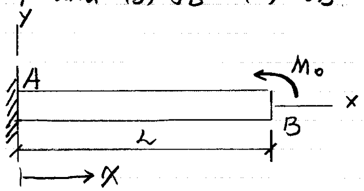
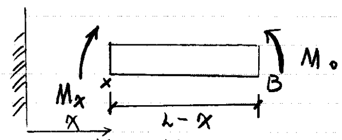
8.2, 8.4, 8.8 8.16

8.2. Find (a) y and (b) y_B (c) θ_B



Soln: It's a statically determinate problem

- Solve for moment in AB. Cut at any point between AB, and draw the FBD of the right cut:



$+\circlearrowleft \Sigma M_B = 0 \Rightarrow \quad M_B - M_x = 0 \Rightarrow$

$M_x = M_o$

- Part a.

$EI \frac{d^2y}{dx^2} = M_x = M_o$

$\Rightarrow$

$EI \frac{dy}{dx} = M_o x + C_1$

$\Rightarrow$

$EI\, y = \frac{1}{2} M_o x^2 + C_1 x + C_2$

Plugging B.C.S for AB:

At point A $(x=0)$: $y=0$, $\frac{dy}{dx}=0$

$\Rightarrow \begin{cases} \theta = \frac{dy}{dx} = M_o(0) + C_1 = 0 \\ 0 = \frac{1}{2} M_o (0)^2 + C_1(0) + C_2 = 0 \end{cases}$

$\Rightarrow C_1 = 0, \quad C_2 = 0$

$\Rightarrow$

$y = \frac{1}{EI}\left(\frac{1}{2} M_o x^2\right) = \boxed{\frac{M_o x^2}{2EI}}$ for AB

Cont. 8.2

(b) from (a):

$\theta = \frac{dy}{dx} = \frac{1}{EI} M_o x$

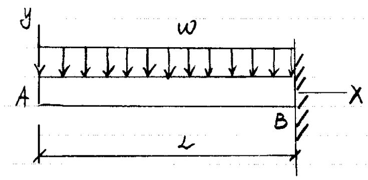
At B: $x = L \Rightarrow$

$\boxed{\theta|_{x=L} = \frac{1}{EI} M_o L}$

(c). At B: $x = L$

$\boxed{y|_{x=L} = \frac{1}{2EI} M_o L^2}$

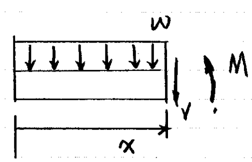
8.4. Solve for the beam below for the questions at 8.2



Soln:

- Solve for the moment in the beam

Cutting at any point between AB and draw the FBD for the left part



$+\circlearrowleft \Sigma M_x = 0 \Rightarrow \quad M + (wx)(\frac{1}{2}x) = 0 \Rightarrow$

$M = -\frac{1}{2} w x^2$

Cont. 8.4

(a).

$EI \frac{d^2y}{dx^2} = M = -\frac{1}{2} w x^2$

$\Rightarrow EI \frac{dy}{dx} = -\frac{1}{6} w x^3 + C_1 \quad (1)$

$\Rightarrow EI\, y = -\frac{1}{24} w x^4 + C_1 x + C_2 \quad (2)$

Plugging B.C.S into them:

At B $(x = L)$:

$\begin{cases} \theta = \frac{dy}{dx} = 0 & \text{to } (1) \\ y = 0 & \text{to } (2) \end{cases}$

$\Rightarrow \begin{cases} -\frac{1}{6} w L^3 + C_1 = 0 & (3) \\ -\frac{1}{24} w L^4 + C_1 L + C_2 = 0 & (4) \end{cases}$

From (3) $\Rightarrow C_1 = \frac{1}{6} w L^3$

From (4) $\Rightarrow$

$C_2 = \frac{1}{24} w L^4 - \frac{1}{6} w L^3 L$

$= -\frac{1}{8} w L^4$

$\Rightarrow$

$y = \frac{1}{EI}\left(-\frac{1}{24} w x^4 + \frac{1}{6} w L^3 x - \frac{1}{8} w L^4\right)$

$\boxed{y = -\frac{w}{24EI}\left(x^4 - 4L^3 x + 3L^4\right)}$

(b) from (a).

$\frac{dy}{dx} = \frac{1}{EI}\left(-\frac{1}{6} w x^3 + \frac{1}{6} w L^3\right)$

$\Rightarrow \left.\frac{dy}{dx}\right|_{x=0} = \frac{wL^3}{6EI} \qquad \boxed{\theta|_A = \frac{wL^3}{6EI}}$

Cont. 8.4 (c)

(c).

$$y\Big|_{x=0} = -\frac{w}{24EI}(0-0+3L^4)$$

$$\boxed{y = -\frac{wL^4}{8EI} \quad \text{at A}}$$

8.8 Solve (b) and (c) for 8.4, Given AB is W12x35, $w_0 = 3$ kips/ft, $L=12$ft, $E = 29\times10^6$ psi

Soln: From 8.4 we have

$$y = -\frac{w}{24EI}(x^4 - 4L^3x + 3L^4)$$

$$\frac{dy}{dx} = -\frac{w}{6EI}(x^3 - L^3)$$

(b) $$\frac{dy}{dx}\Big|_{x=0} = \frac{wL^3}{EI}$$

From appendix C:

$$I = I_x = 285 \text{ in}^4$$

$$\Rightarrow \frac{dy}{dx}\Big|_{x=0} = \frac{(3 \text{ kips/ft})(12 \text{ ft}^3)}{(29\times10^6 \text{ psi})(285 \text{ in}^4)}$$

$$= \frac{(36\times10^3 \text{ lb})(12\times12 \text{ in})^2}{(29\times10^6 \text{ psi})(285 \text{ in}^4)}$$

$$\boxed{\theta = 3.34\times10^{-3} \text{ rad}}$$

(c).

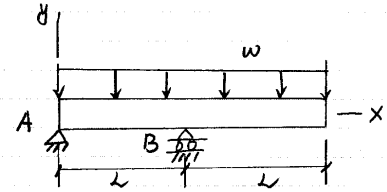
$$y\Big|_{x=0} = -\frac{L^4}{8EI}$$

$$= -\frac{(12 \text{ ft})^4}{8(29\times10^6 \text{psi})(285 \text{ in}^4)}$$

$$= -0.1804 \text{ in}$$

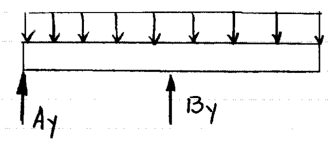
$$\boxed{y = -0.1804 \text{ in} \quad \text{at free edge}}$$

8.16. Find (a) y for AB
(b) θ at A
(c) θ at B



Soln:

- Solve for reactions at first



$+\circlearrowleft \Sigma M_A = 0 \Rightarrow$

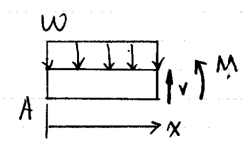
$$B_y(L) - w(2L)(L) = 0$$

$$\Rightarrow B_y = 2wL$$

$$\Sigma \vec{F} = 0 \Rightarrow A_y = 0$$

- Solve for moment in AB:

cut at any point between AB, draw the FBD of the left part:



$+\circlearrowleft \Sigma M_x = 0 \Rightarrow$

$$M + (wx)(\tfrac{1}{2}x) = 0$$

$$\Rightarrow M = -\tfrac{1}{2}wx^2$$

- Solve for y

Cont. 8.16.

$$EI\frac{d^2y}{dx^2} = M = -\tfrac{1}{2}wx^2$$

$$\Rightarrow EI\frac{dy}{dx} = -\tfrac{1}{6}wx^3 + C_1$$

$$\Rightarrow EIy = -\tfrac{1}{24}wx^4 + C_1x + C_2 \quad (1)$$

Plug in B.C.S at A & B to (1)

At A ($x=0$) $y=0$

$$\Rightarrow 0 = 0 + 0 + C_2$$

$$\Rightarrow C_2 = 0$$

At B ($x=L$), $y=0$

$$\Rightarrow 0 = -\tfrac{1}{24}wL^4 + C_1L$$

$$\Rightarrow C_1 = \tfrac{1}{24}wL^3$$

Plug in C_1 & C_2 to (1):

$$y = \frac{1}{EI}\left(-\tfrac{1}{24}wx^4 + \tfrac{1}{24}wL^3x\right)$$

$$y = -\frac{w}{24EI}(x^4 - L^3x)$$

(a). $$\boxed{y = -\frac{wx}{24EI}(x^3 - L^3)}$$

(b). From above:

$$\frac{dy}{dx} = \frac{1}{EI}\left(-\tfrac{1}{6}wx^3 + \tfrac{1}{24}wL^3\right)$$

$$= -\frac{w}{24EI}(4x^3 - L^3)$$

$\Rightarrow$ when $x=0$

$$\boxed{\theta = \frac{wL^3}{24EI} \quad \text{at A}}$$

Cont 8.16.

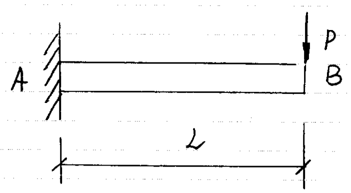
(c):

at B, $x = L$

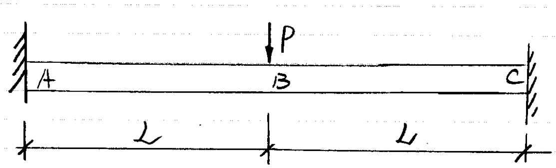
$$\theta = -\frac{w}{24EI}(4L^3 - L^3)$$

$$\Rightarrow \boxed{\theta = -\frac{wL^3}{8EI} \text{ at } B}$$

5. Compare the deflection at for a beam support a weight P mid away from the end. for case (a) and case (b)



case (a) supported as cantilever beam



case (b)
both ends are clamped.

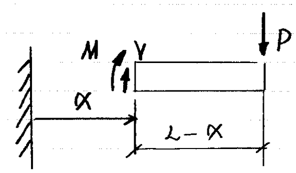
Cont. 5.

Soln:

First we solve it in a regular method

Case (a):

- Finding moment in the beam. cut at any point between AB and draw the FBD of the right part



$$+\circlearrowleft \Sigma M_x = 0 \Rightarrow P(L-x) + M = 0$$

$$\Rightarrow M = -P(L-x)$$

- Solve for elastic curve

$$EI\frac{d^2y}{dx^2} = M = -P(L-x)$$

$$\Rightarrow EI\frac{dy}{dx} = -PLx + \frac{P}{2}x^2 + C_1$$

$$\Rightarrow EI\,y = -\frac{1}{2}PLx^2 + \frac{1}{6}Px^3 + C_1x + C_2$$

$$\Rightarrow y = \frac{1}{EI}\left(-\frac{PLx^2}{2} + \frac{1}{6}Px^3 + C_1x + C_2\right)$$

- plug in B.Cs at $x = 0$: ($\theta = 0$, $y = 0$)

$$y|_{x=0} = C_2 = 0$$

$$\theta|_{x=0} = C_1 = 0$$

$$\Rightarrow y = -\frac{Px^2}{6EI}(3L - x)$$

when $x = L$:

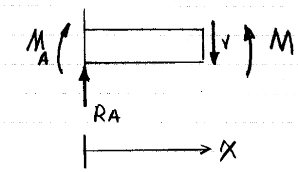
$$\boxed{y = -\frac{PL^3}{3EI} \text{ for } x = L}$$

Con't. 5.

case (b)

It's symmetric about the center, so we only need to study AB.

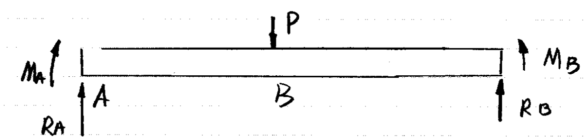
- Solve for M in AB. Cut at any point between AB and draw the FBD of the left part



$$+\circlearrowleft \Sigma M_x = 0 \Rightarrow M - M_A - R_A x = 0$$

$$\Rightarrow M = M_A + R_A x$$

- Solve for reactions R_A and M_A



From symmetry: $\begin{cases} R_A = R_B = \frac{P}{2} \\ M_A = M_B = M_0 \end{cases}$

- Solve for elastic curve equation for AB

$$EI\frac{d^2y}{dx^2} = M = M_0 + \frac{1}{2}Px$$

$$\Rightarrow EI\frac{dy}{dx} = M_0x + \frac{1}{4}Px^2 + C_1$$

$$\Rightarrow EI\,y = \frac{1}{2}M_0x^2 + \frac{1}{12}Px^3 + C_1x + C_2$$

- plug in BCS at

$A(x=0)$: $y = 0 \Rightarrow C_2 = 0$

$\theta = 0 \Rightarrow C_1 = 0$

$B(x=L)$: $\theta = 0 \Rightarrow M_0L + \frac{1}{4}PL^2 + C_1 = 0$

Cont 5.

$\Rightarrow M_0 = -\frac{1}{4}PL$

$$\Rightarrow \quad y = \frac{1}{EI}\left(-\frac{1}{8}PLx^2 + \frac{1}{12}Px^3\right)$$

$$y = -\frac{Px^2}{24EI}(3L - 2x)$$

when $x = L$:

$$\boxed{Y = -\frac{PL^3}{24EI} \quad \text{for } x = L}$$

Comparing the Y at $x=L$ for case a and (b):

$$\boxed{y^{(b)} = \frac{1}{8}y^{(a)}}$$

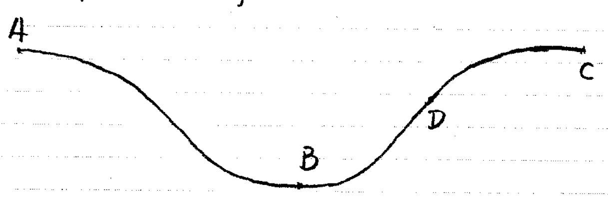
which mean the deflection of the cantilever case is 8 times the clamped on.

Now we slove by shortcut

for case (a), we already got:

$$y = -\frac{Px^2}{6EI}(3L - x)$$

The deformation of case (b) is:



This means $\theta = 0$ at L. and the curve for AB are symmetric to BC

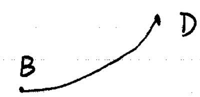
(Beam AC is symmetric about B)

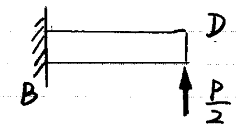
Also, curve for BC is symmetric about D, where D is the center of BC

$\Rightarrow$

The curve for BD is equal to the curve caused by case C:

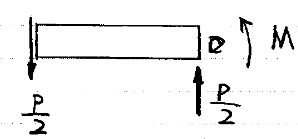
Cont. 5





case (C)

It's $\frac{P}{2}$ because when you cut at D and draw the FBD of the right part



Compare case (a) and case (c), they're all cantilever beam except the length of (c) is half of case (a) and the load in case (c) is half of P.

Using the eqn for cantilever beam (a)

$$y_D = -\frac{PL^3}{3EI} \qquad \text{here } P = \frac{P}{2}, \; L = \frac{L}{2}$$

$$= -\frac{\left(\frac{P}{2}\right)\left(\frac{L}{2}\right)^3}{3EI}$$

$$= -\frac{1}{2}\left(\frac{PL^3}{24EI}\right)$$

but Y_C is only half of the deflection of the center B (look at the deformation curve)

$$\Rightarrow \quad Y_B = 2Y_D = -\frac{PL^3}{24EI} = \frac{1}{8}y_B^{(a)}$$