2.34, 2.38, 2.48, 2.64,

2.34.

Given: $P = 30\,kN$, $E_a = 70\,GPa$, $E_b = 105\,GPa$

Find: the normal stress
(a) in the aluminum layers
(b) in the brass layer.

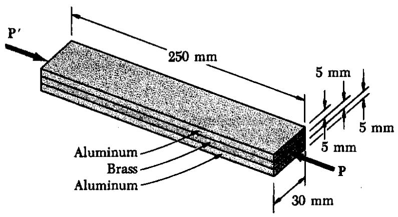
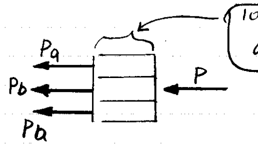


Fig. P2.34

Soln:

We need the axial force in each of the layer cut the whole and draw the F.B.D



long enough so end effects are not important

$\rightarrow \hat{i}$

$\Sigma F_x = 0$: $-P = 2P_a + P_b = -30\ kN$ (1)

Need another condition to solve for forces:

$\delta_a = \delta_b$ (they're attached to each other)

$\delta_a = \dfrac{P_a L}{E_a A_a}$

$\delta_b = \dfrac{P_b L}{E_b A_b}$ $A_a = A_b$

$\Rightarrow \dfrac{P_a}{P_b} = \dfrac{E_a A_a}{E_b A_b} = \dfrac{E_a}{E_b} = \dfrac{70}{105}$ (2)

Solving (1) & (2):

from (2) $\Rightarrow$ $P_a = 0.67\,P_b$ put it to (1) $\Rightarrow$

$2 \times 0.67 P_b + P_b = -30\ kN$

$\Rightarrow$ $P_a = -8.6$ kN
$P_b = -12.8$ kN

Cont. 2.34

$\sigma_a = \dfrac{P_a}{A_a} = \dfrac{-8.6\ kN}{(5mm)(30mm)} = \dfrac{-8.6 \times 10^3\ N}{150 \times 10^{-6}\ m}$
$= -57.3\ MPa$

$\sigma_b = \dfrac{P_b}{A_b} = \dfrac{-12.8\ kN}{(5mm)(30mm)} = \dfrac{-12.8 \times 10^3\ N}{150 \times 10^{-6}\ m}$
$= -85.3\ MPa$

$$\boxed{\begin{aligned}\sigma_a &= -57.3\ MPa\\ \sigma_b &= -85.3\ MPa\end{aligned}}$$

2.38

Given: $E_s = 200\,GPa$, $E_b = 105\,GPa$

Find: (a) the reactions at A & E
(b) the deflection of point C.

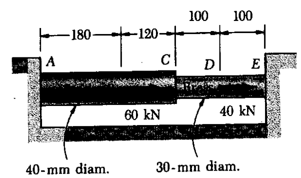
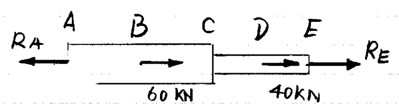


Fig. P2.38

Soln:

(a). We can get one eqn. from the force analysis.



$\rightarrow \hat{i}$

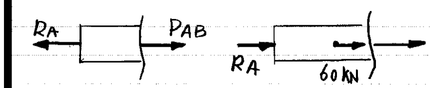
F.B.D of whole bar

$\Sigma F_x = 0$:
$-R_A + 100\ kN + R_E = 0$

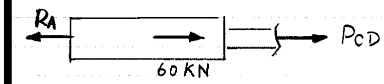
Two unknowns, one eqn, we need the geometry condition.

$0 = \delta_E = \dfrac{P_{AB} L_{AB}}{E_s A_s} + \dfrac{P_{BC} L_{BC}}{E_s A_s} + \dfrac{P_{CD} L_{CD}}{E_b A_b} + \dfrac{P_{DE} L_{DE}}{E_b A_b}$
$= \delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE}$ (1)

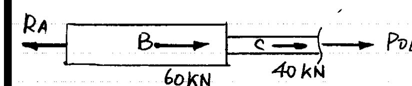
Need P_{AB}, P_{BC}, P_{DC}, P_{DE}: draw the F.B.D for each cutting:



FBD for AB. FBD for AC



FBD for AD



F.B.D for AE

From the above F.B.Ds, we have:

$P_{AB} = +R_A$
$P_{BC} = -60\,kN + R_A$
$P_{CD} = +R_A - 60\,kN$
$P_{DE} = +R_A - 100\,kN$

Plugging all these force back into (1) $\delta = 0 \Rightarrow$

$\dfrac{(+R_A)(180\,mm)}{E_s A_s} + \dfrac{(+R_A - 60kN)(120mm)}{E_s A_s} +$

$\dfrac{(+R_A - 60\,kN)(100mm)}{E_b A_b} + \dfrac{(+R_A - 100\,kN)(100mm)}{E_b A_b}$

$= 0$

$\Rightarrow \dfrac{1}{E_s A_s}\left[+R_A(0.3m)\right] + \dfrac{1}{E_b A_b}\left[+R_A(0.2m)\right] +$

$\dfrac{1}{E_s A_s}\left[-7200\,N\right] + \dfrac{1}{E_b A_b}\left[-16000\,N\right] = 0$

$\Rightarrow$ $R_A = +62.8\ KN$

$R_E = -R_A + 100\,kN = 37.2\ KN$

$$\boxed{\begin{aligned}\vec{R}_A &= -62.8\ kN\ \hat{i}\\ \vec{R}_E &= 37.2\ kN\ \hat{i}\end{aligned}}$$

(b). Finding δ_C:

$\delta_C = \dfrac{P_{AB} L_{AB}}{E_s A_s} + \dfrac{P_{BC} L_{BC}}{E_s A_s}$
$= \dfrac{(62.8\,kN)(180mm) + (+2.8\,kN)(120\,mm)}{(200\,GPa)(\pi\, 20^2 mm^2)}$
$= 46.3\ \mu m$
$(1\ \mu m = 10^{-6}\ m)$

$\boxed{\delta_C = 46.3\,\mu m}$

2.48:

Given: $E_a = 29\times10^6$ psi for link BC and DE, $\frac{1}{2}$" wide, $\frac{1}{4}$" thick, $P = 600$ lb, AF rigid body

Find: (a) Force in each link

(b) deflection A

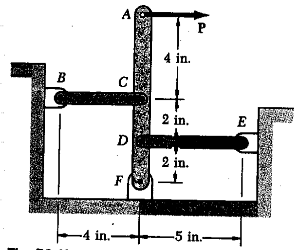
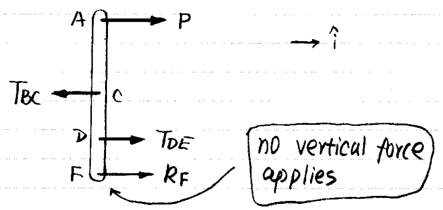


Fig. P2.48

Soln:

(a) Draw the FBD of AF:

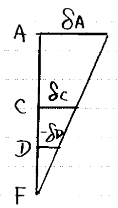


$\curvearrowleft + \Sigma M_F = 0$: $-T_{DE}(2") - P(8") + T_{BC}(4") = 0$

$\Rightarrow$

$$-T_{DE} + 2T_{BC} = 4P = 2.4 \text{ klb} \quad (1)$$

We have two unknowns, but one equation, looking for geometry:



It's $-\delta_D$ here, because D will actually move to the left due to the tension in DE.

$$\frac{\delta_C}{-\delta_D} = \frac{4"}{2"} = 2 \Rightarrow$$

$$\frac{T_{BC} L_{BC}}{E_a A} = -2\frac{T_{DE} L_{DE}}{E_a A} \Rightarrow$$

$$2T_{BC} + 5T_{DE} = 0 \quad (2)$$

Cont. 2.48.

Solving (1) and (2):

$$T_{DE} = -400 \text{ lb}, \quad T_{BC} = 1000 \text{ lb} \Rightarrow \boxed{T_{DE} = -400 \text{ lb}, \; T_{BC} = 1000 \text{ lb}}$$

(b). Finding deflection in A:

From the geometry figure:

$$\frac{\delta_A}{\delta_C} = \frac{8"}{4"} = 2$$

$\Rightarrow$

$$\delta_A = 2\delta_C = \frac{2 T_{BC} L_{BC}}{E_a A} = \frac{2(1000 \text{ lb})(4")}{(29\times10^6 \text{ psi})(\frac{1}{2}" \times \frac{1}{4}")} = 2.2\times10^{-3} \text{ in}$$

$$\boxed{\delta_A = 2.2\times10^{-3} \text{ in}}$$

2.64.

Given: Rod AB wl. $E_b = 15\times10^6$ psi, $\alpha_b = 11.6\times10^{-6}$ /°F

Rod CD wl. $E_a = 10.1\times10^6$ psi, $\alpha_a = 13.1\times10^{-6}$ /°F

Gap of 0.02" is at 60°F

Find: (a) the temperature when $\sigma_{AB} = -20$ ksi

(b) the deformation of AB at that time

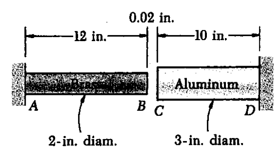


Fig. P2.63

Soln:

$$\delta_{AB} + \delta_{CD} = 0.02 \text{ in} \quad (1)$$

$P_{AB} = P_{CD}$ (tension/compress in the rod)

$$\delta_{AB} = \frac{P_{AB} L_{AB}}{E_{AB} A_{AB}} + \alpha_{AB} \Delta T \, l_{AB} \quad (2)$$

Cont. 2.64

$$\delta_{CD} = \frac{P_{CD} l_{CD}}{E_{CD} A_{CD}} + \alpha_{CD} \Delta T \, l_{CD} \quad (3)$$

We also have:

$$\sigma_{AB} = \frac{P_{AB}}{A_{AB}}, \quad \sigma_{CD} = \frac{P_{CD}}{A_{CD}}$$

$\Rightarrow$

$$P_{AB} = P_{CD} = \sigma_{AB} A_{AB} = P = (-20 \text{ ksi})[\pi(1")^2] = -62.83 \text{ kips}$$

$\Rightarrow$ If we plug in (2) and (3) into (1), one eqn, one unknow which is ΔT

$$0.02 \text{ in} = P\left(\frac{l_{AB}}{E_{AB} A_{AB}} + \frac{l_{CD}}{E_{CD} A_{CD}}\right) + \Delta t(\alpha_{AB} l_{AB} + \alpha_{CD} l_{CD})$$

$$\Rightarrow \Delta T = \frac{0.02 \text{ in} - P\left(\frac{l_{AB}}{E_{AB} A_{AB}} + \frac{l_{CD}}{E_{CD} A_{CD}}\right)}{\alpha_{AB} l_{AB} + \alpha_{CD} l_{CD}}$$

$$= \frac{0.02" - (-62.83 \text{ kips})\left(\frac{12"}{(15\times10^6 \text{ psi})\pi(1")^2} + \frac{10"}{(10.1\times10^6 \text{ psi})\pi(1.5")^2}\right)}{(11.6\times10^{-6}/°F)(12") + (13.1\times10^{-6}/°F)(10")}$$

$$= 165.8 \text{ °F}$$

$\Rightarrow$

$$T = \Delta T + 60°F = 226°F$$

$$\boxed{T = 226° F}$$

(b). Finding δ_{AB}

$$\delta_{AB} = \delta_T^{AB} + \delta_P^{AB}$$

$$\delta_T^{AB} = \alpha_b \Delta T L_{AB} = (11.6\times10^{-6}/°F)(165.8)°F(12") = 0.0231"$$

$$\delta_P^{AB} = \frac{P L_{AB}}{E_b A_b} = \frac{(-62.83 \text{ kips})(12")}{(15\times10^6 \text{ psi})\pi(1")^2} = -0.016"$$

$\Rightarrow$

$$\delta_{AB} = 0.007" \qquad \boxed{\delta_{AB} = 0.007"}$$

2.66.

Given: $P = 30\,kN$, $E = 70\,GPa$, $\nu = 0.35$

Find: (a) δ of the rod

(b) the change in diameter of the rod

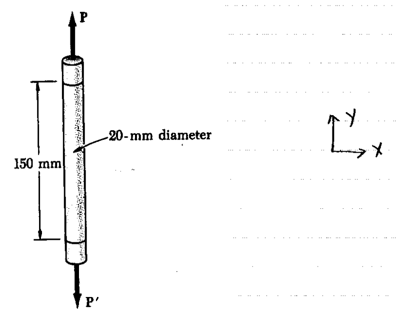


Fig. P2.66 and P2.67

Solution:

(a) Finding δ:

$$\delta = \frac{PL}{EA} = \frac{(30\times10^3\ N)(150\times10^{-3}\ m)}{(70\times10^9\ Pa)\,\pi\,(10\,mm)^2}$$
$$= 0.205\ mm$$

$$\boxed{\delta = 0.205\ mm}$$

(b) Finding the change in diameter:

$$\epsilon_y = \frac{\delta}{L} = \frac{0.205\ mm}{150\ mm} = 1.36\times10^{-3}$$

$$\epsilon_x = -\nu\epsilon_y = -0.35\times1.36\times10^{-3} = -477\times10^{-6}$$

change in diameter $\Delta d = \epsilon_x d$:

$$\Delta d = -477\times10^{-6}\times 20\ mm$$
$$= -9.54\times10^{-6}\ m$$
$$= -9.54\ \mu m$$

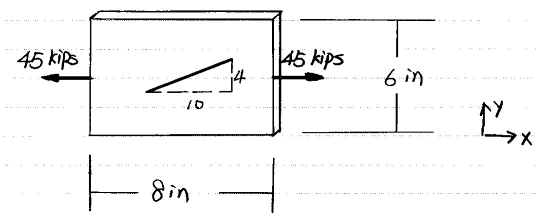
$$\boxed{\Delta d = -9.54\ \mu m}$$

2.68.

Given: the data available in Appendix B for yellow-brass plate w/ $\frac{1}{4}$" thick

Find: the slope of the line when $P = 45$ kips.

Cont. 2.68.



Soln:

From Appendix B, for a yellow brass:

$$E = 15\times10^6\ psi,\quad G = 5.6\times10^6\ psi$$

We have:

$$G = \frac{E}{2(1+\nu)} \Rightarrow$$

$$\nu = \frac{E}{2G} - 1 = \frac{15\times10^6\ psi}{2\times5.6\times10^6\ psi} - 1 = 0.3393$$

The slope of the line under $P = 45$ kips is:

$$Slope = \frac{4(1+\epsilon_y)}{10(1+\epsilon_x)} \quad (\Delta L = \epsilon L)$$

We need to find ϵ_x, ϵ_y.

$$\sigma_x = \frac{P}{A} = \frac{45\times10^3\ lb}{\frac{1}{4}"\times 6"} = 30\times10^3\ psi$$

$$\Rightarrow \epsilon_x = \frac{\sigma_x}{E} = \frac{30\times10^3\ psi}{15\times10^6\ psi} = 0.002$$

$$\epsilon_y = -\nu\epsilon_x = -0.3393\times0.002 = 6.8\times10^{-4}$$

$$\Rightarrow slope = \frac{4(1+6.8\times10^{-4})}{10(1+0.002)} = 0.39947$$

$$\boxed{slope = 0.39947}$$

2.86.

Given: Two blocks of rubber $G = 1.75$ ksi are bonded to rigid supports and plate AB.

$c = 4"$, $P = 10$ kips

Find:

the smallest a and b if τ in the rubber $\le$ 200 psi and the deflection of the plate $\ge \frac{3}{16}$".

Cont 2.86.

Solution:

plate AB is under double shear from the geometry.

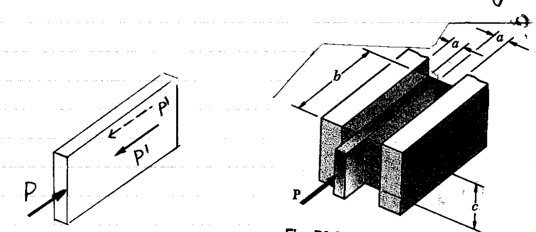


Fig. P2.86 and P2.87

$$\Rightarrow P' = \frac{1}{2}P = 5\ kips$$

$$\tau = \frac{P'}{A} = \frac{5\ kips}{cb} = \frac{5\ kips}{4"\ b} \le 200\ psi$$

$$\Rightarrow b \ge \frac{5\times10^3\ lb}{4"\times200\ psi} = 6.25\ in$$

The deflection of the block = δ of the plate.

$$\delta = \gamma a = \frac{\tau}{G}a \ge \frac{3}{16}"$$

$$\Rightarrow a \ge \frac{3}{16}"\ \frac{G}{\tau}$$
$$\ge \frac{3}{16}"\ \frac{1.75\ ksi}{200\ psi}$$
$$\ge 1.64\ in$$

$$\boxed{\begin{array}{l} a \ge 1.64\ in \\ b \ge 6.25\ in. \end{array}}$$

*: Figure for δ for the block.