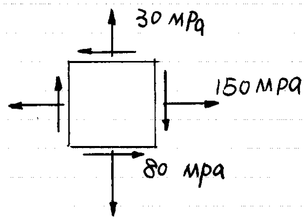
P6.6

Find: (a) the principle planes

(b) the principle stresses



Soln:

$\sigma_x = 150$ MPa

$\sigma_y = 30$ MPa

$\tau_{xy} = -80$ MPa

(a) The orientation of the principle plane is:

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$= \frac{2(-80 \text{ MPa})}{150 \text{ MPa} - 30 \text{ MPa}}$$

$$= -1.33$$

$\Rightarrow$ $2\theta_p = -53.13°$ or

$2\theta_p = 180° - 53.13°$

$= 126.87°$

$$\Rightarrow \boxed{\begin{aligned}\theta_p &= -26.6° \text{ and} \\ \theta_p &= 63.4°\end{aligned}}$$

(b) Principle stresses:

$$\sigma_{max,min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \left(\frac{150+30}{2} \pm \sqrt{\left(\frac{150-30}{2}\right)^2 + (-80)^2}\right) \text{ MPa}$$

$$= (90 \pm 100) \text{ MPa}$$

$$\Rightarrow \boxed{\begin{aligned}\sigma_{max} &= 190 \text{ MPa} \\ \sigma_{min} &= -10 \text{ MPa}\end{aligned}}$$

6.10 (For the state of stress of 6.6)

Find:

(a) the maximum τ plane

(b) τ_{max}

(c) the corresponding normal stress

Soln:

(a) Orientation of τ_{max} plane:

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

$$= -\frac{(150-30) \text{ MPa}}{2(-80) \text{ MPa}}$$

$$= 0.75$$

$\Rightarrow$ $2\theta_s = 36.87°$ and

$2\theta_s = -(180° - 36.87°)$

$= -143.13°$

$$\Rightarrow \boxed{\begin{aligned}\theta_s &= 18.4° \text{ and} \\ \theta_s &= -71.5°\end{aligned}}$$

(b)

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \left(\sqrt{\left(\frac{150-30}{2}\right)^2 + (-80)^2}\right) \text{ MPa}$$

$$\Rightarrow \boxed{\tau_{max} = 100 \text{ MPa}}$$

(c)

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = \frac{150+30}{2}$$

$$\Rightarrow \boxed{\sigma_{ave} = 90 \text{ MPa}}$$

6.30 Solve 6.6 and 6.10, using Mohr's circle

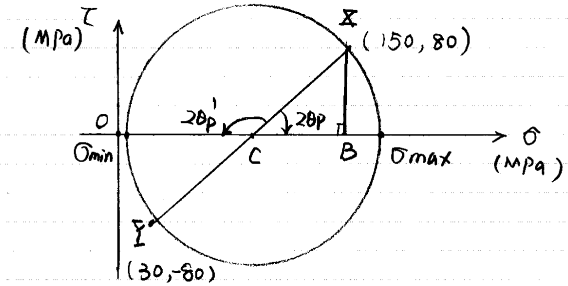
Soln:

$\sigma_x = 150$ MPa

$\sigma_y = 30$ MPa

$\tau_{xy} = -80$ MPa

Connecting $(\sigma_x, -\tau_{xy})$ and (σ_y, τ_{xy}) to get the diameter and center of the Mohr's circle:



(a) principle plane

$$\tan 2\theta_p = \frac{BX}{CB}$$

$BX = \tau_{xy} = 80$ mpa

$$CB = \frac{\sigma_x - \sigma_y}{2} = \frac{150-30}{2} = 60 \text{ MPa}$$

$$\Rightarrow \tan 2\theta_p = \frac{80 \text{ MPa}}{60 \text{ MPa}} = 1.33$$

$\Rightarrow$ $2\theta_p = 53.13°$ ↓

$\Rightarrow$ $\theta_p = 26.6°$ ↓

$= -26.6°$

We can also rotate XY couterclockwisely to the σ axis:

$$2\theta_p' + 2\theta_p = 180°$$

$\Rightarrow$ $\theta_p' = (180° - 53.13)/2$

$= 63.4°$ ↑

$= 63.4°$

Cont. 6.30

$$\boxed{\begin{aligned}\theta_p &= -26.6^\circ \text{ and} \\ \theta_p &= 63.4^\circ\end{aligned}}$$

(b). Obviously, σ_{max} and σ_{min} are the right and left side of the circle:

$$\sigma_{max} = OC + R$$

$$OC = \frac{\sigma_x + \sigma_y}{2} = \left(\frac{150+30}{2}\right) MPa = 90\ MPa$$

$$R = CX = \sqrt{CB^2 + BX^2}$$

$$= \sqrt{60^2 + 80^2}\ MPa$$

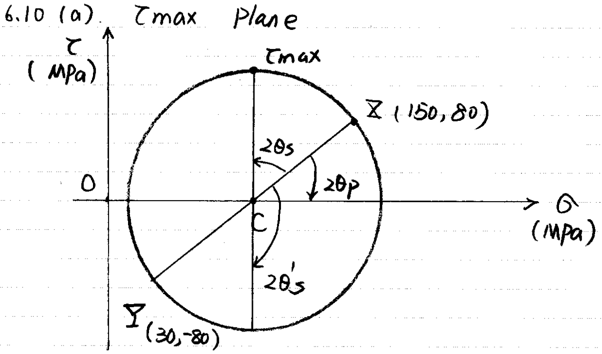
$$= 100\ MPa$$

$\Rightarrow$ $\sigma_{max} = (90 + 100)\ MPa = 190\ MPa$

$$\sigma_{min} = OC - R$$
$$= (90 - 100)\ MPa$$
$$= -10\ MPa$$

$$\boxed{\begin{aligned}\sigma_{max} &= 190\ MPa \\ \sigma_{min} &= -10\ MPa\end{aligned}}$$

6.10 (a). τ_{max} Plane



Cont. 6.30

$$2\theta_s = 90^\circ - 2\theta_p = 90^\circ - 53.13^\circ = 36.87^\circ \uparrow$$

$\Rightarrow$ $\theta_s = 18.43^\circ \uparrow$

$$2\theta_s' = 180^\circ - 2\theta_s = 180^\circ - 36.87^\circ \downarrow$$
$$= 71.56^\circ \downarrow$$

$\Rightarrow$ $\theta_s' = -71.56^\circ$

$$\boxed{\begin{aligned}\theta_s &= 18.43^\circ \text{ and} \\ \theta_s &= -71.56^\circ\end{aligned}}$$

(6.10) b:

$$\boxed{\tau_{max} = R = 100\ MPa}$$

(6.10) c:

$$\boxed{\sigma_{ave} = CB = 90\ MPa}$$

6.44. Solve 6.24 by Mohr's circle.

Given: $d_{DE} = 50\ mm$

Find: τ_{max}, $\sigma_{max,min}$ at
(a) H
(b) K.

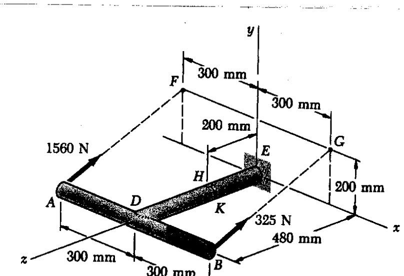
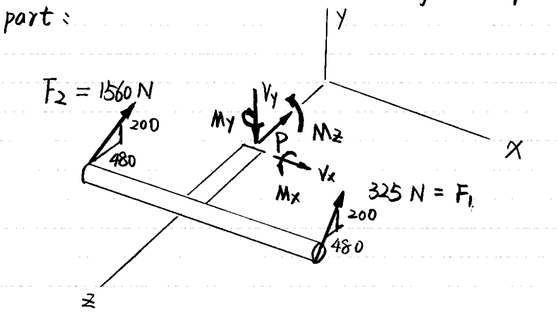


Fig. P6.24

Cont. 6.44

- Solve internal forces at H-K section at first cut at H-K and draw the FBD of the left part:



$$\hat{\lambda}_{F_1} = \hat{\lambda}_{BG} = \frac{\overrightarrow{BG}}{|\overrightarrow{BG}|} = \frac{(200\hat{j} - 480\hat{k})\ mm}{\sqrt{200^2 + 480^2}\ mm}$$

$$= \frac{200\hat{j} - 480\hat{k}}{520}$$

$$= 0.385\hat{i} - 0.92\hat{k}$$

$\Rightarrow$ $\vec{F_1} = (325\ N)(0.385\hat{i} - 0.92\hat{k})$

$$= (125\hat{j} - 300\hat{k})\ N$$

$$\hat{\lambda}_{F_2} = \hat{\lambda}_{F_1}$$

$\Rightarrow$ $\vec{F_2} = (1560\ N)(0.385\hat{i} - 0.92\hat{k})$

$$= (600\hat{j} - 1435\hat{k})\ N$$

$(\hat{i}) \cdot \Sigma\vec{F} = 0$

$\Rightarrow$ $\Sigma F_x = V_x = 0$

$(\hat{j} \cdot \Sigma\vec{F}) = 0$

$\Rightarrow$ $\Sigma F_y = F_{2y} + F_{1y} - V_y$
$= (125 + 600)\ N - V_y$

$\Rightarrow$ $V_y = 725\ N$

$(\hat{k} \cdot \Sigma\vec{F}) = 0$

$\Rightarrow$ $\Sigma F_z = F_{2z} + F_{1z} + P = -300\ N - 1435\ N - P$

$\Rightarrow$ $P = -1735\ N$

Cont. 6.44

$\circlearrowleft + \Sigma M_x = 0$ (At H-K section)

$\Rightarrow$

$$M_x - (F_{2y})(480\ mm - 200\ mm) - (F_{1y})(480\ mm - 200\ mm) = 0$$

$\Rightarrow$

$$M_x = (125 + 600)\ N\ (280\ mm) = 203\ \ N\cdot m$$

$\circlearrowleft \Sigma M_y = 0$ (At H-K section)

$\Rightarrow$

$$M_y - (F_{2z})\ 300\ mm + (F_{1z})(300\ mm) = 0$$

$\Rightarrow$

$$M_y = -(-300\ N)\ 300\ mm + (-1435\ N)(300\ mm) = -340\ \ N\cdot m$$

$\circlearrowleft \Sigma M_z = 0$

$\Rightarrow$

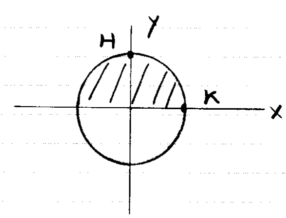
$$M_z - (F_{2y})(300\ mm) + (F_{1y})(300\ mm) = 0$$

$\Rightarrow$

$$M_z = (600\ N)(300\ mm) - (125\ N)(300\ mm) = 142\ \ N\cdot m$$

$P = -1735\ N$
$V_y = 725\ N$
$V_x = 0$
$M_x = 203\ N\cdot m$
$M_y = -340\ N\cdot m$
$M_z = 142\ N\cdot m$

- Solve for bar geometry properties



For x-y cross at H-K section

Cont. 6.44

$$I_x = I_y = \frac{1}{4}\pi\left(\frac{d}{2}\right)^4$$
$$= \frac{1}{4}\pi\left(\frac{50\,mm}{2}\right)^4$$
$$= 306.8 \times 10^{-9}\ \ m^4$$

$$A = \pi\left(\frac{d}{2}\right)^2$$
$$= \pi\left(\frac{50\,mm}{2}\right)^2$$
$$= 1.96 \times 10^{-3}\ \ m^2$$

$$J = 2I = 613.6 \times 10^{-9}\ \ m^4$$

$Q_H = 0$ (about x axis)

$$Q_k = (A_{shade})(\bar{y})$$
$$= \left(\frac{1}{2}A\right)\left(\frac{4r}{3\pi}\right)$$
$$= \left(\frac{1}{2} \times 1.96 \times 10^{-3}\ m^2\right)\left(\frac{4\cdot 25\,mm}{3\pi}\right)$$
$$= 10.4 \times 10^{-6}\ \ m^3$$

$$t_k = d = 50\ mm = 0.05\ \ m.$$

$\Rightarrow$

$I_x = I_y = 306.8 \times 10^{-9}\ \ m^4$
$J = 613.6 \times 10^{-9}\ \ m^4$
$A = 1.96 \times 10^{-3}\ \ m^2$
$Q_k = 10.4 \times 10^{-6}\ \ m^3$
$t_k = 0.05\ \ m$
$Q_H = 0$

- Solve for stresses now

$$\sigma = \sigma^P + \sigma^{Mx} + \sigma^{My}$$

$$\tau = \tau^V + \tau^{Mz}$$

Cont. 6.44

$$\sigma^P = \frac{P}{A} = \frac{-1735\ N}{1.96 \times 10^{-3}\ m^2} = -885\ \ kPa$$

At point H:

$$\sigma^{Mx} = -\frac{M_x y_H}{I_x}$$
$$= -\frac{(+203\ N\cdot m)(0.025\ m)}{306.8 \times 10^{-9}\ m^4}$$
$$= -16.542\ \ MPa$$

$$|\sigma^{My}| = \left|-\frac{M_y x_H}{I_y}\right| = 0 \quad (x_H = 0)$$

$\Rightarrow$

$$\sigma_H = \sigma^P + \sigma^{Mx} + \sigma^{My}$$
$$= -885\ kPa - 16.542\ \ MPa$$
$$= -17.427\ \ MPa$$

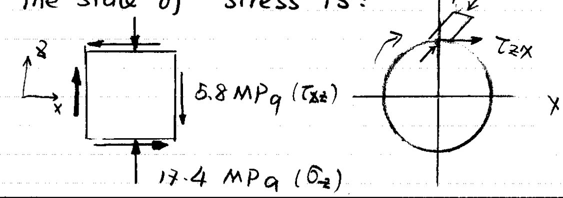
$$\tau^V = \frac{VQ}{It} = 0 \quad (Q^H = 0)$$

$$\tau^{Mz} = \frac{Tc}{J} = \frac{(142)\ N\cdot m\ (0.025\ m)}{613.6 \times 10^{-9}\ m^4}$$
$$= 5.78\ \ MPa$$

$\Rightarrow$

$\sigma_H = -17.4\ \ MPa$
$\tau_H = 5.8\ \ MPa$

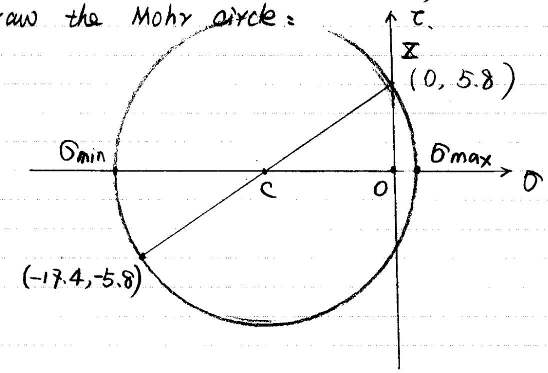
The state of stress is:



Cont. 6.44

$\sigma_x = 0$
$\tau_{xz} = -5.8$ MPa
$\sigma_z = -17.4$ MPa

Decide $(\sigma_x, -\tau_{xz})$ and (σ_z, τ_{zx}), then draw the Mohr circle:



From the circle:

$$\sigma_C = \frac{\sigma_x + \sigma_z}{2} = \frac{0 - 17.4}{2} = -8.7 \text{ MPa}$$

$$OC = \frac{\sigma_x - \sigma_z}{2} = \frac{0-(17.4)}{2} = 8.7 \text{ MPa}$$

$\Rightarrow$ $R = \sqrt{OC^2 + OX^2}$
$= \sqrt{8.7^2 + 5.8^2}$
$= 10.5$ MPa

$\Rightarrow$ $\sigma_{max} = R - OC$
$= (10.5 - 8.7)$ MPa
$= 1.8$ MPa

$\sigma_{min} = -(R + OC)$
$= -(10.5 + 8.7)$ MPa
$= -19.2$ MPa

$\tau_{max} = R = 10.5$ MPa

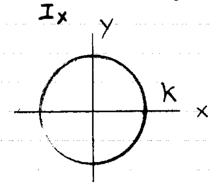
Cont. 6.44

$$\boxed{\begin{array}{l} \sigma_{max} = 1.8 \text{ MPa} \\ \sigma_{min} = -19.2 \text{ MPa} \quad \text{at H.} \\ \tau_{max} = 10.5 \text{ MPa} \end{array}}$$

At point k:

$\sigma^P = -885$ kPa

$\sigma^{Mx} = -\dfrac{M_x y_k}{I_x} = 0 \quad (y_k = 0)$



$|\sigma^{My}| = \left|\dfrac{M_y x_k}{I_y}\right| \quad (x_k = 0.025 \text{ m})$

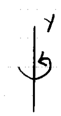
$= \left|\dfrac{(-340 \text{ N·m})(0.025 \text{m})}{306.8 \times 10^{-9} \text{ m}^4}\right|$

$= +27.7$ MPa

*: We don't know which My is for a smiling beam here, so we take the magnitude.

We decide the (+) or (-) for σ^{My} by the analysis of "stretch" or "compression" at k caused by My:

$M_y = -340$ N·m so the actual rotation of My is



This meas when $x<0$, it's under compression. $\Rightarrow \sigma < 0$

$\Rightarrow$ $\sigma^{My}_k = 27.7$ (MPa) $(x > 0)$

Cont. 6.44

$\sigma_k = \sigma_k^P + \sigma_k^{Mx} + \sigma_k^{My}$
$= -885$ kPa $+ 27.7$ MPa
$= 26.8$ MPa

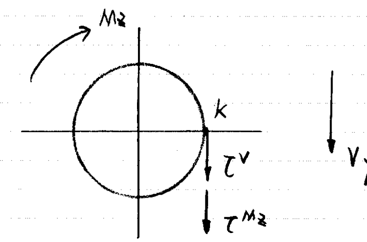
$\tau^V = \dfrac{VQ}{It}$

We already have: $Q_k = 10.4 \times 10^{-6}$ m³
$t_k = 0.05$ m

$\Rightarrow$ $\tau^V = \dfrac{(725 \text{ N})(10.4 \times 10^{-6} \text{ m}^3)}{(306.8 \times 10^{-9} \text{m}^4)(0.05 \text{m})}$
$= 491$ kPa

$\tau^{Mz} = \dfrac{M_z C_k}{J} = \tau_H^{Mz} \quad (C_k = C_H)$
$= 5.78$ MPa

Study the direction of τ at k:



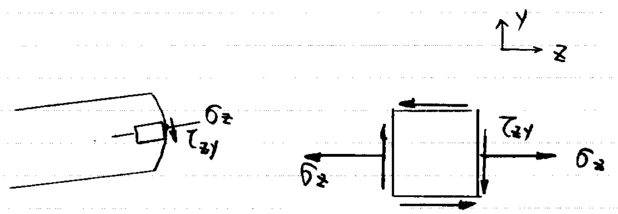
$\Rightarrow$ $\tau = \tau^V + \tau^{Mz}$
$= 491$ kPa $+ 5.78$ MPa
$= 6.3$ MPa

Now:

$$\boxed{\begin{array}{l} \sigma^k = 26.8 \text{ MPa} \\ \tau = 6.3 \text{ MPa} \downarrow \end{array}}$$

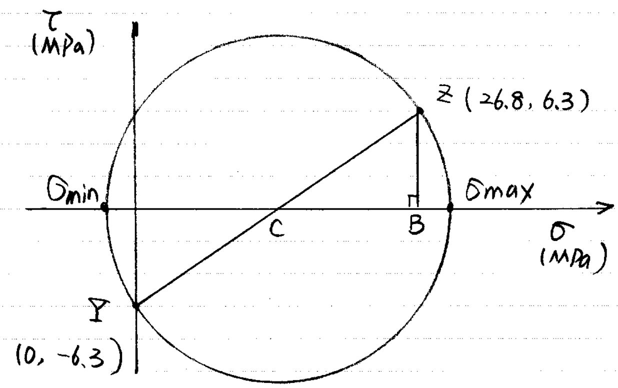
Cont. 6.44.

Draw the state the stress at point k:



$\sigma_z = 26.8$ MPa
$\sigma_y = 0$
$\tau_{zy} = -6.3$ MPa

Draw $(\sigma_z, -\tau_{zy})$ and (σ_y, τ_{yz}) on $\sigma-\tau$ then draw the Mohr circle.



$\sigma_{max} = \sigma_c + R$, $\sigma_{min} = \sigma_c - R$

$$\sigma_c = \frac{\sigma_z + \sigma_y}{2} = \frac{26.8}{2} \text{MPa} = 13.4 \text{ MPa}$$

$$CB = \frac{\sigma_z - \sigma_y}{2} = 13.4 \text{ MPa}$$

$$\Rightarrow R = \sqrt{CB^2 + (6.3\text{MPa})^2}$$

$$= \sqrt{13.4^2 + 6.3^2} \text{ MPa}$$

$$= 14.8 \text{ MPa} = \tau_{max}$$

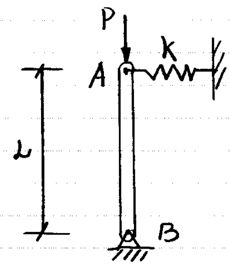
$$\Rightarrow \sigma_{max} = (13.4 + 14.8) \text{ MPa} = 28.2 \text{ MPa}$$

$$\sigma_{min} = (13.4 - 14.8) \text{ MPa} = -1.4 \text{ MPa}$$

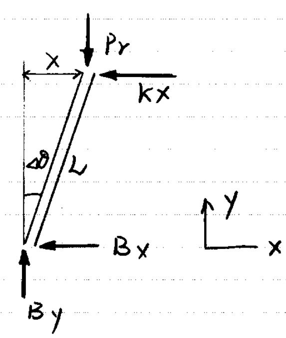
Cont. 6.44:

$$\boxed{\begin{array}{l} \sigma_{max} = 28.2 \text{ MPa} \\ \sigma_{min} = -1.4 \text{ MPa} \\ \tau_{max} = 14.8 \text{ MPa} \end{array}}$$

11.1: Find the critical load P_{cr}.



Soln: assume the rod has a small rotation $\Delta\theta$.



$$+\circlearrowleft \Sigma M_B = 0 \Rightarrow$$

$$(kx) L \cos(\Delta\theta) - P L \sin\Delta\theta = 0$$

if $\Delta\theta << 1$

$$\Rightarrow \cos(\Delta\theta) = 1$$
$$\sin(\Delta\theta) = \Delta\theta$$

$$\Rightarrow k L\Delta\theta L - P L \Delta\theta = 0$$

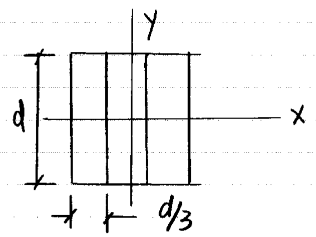
$$\Rightarrow P = kL$$

$$\boxed{P_r = kL}$$

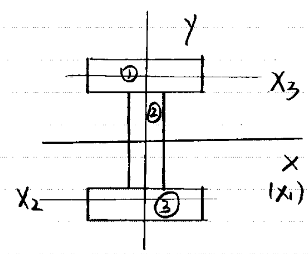
11.24.

Given: L of the length, cross (a) and (b)

Find: the ratio of the critical load for (a) and (b).



(a)



(b)

Soln:

We know:

$$P_{cr} = \frac{\pi^2 E I}{L_e^2}$$

$$\Rightarrow \frac{P_{cr}^{(a)}}{P_{cr}^{(b)}} = \frac{I^{(a)}}{I^{(b)}}$$

$$I^{(a)} = \frac{1}{12} d^4 \quad \text{for } I_x \text{ and } I_y$$

$$I_x^{(b)} = \sum_1^3 \left(I_{xi} + A_i d_i^2\right)$$

$$I_{x1} = \frac{1}{12}\left(\frac{d}{3}\right)(d)^3 = \frac{1}{36} d^4$$

$$I_{x2} = I_{x3} = \frac{1}{12}(d)\left(\frac{d}{3}\right)^3 = \frac{1}{108} d^4$$

$$d_1 = 0, \quad d_2 = d_3 = \frac{d}{2} + \frac{d}{6} = \frac{2}{3} d$$

$$\Rightarrow I_x^{(a)} = \frac{1}{36} d^4 + 2\left[\frac{1}{108} d^4 + d\left(\frac{d}{3}\right)\left(\frac{2}{3} d\right)^2\right]$$

$$= \left(\frac{1}{36} + \frac{17}{54}\right) d^4$$

$$= \frac{37}{108} d^4$$

Cont. 11.24

$$I_y^{(b)} = \sum_1^3 \left(I_{yi} + A_i d_i^2\right)$$

$$I_{y1} = \frac{1}{12} d\left(\frac{d}{3}\right)^3 = \frac{1}{324} d^4$$

$$I_{y2} = I_{y3} = \frac{1}{12}\left(\frac{d}{3}\right) d^3 = \frac{1}{36} d^4$$

$$d_i = 0$$

$$\Rightarrow I_y^{(b)} = \frac{1}{324} d^4 + 2\left(\frac{1}{36} d^4\right)$$

$$= \frac{19}{324} d^4$$

Comparing $I_y^{(b)}$ and $I_x^{(b)}$

$$I_y^{(b)} < I_x^{(b)}$$

So we need to use the smaller I to get $P_{cr}^{(b)}$

$$\Rightarrow P_{cr}^{(b)} = \frac{\pi^2 E I_y^{(b)}}{L_e^2}$$

$$\Rightarrow \frac{P_{cr}^{(a)}}{P_{cr}^{(b)}} = \frac{I_y^{(a)}}{I_y^{(b)}}$$

$$= \frac{\frac{1}{12} d^4}{\frac{19}{324} d^4}$$

$$= 1.421$$

$$\boxed{\frac{P_{cr}^{(a)}}{P_{cr}^{(b)}} = 1.421}$$