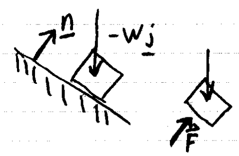
2.41, 2.65, 2.81, 2.83



2.41.

Given: $\hat{n} = 0.74\hat{i} + 0.67\hat{j}$

$\vec{W} = -W\hat{j}$

$\vec{F} = 1000\,N\,\hat{n}$

Find: the angle between vector $\vec{F}$ and $\vec{W}$

Solution:

- the angle between two vectors is also the angle between the unit vector $\hat{\lambda}_F$ and $\hat{\lambda}_W$
- from the definition of dot product between two vectors

$\hat{\lambda}_F \cdot \hat{\lambda}_W = |\hat{\lambda}_F||\hat{\lambda}_W| \cos\theta$ $\quad (\hat{\lambda}_F = \hat{n})$

$$\Rightarrow \cos\theta = \frac{\hat{\lambda}_F \cdot \hat{\lambda}_W}{|\hat{\lambda}_F||\hat{\lambda}_W|}$$

$$= (0.74\hat{i} + 0.67\hat{j}) \cdot (-\hat{j}) / 1$$

$$= -0.67$$

$$\Rightarrow \theta = \cos^{-1}(-0.67)$$

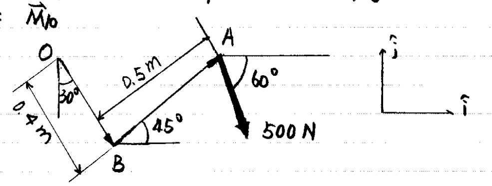
$$= 132.07^\circ$$

$$\boxed{\theta = 132.07^\circ}$$

2.65

Given: the force and the point as the figure below

Find: $\vec{M}_O$



Soln: the moment of $\vec{F}$ about point O is

$\vec{M}_O = \vec{r}_{OA} \times \vec{F}$ (just need to find this)

$\vec{r}_{OA}$ and $\vec{F}$ can be found from geometry

- $\vec{r}_{OA} = \vec{r}_{OB} + \vec{r}_{BA}$

$= r_{OB}\,\hat{\lambda}_{OB} + r_{BA}\,\hat{\lambda}_{BA}$

$= 0.4\ (\sin 30^\circ\,\hat{i} - \cos 30^\circ\,\hat{j})\,m +$

$\ \ 0.5\ (\cos 45^\circ\,\hat{i} + \sin 45^\circ\,\hat{j})\,m$

$= [(0.4 \cdot \frac{1}{2} + 0.5 \cdot \frac{\sqrt{2}}{2})\,\hat{i} + (-0.4 \cdot \frac{\sqrt{3}}{2} + 0.5 \cdot \frac{\sqrt{2}}{2})\,\hat{j}]\,m$

$= [0.55\hat{i} + 0.007\hat{j}]\,m$

$\vec{F} = F\ \hat{\lambda}_F$

$= 500\ (\cos 60^\circ\,\hat{i} - \sin 60^\circ\,\hat{j})\,N$

$= 500\ (\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j})\,N$

$= 250\ (\hat{i} - \sqrt{3}\hat{j})\,N$

next page

Cont 2.65

$\vec{M}_O = \vec{r}_{OA} \times \vec{F}$

$= (0.55\hat{i} + 0.007\hat{j}) \times 250(\hat{i} - \sqrt{3}\hat{j})\,mN$

$= 250\,(-0.55 \cdot \sqrt{3} \cdot (\hat{i} \times \hat{j}) + 0.007(\hat{j} \times \hat{i}))\,mN$

$= 250\,(-0.95\hat{k} - 0.007\hat{k})\,mN$

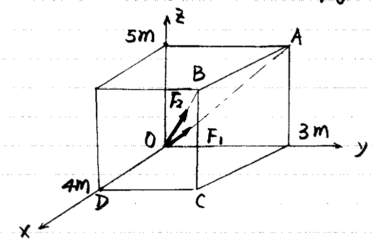
$= -239.25\,\hat{k}\,mN$

$$\boxed{\vec{M}_O = -239.25\,\hat{k}\,mN}$$

2.81.

Given: $F_1 = 5N$, $F_2 = 7N$

$\vec{r}_A$, $\vec{r}_C$, $\vec{r}_D$ as in the figure



Find:

a) a unit vector in the direction OB

$\hat{\lambda}_B = \dfrac{\vec{r}_B}{r_B}$

$= \dfrac{(4\hat{i} + 3\hat{j} + 5\hat{k})\ m}{\sqrt{3^2 + 4^2 + 5^2}\ m}$

$= \dfrac{1}{5\sqrt{2}}\,(4\hat{i} + 3\hat{j} + 5\hat{k})$

b). a unit vector in the direction OA

$\hat{\lambda}_A = \dfrac{\vec{r}_A}{r_A}$

$= \dfrac{(0\hat{i} + 3\hat{j} + 5\hat{k})\ m}{\sqrt{0^2 + 3^2 + 5^2}\ m}$

$= \dfrac{1}{\sqrt{34}}\,(3\hat{j} + 5\hat{k})$

c). write $\vec{F}_1$ and $\vec{F}_2$ in terms of $F\hat{\lambda}$

$\vec{F}_1 = F_1\hat{\lambda}_{F_1} = F_1\hat{\lambda}_A$

$= \dfrac{5N}{\sqrt{34}}\,(3\hat{j} + 5\hat{k})$

next page

Cont c), 2.81

$\vec{F}_2 = F_2 \cdot \hat{\lambda}_{F_2} = F_2 \cdot \hat{\lambda}_B$

$= \dfrac{7N}{5\sqrt{2}}\,(4\hat{i} + 3\hat{j} + 5\hat{k})$

d). what's the angle AOB

the angle AOB can be found from the dot product of $\hat{\lambda}_A$ and $\hat{\lambda}_B$

$\hat{\lambda}_A \cdot \hat{\lambda}_B = |\hat{\lambda}_A| \cdot |\hat{\lambda}_B| \cos(AOB)$

$= \cos(AOB)$

$\Rightarrow \cos(AOB) = \dfrac{1}{\sqrt{34}}(3\hat{j} + 5\hat{k}) \cdot \dfrac{1}{5\sqrt{2}}(4\hat{i} + 3\hat{j} + 5\hat{k})$

$= \dfrac{1}{10\sqrt{17}}\,(3\hat{j} \cdot 3\hat{j} + 5\hat{k} \cdot 5\hat{k})$

$= \dfrac{1}{10\sqrt{17}}\,(9 + 25)$

$= \dfrac{\sqrt{17}}{5}$

$\Rightarrow AOB = \cos^{-1}\left(\dfrac{\sqrt{17}}{5}\right) = \cos^{-1}(0.8246) = 34.4^\circ$

$\angle AOB = 34.4^\circ$

e). what's F_{1x}

$F_{1x} = \hat{i} \cdot \vec{F}_1$

$= \hat{i} \cdot \dfrac{5}{\sqrt{34}}\,(3\hat{j} + 5\hat{k})$

$= 0$

$F_{1x} = 0$

f). what's $\vec{r}_{DO} \times \vec{F}_1$

$\vec{r}_{DO} = -\vec{r}_{OD} = -4\hat{i}\,m$

$\Rightarrow \vec{r}_{DO} \times \vec{F}_1 = -4\hat{i} \times \left(\dfrac{5}{\sqrt{34}}(3\hat{j} + 5\hat{k})\right)\,mN$

$= \dfrac{-20}{\sqrt{34}}\,(3\,\hat{i} \times \hat{j} + 5\,\hat{i} \times \hat{k})\,mN$

$= \dfrac{-20}{\sqrt{34}}\,(3\hat{k} - 5\hat{j})\,mN$

g). what's M of $\vec{F}_2$ about axis DC ?

next page

Cont. 9). 2.81

the M of $\vec{F}_2$ about axis DC with $\hat{\lambda}_{DC}$ is

$$M_{\hat{\lambda}_{DC}} = \hat{\lambda}_{DC} \cdot (\vec{r} \times \vec{F}_2)$$

· Finding $\hat{\lambda}_{DC} = \frac{\vec{r}_{DC}}{r_{DC}} = \hat{j}$

· Finding $\vec{r}$:

Since $\vec{r}$ is the point of action to any point on the axis, we choose the easiest one:

$$\vec{r}_{OD} = 4\hat{i}\ m$$

· Finding $M_{\hat{\lambda}_{DC}}$

$$M_{\hat{\lambda}_{DC}} = \hat{j} \cdot (4\hat{i}) \times \left(\frac{7}{5\sqrt{2}}(4\hat{i}+3\hat{j}+5\hat{k})\right) mN$$
$$= \frac{7}{5\sqrt{2}}\hat{j} \cdot 4\hat{i} \times (4\hat{i}+3\hat{j}+5\hat{k})\ mN$$
$$= \frac{7}{5\sqrt{2}}\hat{j} \cdot (12\hat{k} - 20\hat{j})\ mN$$
$$= \frac{7}{5\sqrt{2}}(0 - 20)\ mN$$
$$= -14\sqrt{2}\ mN$$

$$M_{\hat{\lambda}_{DC}} = -14\sqrt{2}\ mN$$

b). using a different point on DC or the line of action

choose C instead of D

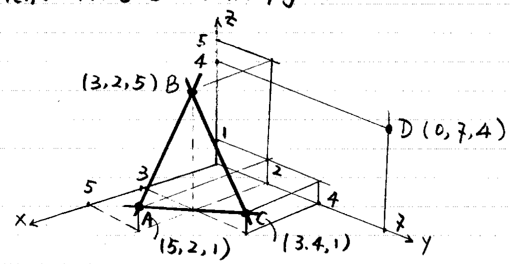
$$\vec{r} = \vec{r}_{OC} = (4\hat{i} + 3\hat{j})\ m$$

$$M_{\hat{\lambda}_{DC}} = \hat{j} \cdot (4\hat{i}+3\hat{j}) \times \frac{7}{5\sqrt{2}}(4\hat{i}+3\hat{j}+5\hat{k})\ mN$$
$$= \frac{7}{5\sqrt{2}}\hat{j} \cdot [(0+12\hat{k}-20\hat{j}) + (-12\hat{k}+0+15\hat{i})]\ mN$$
$$= \frac{7}{5\sqrt{2}}\hat{j} \cdot (15\hat{i} - 20\hat{j})\ mN$$
$$= \frac{7}{5\sqrt{2}}(0-20)\ mN$$
$$= -14\sqrt{2}\ mN$$

the answer is the same as a.

2.83.

Given: A, B, C in the figure



Find:

a). a unit normal vector to the plane ABC

From the definition of cross product about two vectors, we know that:

the cross product, which is also a vector, is perpendicular to the plane of the two vectors.

⇒ we can choose any two vectors inside plane ABC to find a unit normal vector from the cross product.

choose $\vec{r}_{AB}$ and $\vec{r}_{AC}$:

· Finding $\vec{r}_{AB}$ and $\vec{r}_{AC}$

$$\vec{r}_{AB} = \vec{r}_B - \vec{r}_A = (3\hat{i}+2\hat{j}+5\hat{k}) - (5\hat{i}+2\hat{j}+\hat{k}) = -2\hat{i}+4\hat{k}$$

$$\vec{r}_{AC} = \vec{r}_C - \vec{r}_A = (3\hat{i}+4\hat{j}+\hat{k}) - (5\hat{i}+2\hat{j}+\hat{k}) = -2\hat{i}+2\hat{j}$$

· Finding $\vec{r} = \vec{r}_{AC} \times \vec{r}_{AB}$

$$\vec{r} = (-2\hat{i}+2\hat{j}) \times (-2\hat{i}+4\hat{k})$$
$$= -2\hat{i}\times(-2\hat{i}+4\hat{k}) + 2\hat{j}\times(-2\hat{i}+4\hat{k})$$
$$= 0 + 8\hat{j} + 4\hat{k} + 8\hat{i}$$
$$= 8\hat{i} + 8\hat{j} + 4\hat{k}$$

· Finding $\hat{\lambda} = \frac{\vec{r}}{r}$

$$\hat{\lambda} = \frac{8\hat{i}+8\hat{j}+4\hat{k}}{\sqrt{8^2+8^2+4^2}} = \boxed{\frac{1}{3}(2\hat{i}+2\hat{j}+\hat{k})}$$

b). Find the distance from plane ABC to D

the distance from a point to a plane can be found by the dot product between the unit normal vector and vector $\vec{r}$, where $\vec{r}$ goes from any point in the plane to the point we concerned

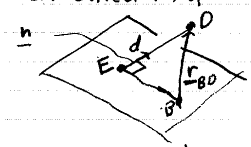
choose point $\vec{B}$ in the plane

· Finding $\vec{r}_{BD}$

$$\vec{r}_{BD} = \vec{r}_D - \vec{r}_B = (0\hat{i}+7\hat{j}+4\hat{k}) - (3\hat{i}+2\hat{j}+5\hat{k}) = -3\hat{i}+5\hat{j}-\hat{k}$$

· choose the unit normal vector solved in a

$$\hat{n} = \frac{1}{3}(2\hat{i}+2\hat{j}+\hat{k})$$



· Finding $d = |\vec{r}_{BD} \cdot \hat{n}|$

$$d = \left|(-3\hat{i}+5\hat{j}-\hat{k}) \cdot \left(\frac{1}{3}(2\hat{i}+2\hat{j}+\hat{k})\right)\right|$$
$$= \left|\frac{1}{3}(-6+10-1)\right|$$
$$= 1$$

$$\boxed{d = 1}$$

c) what're the coordinates of the point on the plane closest to point D.

The point closest to D will be the intercection of the plane and the normal line through D.

let this point be E:

$$\vec{r}_E = \vec{r}_D - d\hat{n}$$
$$= (0\hat{i}+7\hat{j}+4\hat{k}) - 1\left(\frac{1}{3}(2\hat{i}+2\hat{j}+\hat{k})\right)$$
$$= -\frac{2}{3}\hat{i} + \frac{19}{3}\hat{j} + \frac{11}{3}\hat{k}$$

$$\boxed{\vec{r}_E = -\frac{2}{3}\hat{i} + \frac{19}{3}\hat{j} + \frac{11}{3}\hat{k}}$$

* please add the unit after the value to get full credit.