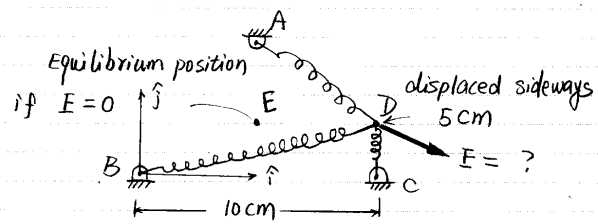
1). Three identical springs hold a mass. Find the force $\underline{F}$ if:

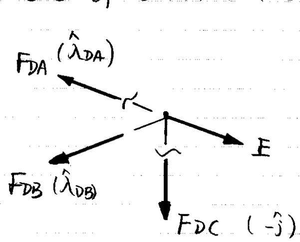
a). $l_0 = 0$, $k = 10\ N/cm$

b). $l_0 = 2\,cm$, $k = 10\ N/cm$

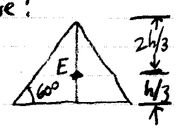


Solution:

This is an equilateral triangle: draw the F.B.D. of small mass:



Note: pt E is 1/3 of the way up from base:



$\Sigma \vec{F} = \vec{0}$, thus we have

$$\vec{F} + \vec{F}_{DC} + \vec{F}_{DB} + \vec{F}_{DA} = \vec{0}$$

- Finding $\vec{F}_{DC}$

$$\vec{F}_{DC} = F_{DC}(-\hat{j})$$
$$= K\Delta L\ (-\hat{j})$$
$$= K(L - l_0)\ (-\hat{j})$$
$$= k(|\vec{r}_{DC}| - l_0)\ (-\hat{j})$$

$$|\vec{r}_{DC}| = 10\ cm\ (\sin 60°)\ \frac{1}{3} = h/3$$
$$= \frac{5}{3}\sqrt{3}\ \ cm$$

- Finding $\vec{F}_{DB}$

$$\vec{F}_{DB} = F_{DB}\hat{\lambda}_{DB}$$
$$= k\,\Delta L\,\hat{\lambda}_{DB}$$
$$= k(|\vec{r}_{DB}| - l_0)\frac{\vec{r}_{DB}}{|\vec{r}_{DB}|}$$

Cont. 1).

$$|\vec{r}_{DB}| = 10.4\ \ cm$$

$$\vec{r}_{DB} = \vec{r}_B - \vec{r}_D$$
$$= 0 - (10\hat{i} + \tfrac{5}{3}\sqrt{3}\,\hat{j})\ cm$$
$$= -10\hat{i} - \tfrac{5}{3}\sqrt{3}\,\hat{j}\ \ \ cm$$

- Finding $\vec{F}_{DA}$

Similarity:

$$\vec{F}_{DA} = k(|\vec{r}_{DA}| - l_0)\frac{\vec{r}_{DA}}{|\vec{r}_{DA}|}$$

$$\vec{r}_{DA} = \vec{r}_A - \vec{r}_D$$
$$= 5\,cm\,\hat{i} + 5\sqrt{3}cm\,\hat{j} - (10cm\,\hat{i} + \tfrac{5}{3}\sqrt{3}\,cm\,\hat{j})$$
$$= -5\,cm\,\hat{i} + \tfrac{10}{3}\sqrt{3}\ cm\,\hat{j}$$

$$|\vec{r}_{DA}| = 7.6\ \ cm$$

a): $l_0 = 0$

$$\Rightarrow \vec{F}_{DC} = k|\vec{r}_{DC}|(-\hat{j})$$
$$= k\,\frac{5\sqrt{3}}{3}\,cm(-\hat{j})$$

$$\vec{F}_{DB} = k\,\vec{r}_{DB}$$
$$= k(-10cm\,\hat{i} - \tfrac{5}{3}\sqrt{3}\ cm\,\hat{j})$$

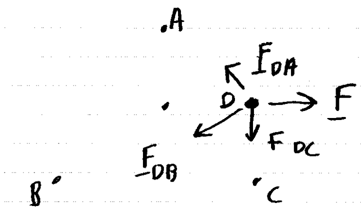
$$\vec{F}_{DA} = k\,\vec{r}_{DA}$$
$$= k(-5cm\,\hat{i} + \tfrac{10}{3}\sqrt{3}\ cm\,\hat{j})$$

$$\vec{F} = F_x\hat{i} + F_y\hat{j}$$

$$\Rightarrow \Sigma\vec{F} = F_x\hat{i} + F_y\hat{j} + k\left(\tfrac{5\sqrt{3}}{3}(-\hat{j}) - 10\hat{i} - \tfrac{5}{3}\sqrt{3}\,\hat{j} - 5\hat{i} + \tfrac{10}{3}\sqrt{3}\,\hat{j}\right)cm$$
$$= [F_x - k\ 15(cm)]\,\hat{i} + F_y\hat{j}$$
$$= \vec{0}$$

$$\Rightarrow \boxed{\begin{array}{l} F_x = k\,15(cm) = 150\ N \\ F_y = 0 \end{array}} \Rightarrow \boxed{\underline{F} = 150N\,\hat{i}}$$

Alt. method for 2a (more concepts, less math)



For "zero length" springs ($l_0 = 0$)

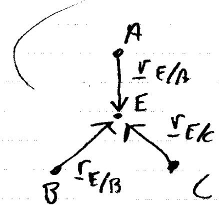
$$\underline{F}_{DB} = -k\,\underline{r}_{D/B}\ ,\ \underline{F}_{DC} = -k\,\underline{r}_{D/C},\ \underline{F}_{DA} = -k\,\underline{r}_{D/A}$$

But
$$\underline{r}_{D/B} = \underline{r}_{D/E} + \underline{r}_{E/B}$$
$$\underline{r}_{D/C} = \underline{r}_{D/E} + \underline{r}_{E/C}$$
$$\underline{r}_{D/A} = \underline{r}_{D/E} + \underline{r}_{E/A}$$

So

$$\underline{F} = -\underline{F}_{DA} + -\underline{F}_{DB} + -\underline{F}_{DA}$$
$$= k\left[3\underline{r}_{D/E} + \underbrace{\underline{r}_{E/B} + \underline{r}_{E/C} + \underline{r}_{E/A}}_{= \underline{0}}\right]$$
$$= 3k\,\underline{r}_{D/E}$$
$$= 3\ (10N/cm)\ 5cm\,\underline{i}$$

$$\boxed{\underline{F} = 150N\,\underline{i}}$$



Cont. 1).

b) $\sum \vec{F}$

$$= F_x\hat{i} + F_y\hat{j} + k\left[\left(\tfrac{5}{3}\sqrt{3}-2\right)cm(-\hat{j}) + (10.4-2)\,cm\,\frac{(-10\hat{i}-\frac{5}{3}\sqrt{3}\hat{j})\,cm}{10.4\ cm} + (7.6-2)cm\,\frac{(-5cm\,\hat{i}+\frac{10}{3}\sqrt{3}cm\,\hat{j})}{7.6\ cm}\right]$$

$$= F_x\hat{i} + F_y\hat{j} + k\left[-0.89\hat{j} + (-8.08\hat{i} - 2.33\hat{j}) + (-3.68\hat{i} + 4.25\hat{j})\right]cm$$

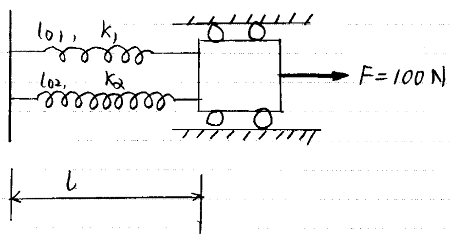
$$= F_x\hat{i} + F_y\hat{j} + \left[-117.6\,\hat{i} + 10.3\hat{j}\right] N$$

$$= (F_x - 117.6\,N)\,\hat{i} + (F_y + 1.03N)\,\hat{j}$$

$$= 0$$

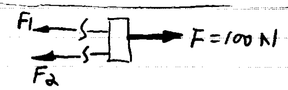
$$\Rightarrow \boxed{\begin{array}{l} F_x = 117.6\ N \\ F_y = -10.3\ N \end{array}} \Rightarrow \boxed{\underline{F} = 117.6N\,\underline{i} - 10.3N\,\underline{j}}$$

2). $l_{01} = 3cm$, $k_1 = 10\ N/cm$; $l_{02} = 6cm$, $k_2 = 5\ N/cm$. } Find l.



~~Soln~~

F.B.D :



$$F = F_1 + F_2$$
$$= k_1(l - l_{01}) + k_2(l - l_{02})$$
$$= (k_1 + k_2)\,l - k_1 l_{01} - k_2 l_{02}$$

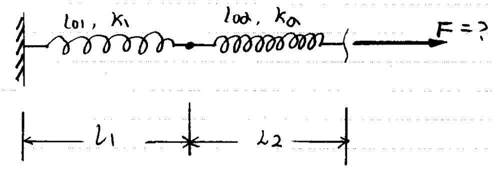
ys.

$$\Rightarrow l = \frac{F + k_1 l_{01} + k_2 l_{02}}{k_1 + k_2}$$

$$= \frac{100N + (10N/cm)(3cm) + (5N/cm)(6cm)}{10\ N/cm + 5N/cm}$$

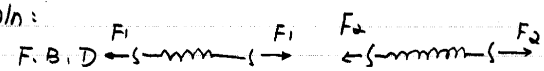
$$= 10.67\ cm$$

3). $l_{01} = 3cm$, $k_1 = 10\ N/cm$; $l_{02} = 6cm$, $k_2 = 5\ N/cm$ } Find F



Soln :

F.B.D



$$l = l_1 + l_2 \qquad (1)$$
$$F_1 = k_1(l_1 - l_{01}) \Rightarrow l_1 = l_{01} + \frac{F_1}{k_1} \qquad (2)$$
$$F_2 = k_2(l_2 - l_{02}) \Rightarrow l_2 = l_{02} + \frac{F_2}{k_2} \qquad (3)$$

put (2) and (3) back into (1)

$$\Rightarrow l_{01} + \frac{F_2}{k_2} + l_{02} + \frac{F_1}{k_1} = l$$

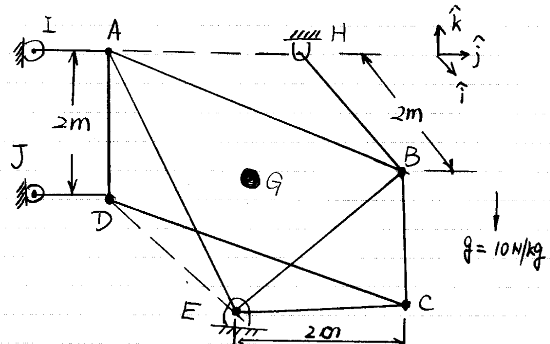
$$\Rightarrow l = l_{01} + l_{02} + F\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

$$\Rightarrow F = \frac{l - (l_{01} + l_{02})}{\frac{1}{k_1} + \frac{1}{k_2}} = \frac{10cm - (6cm + 3cm)}{\frac{1}{10\,N/cm} + \frac{1}{5\,N/cm}}$$

$$= 3.3\ N$$

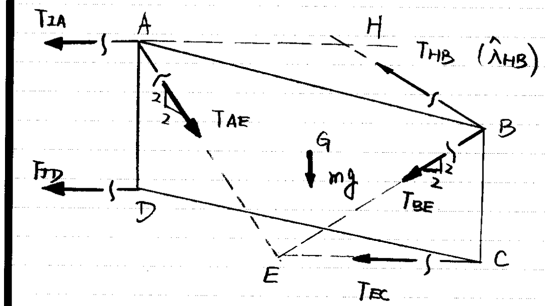
4). Sign held by 6 bars : IA, JD, HB, EC, EB, EA. $g = 10N/kg$. And ABCD is uniformly plate C.O.M at G. $m = 10kg$

Find T_{EC} & T_{HB}.



Cont. 4

Draw the F.B.D of the plate :

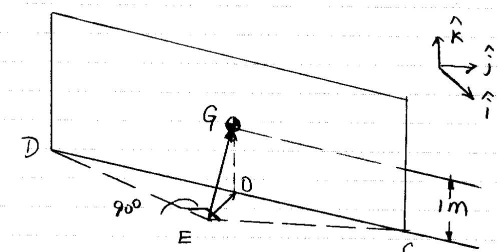


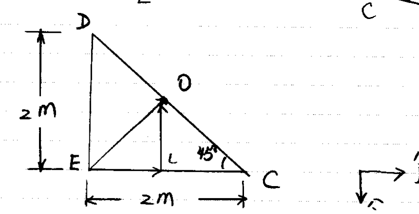
•Finding T_{HB} : take the moment about axis EC: only T_{HB} and $m\vec{g}$ will produce non-zero moment about it

$$\sum M_{\hat{\lambda}_{EC}} = \hat{\lambda}_{EC}\cdot\left(\sum_{i=1}^{2}\vec{r}_i \times \vec{F}_i\right)$$

$$= \hat{j}\cdot\left(\vec{r}_{EG}\times m\vec{g} + \vec{r}_{CB}\times\vec{T}_{HB}\right)$$

$$\vec{r}_{EG} = (-\hat{i} + \hat{j} + \hat{k})\,(m)$$





$$\vec{r}_{EG} = \vec{r}_{OG} + \vec{r}_{EO} = 1m\,\hat{k} + (\vec{r}_{EL} + \vec{r}_{LO})$$
$$= 1m\,\hat{k} + (1m\,\hat{j} - 1m\,\hat{i})$$
$$= (-\hat{i} + \hat{j} + \hat{k})\,(m)$$

Cont. 4

$\Rightarrow$

$\Sigma M_{\hat{\lambda}_{EC}} = \hat{j}\cdot\{[(-\hat{i}+\hat{j}+\hat{k})(m)\times(10\,kg\times 10\,\frac{N}{kg}(-\hat{k})] +$

$2m\,\hat{k}\times T_{HB}(-\hat{i})\}$

$= \hat{j}\cdot\{100(-\hat{i}-\hat{j})\,mN - 2m\,T_{HB}\,\hat{j}\}$

$= -100m\,N - 2m\,T_{HB}$

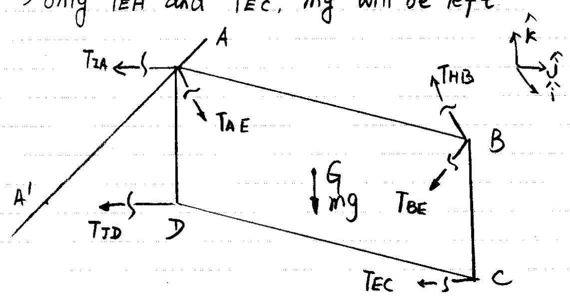
$= 0$

$\Rightarrow \boxed{T_{HB} = \frac{-10\,mN}{2m} = -50N}$

- Finding T_{EC}
  Method 1:

Take the moment about axis through A in $\hat{j}+\hat{k}$ direction

$\Rightarrow$ only T_{EH} and T_{EC}, mg will be left



AA' is in $\hat{j}+\hat{k}$ direction.
T_{JD}, T_{IA}, T_{AE} pass through AA'
T_{BE} // AA'

$\Rightarrow \Sigma M_{\hat{\lambda}_{AA'}} = \hat{\lambda}_{AA'}\cdot(\vec{r}_{AC}\times\vec{T}_{EC} + \vec{r}_{AB}\times\vec{T}_{HB} + \vec{r}_{AG}\times m\vec{g})$

$\hat{\lambda}_{AA'} = \hat{j}+\hat{k}$

$\vec{r}_{AC} = -2m\,\hat{k} + 2m\hat{j} + 2m\hat{i} = 2\vec{r}_{AG}$

$\vec{r}_{AB} = 2m\,\hat{i} + 2m\hat{j}$

to

$\Rightarrow \Sigma M_{\hat{\lambda}_{AA'}}$

$= (\hat{j}+\hat{k})\cdot[(-2m\hat{k}+2m\hat{i}+2m\hat{j})\times T_{EC}(-\hat{j}) + (2m\hat{i}+2m\hat{j})\times T_{HB}(-\hat{i})$

$+ (-1m\,\hat{k}+1m\hat{j}+1m\hat{i})\times mg\,(-\hat{k})]$

$= (\hat{j}+\hat{k})\cdot[-2m\,T_{EC}\,\hat{i} - 2m\,T_{EC}\,\hat{k} + 2m\,T_{HB}\,\hat{k} - 1m(100N)$
$(-\hat{i}-\hat{j})]$

$= (\hat{j}+\hat{k})\cdot[(-2m\,T_{EC} + 100\,mN)\,\hat{i} + (2m\,T_{HB} - 2m\,T_{EC} + 100\,mN)\hat{j}]$

$= 2m\,T_{HB} - 2m\,T_{EC} + 100\,mN$

$= -100mN - 2m\,T_{EC} + 100\,mN$

$= \vec{0}$

$\Rightarrow \boxed{T_{EC} = 0}$

Method (2):

1). Take the moment about axis AB, only T_{EC} and T_{DJ} will produce non-zero moment about it.

$\Sigma M_{\hat{\lambda}_{AB}} = \hat{\lambda}_{AB}\cdot(\vec{r}_{AD}\times\vec{T}_{DJ} + \vec{r}_{BC}\times\vec{T}_{CE})$

$\hat{\lambda}_{AB} = (\hat{i}+\hat{j})$

$\vec{r}_{AD} = -2m\,\hat{k}$, $\vec{r}_{BC} = -2m\,\hat{k}$

$\vec{T}_{DJ} = T_{DJ}(-\hat{j})$, $\vec{T}_{CE} = -T_{CE}\,\hat{j}$

$\Rightarrow \Sigma M_{\hat{\lambda}_{AB}} = (\hat{i}+\hat{j})\cdot[-2m\hat{k}\times T_{DJ}(-\hat{j}) + (-2m\hat{k})\times T_{CE}(-\hat{j})]$

$= (\hat{i}+\hat{j})\cdot[-2m\,T_{DJ}\,\hat{i} - 2m\,T_{CE}\,\hat{i}]$

$= -2m\,T_{DJ} - 2m\,T_{CE}$

$= 0$

$\Rightarrow T_{CE} = -T_{DJ}$

2). Take the moment about axis EH
$\Rightarrow$ only T_{DJ} will produce non-zero moment about it

$\Sigma M_{\hat{\lambda}_{EH}} = \hat{\lambda}_{EH}\cdot(\vec{r}_{ED}\times\vec{T}_{DJ}) = 0$

$\Rightarrow T_{DJ} = 0$

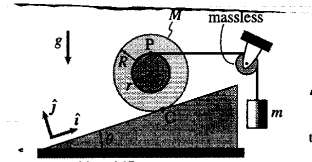
Above all: $\boxed{T_{EC} = 0}$

4.17.

Given: $r = \frac{1}{2}R$, string is in horizontal direction. no slip between the reel and the slope. there is gravity.

Find:

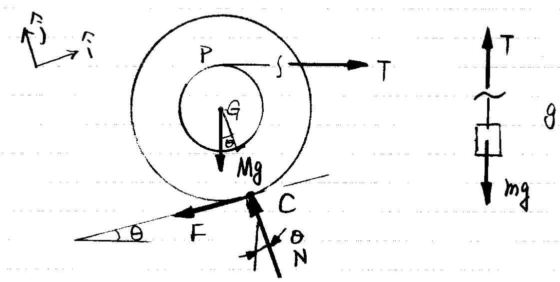
a). the ratio of the masses so that the system is at rest.



problem 4.17:
(Filename:pfigure2.blue.47.3.a)

Soln:

Draw the F.B.D of the reel and the mass

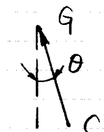


For mass m:

$\Sigma\vec{F} = 0 \Rightarrow T = mg$

For reel M:

$\Sigma\vec{M}_C = \vec{0}$



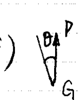
$\Rightarrow \vec{r}_{CG}\times M\vec{g} + \vec{r}_{CP}\times\vec{T} = \vec{0}$

$\Rightarrow R(\hat{j})\times Mg(-\cos\theta\,\hat{j} - \sin\theta\,\hat{i})$

$+ (\vec{r}_{CG} + \vec{r}_{GP})\times T\,(\cos\theta\,\hat{i} - \sin\theta\,\hat{j}) = \vec{0}$

$\vec{r}_{CG} + \vec{r}_{GP} = R(\hat{j}) + r(\cos\theta\,\hat{j} + \sin\theta\,\hat{i})$

$= r\sin\theta\,\hat{i} + (R + r\cos\theta)\,\hat{j}$



$\Rightarrow \Sigma\vec{M}_0 = RMg\sin\theta\,\hat{k} + [r\sin\theta\,\hat{i} + (R + r\cos\theta)\hat{j}]\times T(\cos\theta\,\hat{i} - \sin\theta\,\hat{j})$

$$\Rightarrow \sum \vec{M}_O = RMg\sin\theta\,\hat{k} - T(r+R\cos\theta)\,\hat{k} = \vec{0}$$

$$\Rightarrow RMg\sin\theta = T(r+R\cos\theta) = mg(r+R\cos\theta)$$

$$\Rightarrow \boxed{\frac{m}{M} = \frac{R\sin\theta}{r+R\cos\theta} = \frac{2\sin\theta}{1+2\cos\theta}}$$

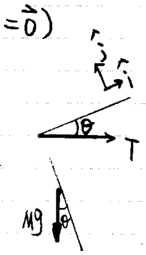
b). Find the correspoing tension in the string

$$\boxed{T = mg = \frac{2Mg\sin\theta}{1+2\cos\theta}} \quad (\text{from a})$$

c). Find the force on the real at point C, (M, g, R, θ)

$$\vec{R} = \vec{F} + \vec{N} = -(\vec{T} + M\vec{g}) \quad (\sum\vec{F} = \vec{0})$$

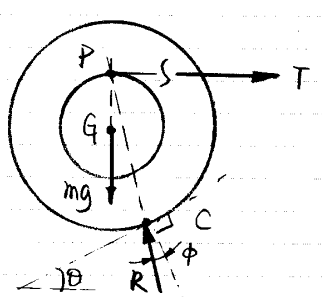
from b)



$$\Rightarrow \vec{R} = -\Big(\frac{2Mg\sin\theta}{1+2\cos\theta}(\cos\theta\,\hat{i} - \sin\theta\,\hat{j}) + Mg(-\cos\theta\,\hat{j} - \sin\theta\,\hat{i})\Big)$$

$$\boxed{= \frac{Mg}{1+2\cos\theta}\Big(+\sin\theta\,\hat{i} + (2+\cos\theta)\,\hat{j}\Big)}$$

d). Find the point where the gravity meet the string force then find $\tan\phi$. Does it agree with c)?



Soln: $\vec{R}$ is in the direction of $\hat{\lambda}_{CP}$

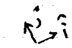
From a): $\vec{r}_{CP} = r\sin\theta\,\hat{i} + (R + r\cos\theta)\,\hat{j}$

$$\Rightarrow \hat{\lambda}_{CP} = \frac{r\sin\theta\,\hat{i} + (R+r\cos\theta)\,\hat{j}}{[r\sin^2\theta + R + r\cos^2\theta]^{1/2}}$$

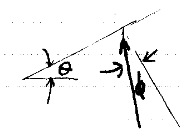
Cont. 4.17

$$\tan\phi = \frac{\lambda_{cx}}{\lambda_{cy}} = \frac{r\sin\theta}{R + r\cos\theta} = \frac{\sin\theta}{2+\cos\theta}$$

From C:



$$\tan\phi = \frac{\sin\theta}{2+\cos\theta}$$

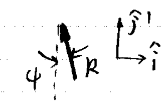


So they agree with each other.

e). what's the relations between the angle ψ at C respect normal to the groud and the mass ratio?

if we change the $\hat{i}$-$\hat{j}$ to $\hat{i}'$-$\hat{j}'$

From c. similarily:



$$\vec{R} = -(\vec{T} + M\vec{g})$$
$$= -\Big(\frac{2Mg\sin\theta}{1+2\cos\theta}\,\hat{i}' + Mg(-\hat{j}')\Big)$$

$$\Rightarrow \tan\psi = \left|\frac{R_x}{R_y}\right| = \frac{2\sin\theta}{1+2\cos\theta}$$

Compared with:

$$\frac{m}{M} = \frac{2\sin\theta}{1+2\cos\theta}$$

They're the same! $\boxed{\tan\psi = \frac{m}{M}}$

f). check $\frac{m}{M}$, $\vec{F}_C$ for $\theta = 0$ and $\theta = \pi/2$

- $\theta = 0 \Rightarrow \frac{m}{M} = 0$, $\vec{R} = \frac{Mg}{1+2}(0 + (2+1)\hat{j})$ $= Mg\,\hat{j}$ (holds up wheel, $T=0$)

- $\theta = \frac{\pi}{2} \Rightarrow \frac{m}{M} = \frac{2}{1+0} = 2$

$$\vec{R} = \frac{Mg}{1+0}(\hat{i} + (2+0)\hat{j})$$
$$= Mg(\hat{i} + 2\hat{j})$$

(again $Mg\,\hat{j}$ holds up wheel, $T = 2Mg$)