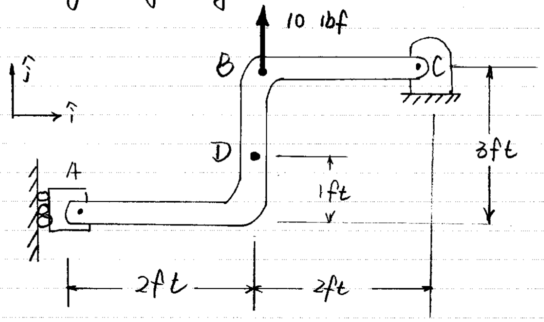
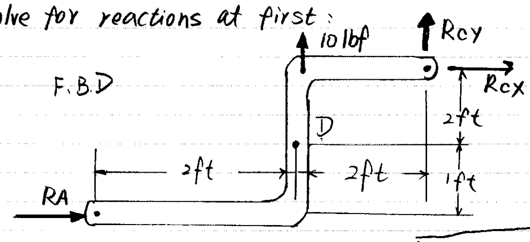
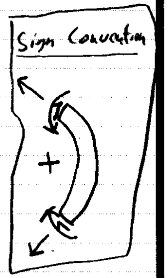
1). Find then tension, shear and bending moment at D. Neglect gravity.



a). Using section AD
b). Using section DBC
c). Do the answers above agree?

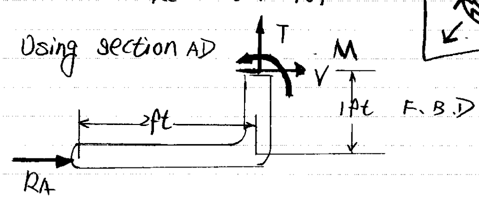
Soln:
Solve for reactions at first:





$\Sigma F_x = 0 \Rightarrow R_A + R_{cx} = 0$
$\Rightarrow R_A = {}^{-}R_{cx}$
$\Sigma F_y = 0 \Rightarrow {}^{-}R_{cy} = 10$ lbf
$+\circlearrowleft \ \Sigma M_c = 0 \Rightarrow R_A(3ft) - 10(2ft) = 0$
$\Rightarrow R_A = 6.7$ lbf
$R_{cx} = {}^{-}6.7$ lbf

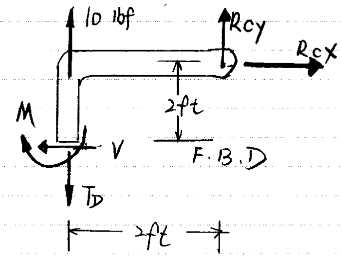
a). Using section AD



$\Sigma F_x = 0 \Rightarrow V = {}^{-}R_A = {}^{-}6.7$ lbf
$\Sigma F_y = 0 \Rightarrow T = 0$
$+\circlearrowleft \ \Sigma M_D = 0 \Rightarrow R_A(1ft) + M = 0$
$\Rightarrow M = -R_A \, 1ft$
$= -6.7$ ft·bf

$\boxed{V = -6.7 \text{ lbf} \quad T = 0 \quad M = -6.7 \text{ ft bf}}$ (*)

b). Using DBC



$\Sigma F_x = 0 \Rightarrow +R_{cx} - V = 0$
$\Rightarrow V = +R_{cx} = {}^{-}6.7$ lbf

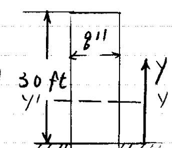
$\Sigma F_y = 0 \Rightarrow 10 - T_D + R_{cy} = 0$
$\Rightarrow T_D = 10lbf - 10lbf$
$= 0$

$+\circlearrowleft \ \Sigma M_D = 0 \Rightarrow R_{cy}(2ft) - R_{cx}(2ft) - M = 0$
$\Rightarrow M = -R_{cx}(2ft) + R_{cy}(2ft)$
$= 6.7(2ft) - 10(2ft)$
$= -6.6$ ft.lbf

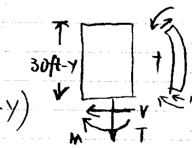
$\boxed{V = {}^{-}6.7 \text{ lbf} \quad M = {}^{-}6.6 \text{ ft.lbf} \quad T = 0}$ } agrees w/ prev. answer at (*)

c). The answers above AGREE!

2. The telephone pole is shown.
$\rho = 60$ lbm per cubic foot
Find the tension, shear and bending moment as a function of y.



Soln:
let's cut at y, take the upper cut, draw the F.B.D of it:



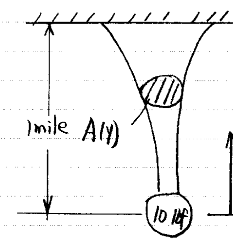
$W = \rho g V = \rho g A(30ft - y)$
$= \rho g \pi \left(\frac{9}{2} in\right)^2 (30ft - y)$
$= \rho g \pi \left(\frac{9}{2} \cdot \frac{1}{12} ft\right)^2 (30ft - y)$
$= \pi \left(60 \frac{lbm}{ft^3}\right)\left(\frac{1bf}{1bm}\right)\left(\frac{81}{24^2}\right) ft^2 (30ft - y)$
$= 8.44\pi(30ft - y) \frac{lbf}{ft}$

Cont. 2.
$\Sigma F_y = 0 \Rightarrow T = -W = -8.44\pi(30ft - y) \frac{lbf}{ft}$
$\Sigma F_x = 0 \Rightarrow V(y) = 0$

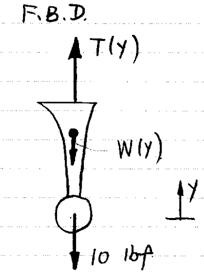
$+\circlearrowleft \ \Sigma M_y = 0 \Rightarrow M(y) = 0$

$\boxed{M(y) = 0 \quad V(y) = 0 \quad T(y) = -8.44\pi(30ft - y) \frac{lbf}{ft}}$

3. W = 10 lbf, L = 1 mile. What's the lightest wire that will do the job?
($\rho = 500$ lbm/ft³, $\sigma^*_{max} = 100,000$ lbf/in²)



F.B.D.



Soln:
To do the job, $\sigma(y) \le \sigma^*$ $\quad 0 \le y \le 1$ mile

$\sigma(y) = \frac{T(y)}{A(y)}$ (According to the definition of σ)

We need to find $\frac{T(y)}{A(y)} = \sigma^* = 100,000$ lbf/in²

- According to the F.B.D. of the lower cut,

W(y) is the total weight of the wire goes from 0 to y.

$W(y) = \int_0^y \rho g A(s)\, ds$
$= \rho g \int_0^y A(s)\, ds$

Why? If $A(y) > \frac{T(y)}{\sigma^*}$ its too much material. If $A(y) < \frac{T(y)}{\sigma^*}$ $\Rightarrow \sigma > \sigma^*$, it breaks

$\Sigma F_y = 0 \Rightarrow$
$T(y) = 10lbf + W(y)$
$\Rightarrow$
$\sigma(y) = \frac{10lbf + W(y)}{A(y)} = \frac{10lbf + \rho g \int_0^y A(s)\, ds}{A(y)}$

- $\sigma(y) = \sigma^* = 100,000$ lbf/in² for the lightest

$\Rightarrow 10lbf + \rho g \int_0^y A(s)\, ds = T = \sigma^* A(y) = 10^5 \frac{lbf}{in^2} A(y)$

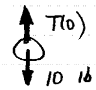
Cont. 3.

$\Rightarrow \frac{d}{dy}[10\text{ lbf} + \rho g\int_0^y A(s)\,ds] = \frac{d}{dy}(\sigma^* A(y))$

$\Rightarrow \rho g A(y) = \sigma^* \frac{dA(y)}{dy}$

$\Rightarrow A(y) = Ce^{\frac{\rho g}{\sigma^*}y}$

• Finding c:



$T(0) = 10\text{ lbf}$

$\frac{T(0)}{A(0)} = \sigma^*$

$\Rightarrow \frac{T(0)}{C} = \sigma^*$

$\Rightarrow C = \frac{T(0)}{\sigma^*} = \frac{10\text{ lbf}}{\sigma^*}$

plugging it back into $A(y)$:

$A(y) = \frac{T(0)}{\sigma^*} e^{(\frac{\rho g}{\sigma^*}y)}$

• Finding the total weight:

$W = \int_0^{1\,mile} \rho g A(y)\,dy = \rho g \int_0^{5280ft} \frac{T(0)}{\sigma^*} e^{(\frac{\rho g}{\sigma^*}y)}dy$

$= (\rho g \frac{T(0)}{\sigma^*}\cdot\frac{\sigma^*}{\rho g}) e^{(\frac{\rho g}{\sigma^*}y)}\Big|_0^{5280ft}$

$= T(0)e^{(\frac{\rho g}{\sigma^*}y)}\Big|_0^{5280ft}$

• Find $\frac{\rho g}{\sigma^*}y\Big|_{5280}$

$\frac{\rho g}{\sigma^*}y\Big|_{5280} = \dfrac{500\frac{lbm}{ft^3}\cdot\frac{1bf}{1bm}\cdot 5280ft}{10^5\frac{lbf}{(\frac{1}{12}ft)^2}}$

$= \frac{500\times5280}{10^5\times12^2}\quad\frac{lbm\cdot lbf\cdot ft\cdot ft^2}{ft^3\cdot lbm\cdot lbf}$

$= 0.18$

There's no unit for it.

• $A(0) = T(0)/\sigma^* = \dfrac{10\text{ lbf}}{10^5\frac{lbf}{(\frac{1}{12}ft)^2}} = 0.0144\ ft^2$

$\Rightarrow d(0) = 0.12\ ft$ Similarly: $d(1\,mile) = 0.13\ ft$

For square cross Area

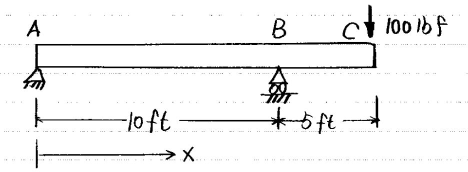
• Back to the total weight:

$W = T(0)e^{(\frac{\rho g}{\sigma^*}y)}\Big|_0^{5280ft}$

$= 10\text{ lbf}(e^{0.18}-1) = \boxed{1.97\text{ lbf}}$

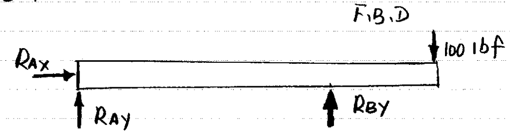
4. Draw V and M diagrams for these beams

a)



Soln.

• Solve for reactions:



$\Sigma F(x) = 0 \Rightarrow R_{Ax} = 0$

$\Sigma F(y) = 0 \Rightarrow R_{Ay} + R_{By} = 100\text{ lbf}$

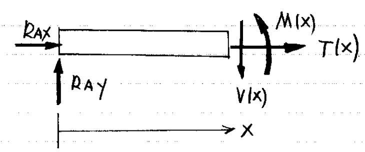
$\circlearrowleft + \Sigma M_A = 0 \Rightarrow$

$R_{By}(10ft) - 100\text{ lbf}(15ft) = 0$

$\Rightarrow R_{By} = 150\text{ lbf}$

$R_{Ax} = 0,\quad R_{Ay} = -50\text{ lbf},\quad R_{By} = 150\text{ lbf}.$

• Study section AB, cut at any point between A, B and take the left hand side.



$\Sigma F_y = 0 \Rightarrow V(x) = R_{Ay} = -50\text{ lbf}$

$\Sigma F_x = 0 \Rightarrow T(x) = -R_{Ax} = 0$

$\circlearrowleft + \Sigma M_{/x} = 0 \Rightarrow R_{Ay}(x) - M(x) = 0$

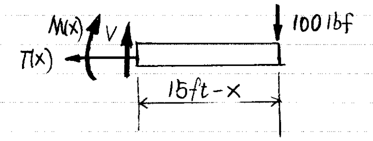
$\Rightarrow M(x) = R_{Ay}(x)$

$= -50x\text{ lbf}$

$V(x) = -50\text{ lbf}$
$T(x) = 0$
$M(x) = -50x\text{ lbf}$

Cont. 4a.

• Study section BC: cut at any point between B & C, take the right part and draw the F.B.D.



$\Sigma F(x) = 0 \Rightarrow T(x) = 0$

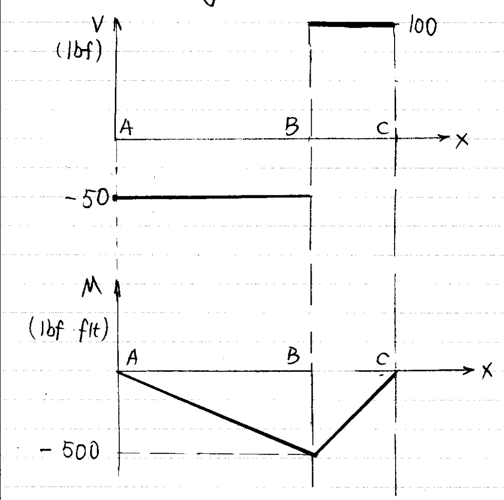
$\Sigma F(y) = 0 \Rightarrow V(x) = 100\text{ lbf}$

$\circlearrowleft + \Sigma M_{/x} = 0 \Rightarrow M(x) + 100(15-x)\,lbf\cdot ft = 0$

$\Rightarrow M(x) = -100(15-x)\ lbf\,ft\ (10 \le x \le 15ft)$

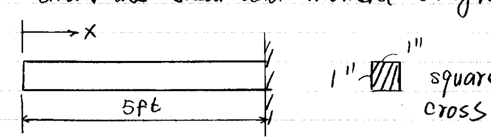
$V(x) = 100\text{ lbf}$
$M(x) = -100(15ft - x)\text{ lbf}\quad 10 \le x \le 15ft$

• Draw the diagram



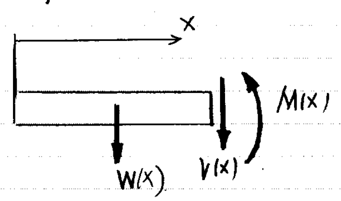
4b: $\rho = 500\ lbm/ft^3$ including gravity

draw the shear and moment diagram.



Cont. 4b.

- Take the whole beam as one part. Cut at any point between $0 < x < 5\,ft$
  take the left side and draw the F.B.D



$\Sigma F_y = 0 \Rightarrow \quad V(x) = -W(x)$

$\Sigma M_{/x} = 0 \Rightarrow \quad M(x) + W(x)\frac{x}{2} = 0$

$\Rightarrow \quad M(x) = -\frac{x}{2}W(x)$

- Finding W(x)

$$W(x) = \rho g V(x) = \rho g A L(x) = \rho g A X$$

$\rho = 500\ lbm/ft^3$. $g = \frac{1\ lbf}{1\ lbm}$

$A = (\frac{1}{12}ft)^2 = \frac{1}{144}ft^2$

$\Rightarrow$

$$W(x) = 500\left(\frac{lbm}{ft^3}\right)\frac{1\ lbf}{1\ lbm}\cdot\frac{1}{144}ft^2\cdot X$$

$$= 3.5X\ \frac{lbf}{ft}$$

- Back to V(x) and M(x)

$V(x) = -W(x)$

$\boxed{= -3.5X\ \frac{lbf}{ft}}$

$M(x) = -\frac{X}{2}W(x)$

$= -\frac{x}{2}\left(3.5x\ \frac{lbf}{ft}\right)$

$\boxed{M(x) = -1.75x^2\ \frac{lbf}{ft}}$

- Draw the graph.

Cont 4b).