

Seelye Martin of the department of oceanography at the University of Washington writes about this department's recent article [March] on sailboats without hulls. "It seems appropriate in a festive season of playthings to follow up this concept with a description of a toy boat that sails directly into the eye of the wind. It is the invention of Peter Kauffman and Eric Lindahl here in Seattle. We call it the 'push-me-pull-you' after the Dr. Dolittle animal with two heads.

"The boat consists of a propeller in the air coupled by a straight shaft to a propeller in the water [*see bottom illustration on opposite page*]. To reduce drag the shaft housing is mounted on a simple frame on top of a pair of pontoons. When the boat is placed on water in a wind with the water propeller upwind, the wind turns the air propeller and thereby the water propeller, which drives the boat into the wind.

"We have tried the boat in both laboratory and field tests. In the laboratory test, where the boat was in a large tank of water and a room fan was the source of wind, we placed the boat in front of the fan with the air propeller motionless. The breeze blew the boat away from the fan and started the propeller turning. As the propeller came up to speed, the boat stopped moving away from the fan and began accelerating toward it until the boat reached the fan.

"We also conducted a field test between two piers on Lake Union on a gusty day in February. The boat moved upwind over a distance of about 50 feet at a speed of about two feet per second in a strong wind with gusts of up to 30 miles per hour. The response of the boat to gusts is to slow down at first, because of the increased wind drag, and then to accelerate as the air propeller speeds up.

"There are some problems with the boat. If it becomes tilted at a direction of about 20 degrees to the wind, the air drag on the frame and the propeller tends to maintain the tilt. Moreover, wave motion can cause the air propeller to strike the water occasionally, so that the water propeller surfaces. Another problem is that too large a propeller velocity can pull the front end of the boat under water. In general, however, the force couple between the propellers manages to keep the boat headed into the wind.

"Why does it work? In the boat that you described in March, J. G. Hagedoorn replaced the sail of a sailboat with a kite and the keel with a hydrofoil. In the push-me-pull-you we replace the sail with a rotating airfoil and the keel with a coupled rotating waterfoil. The boat works for two reasons. First, the air velocity is much greater than the water velocity. Second, the energy of a moving parcel of air or water is proportional to the square of the air velocity, whereas the momentum is proportional simply to the velocity.

"To expand on this idea the accompanying illustration includes a sketch of the push-me-pull-you with the relevant air and water velocities. For simplicity we assume that the boat is initially stationary. We then estimate the drag and thrust forces on the boat caused by a parcel of air moving past the air propeller. Working out the calculations properly involves messy algebraic equations; the following is an extremely simplified version.

"The air propeller extracts energy from the parcel of air by reducing the air velocity immediately downwind of the propeller. If the wind velocity is 5 (in some convenient unit) upwind of the propeller and 4 downwind, the energy extracted is proportional to the difference of the square of the two velocities $(25 - 16 = 9)$, whereas the drag on the propeller is proportional to the momentum difference $(5 - 4 = 1)$.

"If the shaft has no energy loss, the water upwind of the water propeller has zero velocity and the water propeller is perfectly efficient, the water energy or water velocity squared behind the propeller is $(U_2)^2 = 9$, so that $U_2 = 3$, and the water-propeller thrust is proportional to 3. From this oversimplified argument on energy and momentum the thrust is three times the drag, so that if we release the boat, it will accelerate into the wind. Because of the difference in the effects of momentum and energy, the faster the wind is, the better the pu me-pull-you works."

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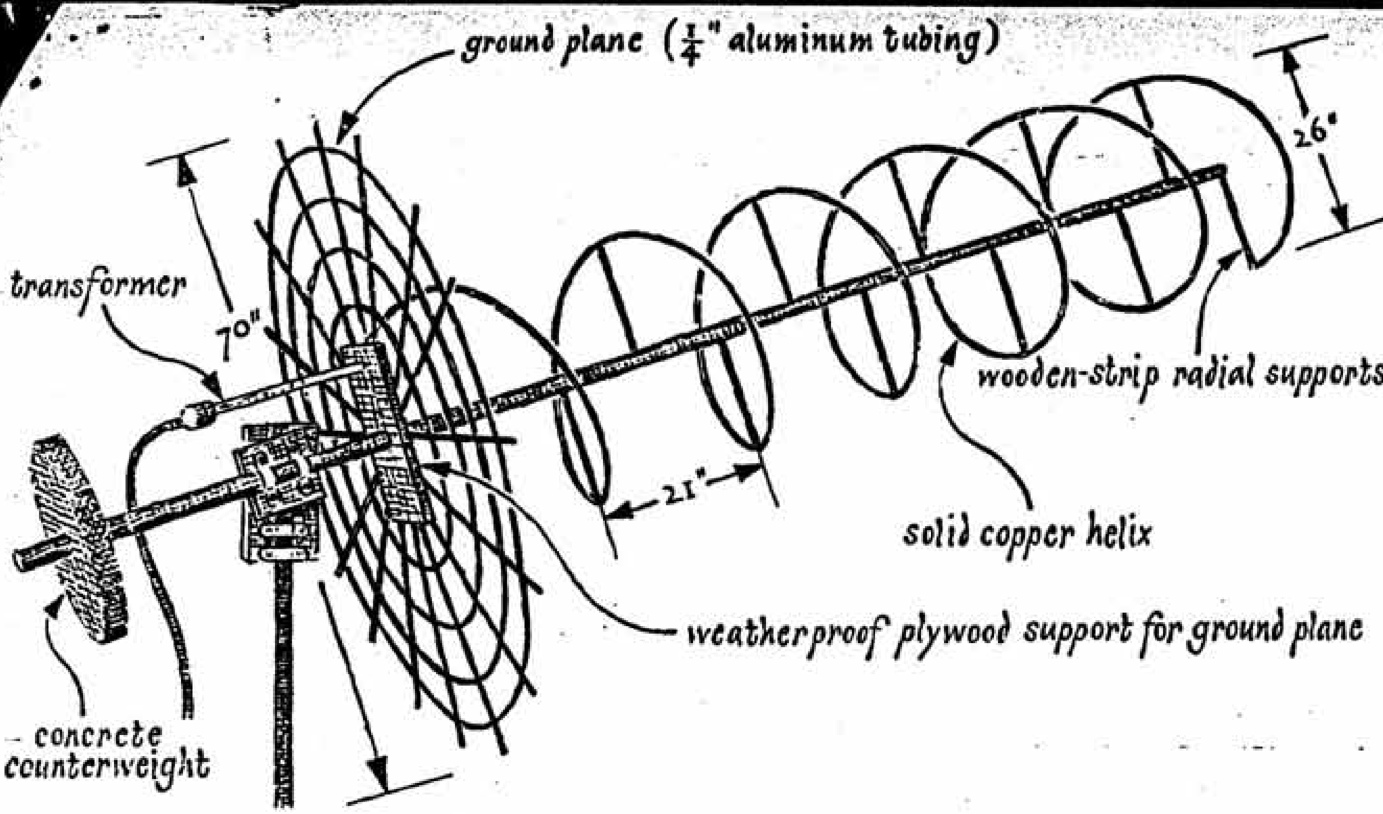
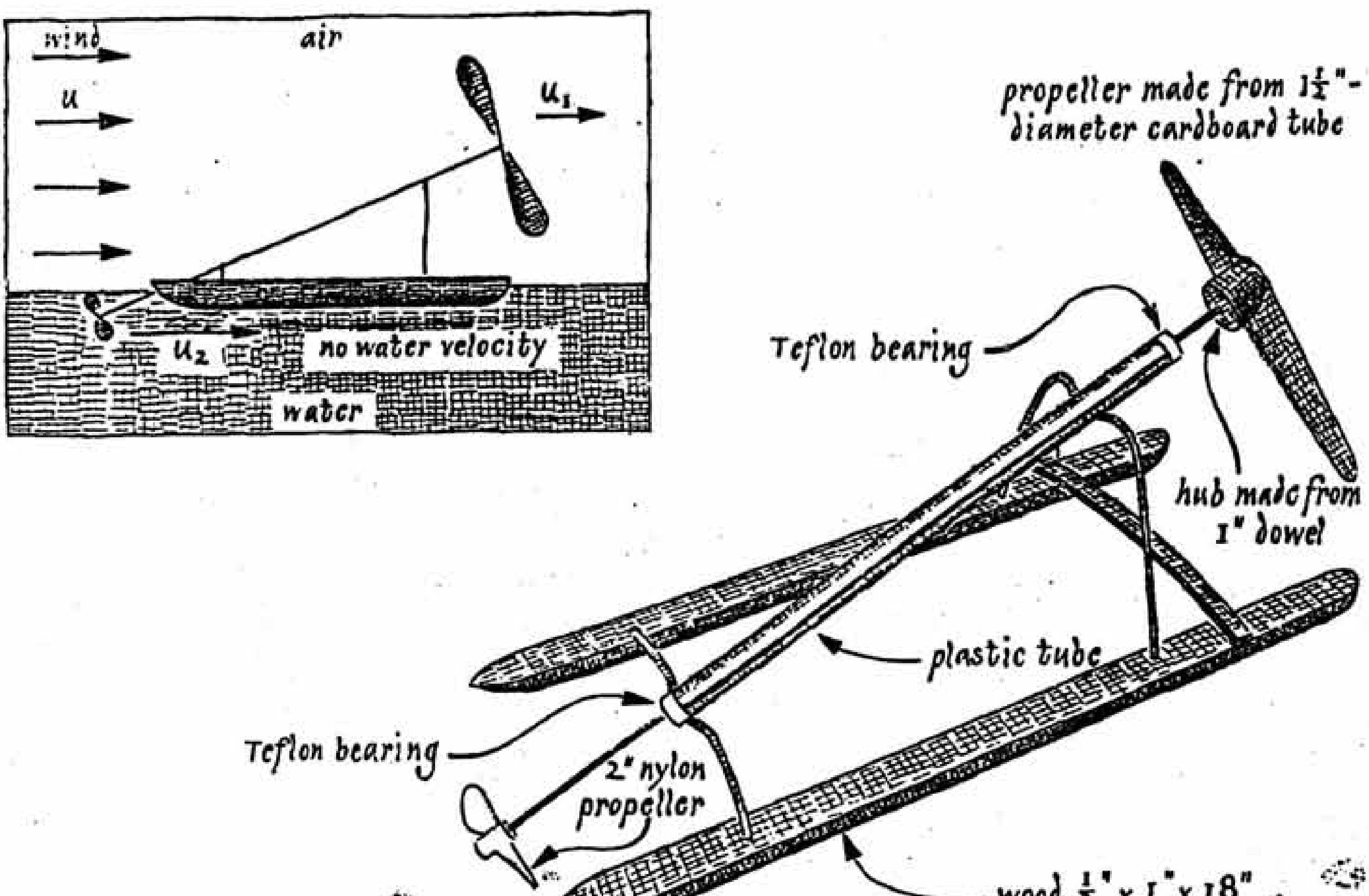


Eugene F. Ruperto's helical antenna



"Push-me-pull-you" boat designed by Peter Kauffman and Eric Lindahl