Ying Wang

P. 5.22, 5.14. 5.58, 5.66

P.5.22: Find the shearing stress at (a) point a, (b). point b. in n-n section

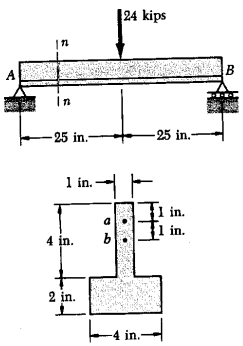


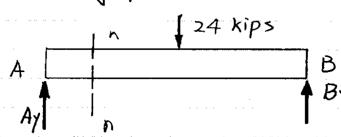
Fig. P5.22

Soln: From the eqn $\tau = \frac{VQ}{It}$

(a).

- Finding V in n-n section

First solving for reaction at point A:



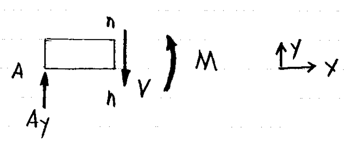
FBD of the whole beam

$+\circlearrowleft \Sigma M_B = 0 \Rightarrow$

$-A_y(50 in) + 24 kips(25 in) = 0$

$\Rightarrow A_y = 12$ kips.

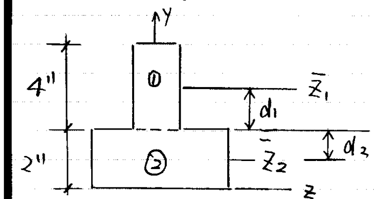
Then cutting at section n-n and draw the FBD of the left part



$\Sigma F_y = 0 \Rightarrow V = A_y = 12$ kips

- Finding $I_{\bar{z}}$

First solving for the N.A



Taking the y-z as shown above:

$\bar{y}_1 = 2in + \frac{4in}{2} = 4in$

$\bar{y}_2 = \frac{2in}{2} = 1 in$

$A_1 = (1 in)(4in) = 4 \ in^2$

$A_2 = (2in)(4in) = 8 \ in^2$

$$\Rightarrow \bar{y} = \frac{A_1\bar{y}_1 + A_2\bar{y}_2}{A_1 + A_2}$$

$$= \frac{(4in^2)(4in) + (8in^2)(1in)}{4 in^2 + 8 in^2}$$

$= 2 \ in$

Then solving for $I_{\bar{z}}$:

$I_{\bar{z}} = (I_{\bar{z}_1} + A_1 d_1^2) + (I_{\bar{z}_2} + A_2 d_2^2)$

$d_1 = 2"$, $d_2 = 1"$

$I_{\bar{z}_1} = \frac{1}{12}(b)(h^3) = \frac{1}{12}(1")(4")^3$

$= 5.3 \ in^4$

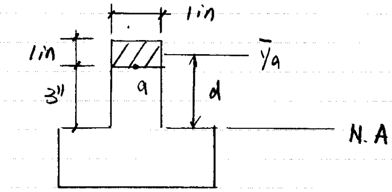
$I_{\bar{z}_2} = \frac{1}{12}(4in)(2in)^3$

$= 2.7 \ in^4$

$\Rightarrow I_{\bar{z}} = [5.3 \ in^4 + (4in^2)(2")^2] + [2.7 \ in^4 + (8in^2)(1")^2]$

$= 32 \ in^4$

- Finding Q_a:



Q_a is about the shaded area above a:

$Q_a = A\bar{y}_a$ where

$A_a = (1")(1") = 1 \ in^2$

$\bar{y}_a = d = 3" + \frac{1"}{2} = 3.5"$

$\Rightarrow Q_a = (1 in^2)(3.5")$

$= 3.5 \ in^3$

- Finding $t_a = 1$ in
- Finding τ_a

$\tau_a = \frac{V Q_a}{I t_a}$

$= \frac{(12 \ kips)(3.5 in^3)}{(32 in^4)(1 in)}$

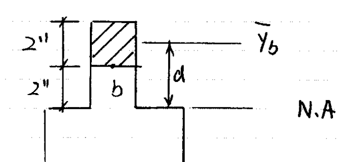
$= 1313$ psi

$\boxed{\tau_a = 1313 \ Psi}$

(b). For point b:

I, V, t are the same as (a)

- solving Q_b.



Q_b is about the shaded area above.

$\bar{y}_b = d = 2" + 2"/2 = 3"$

$A_b = (2")(1") = 2 \ in^2$

$\Rightarrow Q_b = A_b\bar{y}_b = (2in^2)(3") = 6 \ in^3$

Cont. 5.22.

- Finding τ_b

$$\tau_a = \frac{V_b Q_b}{I\ t}$$

$$= \frac{(12\ \text{kips})(6\ \text{in}^3)}{(32\ \text{in}^4)(1'')}$$

$$= 2250\ \text{psi}$$

$$\boxed{\tau_a = 2250\ \text{psi}}$$

5.14.

Find: (a) σ_{max}

(b) τ_{max} and compared with $\tau = \frac{VQ}{It}$ in the middle section

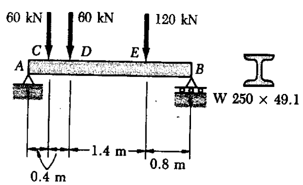
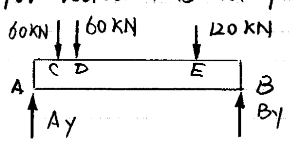


Fig. P5.14

Solution: We need to get the internal forces in the middle at first to solve for $\sigma - n$.

- Finding V, T, M in the middle section.

Solving for reaction B at first



FBD of the whole beam

$\circlearrowleft \Sigma M_A = 0 \Rightarrow$

$(B_y)(3\ m) - (120\ kN)(2.2\ m) - (60\ kN)(0.8\ m + 0.4\ m) = 0$

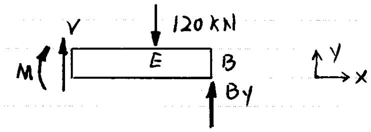
$\Rightarrow$

$B_y = 112\ kN$

Cont. 5.14

Then solving for internal forces:

cut at the middle and draw the FBD of the right part:



$\Sigma F_y = 0$: $V - 120\ kN + 112\ kN = 0 \Rightarrow$

$V = 8\ kN$

$+\circlearrowleft \Sigma M_B = 0$:

$(120\ kN)(0.8\ m) - M - V(1.5\ m) = 0$

$\Rightarrow$

$M = (96\ kN \cdot m) - (12\ kN \cdot m)$

$= 84\ kN \cdot m$

$T = 0$ (axial force)

(a) σ_{max}

$$\sigma_m = +\frac{MC}{I}$$

From the Appendix C, for W250 × 49.1:

$I_x = 70.8 \times 10^6\ mm^4$

$C = \frac{d}{2} = 123.5\ mm$

$\Rightarrow$

$$\sigma_m = \frac{(84\ kN \cdot m)(123.5 \times 10^{-3}\ m)}{(70.8 \times 10^6)(10^{-12})\ m^4}$$

$$= 146.6\ MPa$$

$$\boxed{\sigma_m = 146.6\ MPa}$$

(b) τ_{max}

- First using $\tau_{max} = V / A_{web}$

$A_{web} = (t_w)(d)$

From the Appendix C:

Cont. 5.14 (b)

$t_w = 7.4\ mm$

$d = 247\ mm$

$\Rightarrow$

$$\tau_m = \frac{V}{A_{web}} = \frac{8\ KN}{(7.4 \times 10^{-3})m\ (247 \times 10^{-3})m}$$

$$= 4.38\ MPa$$

$$\boxed{\tau_m^{(1)} = 4.38\ MPa}$$

- Then using $\tau = \frac{VQ}{It}$

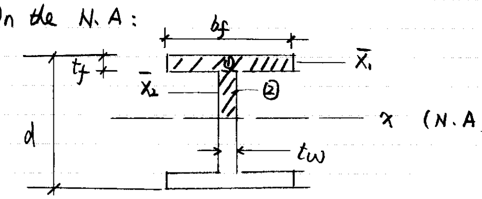
$\tau_m = \frac{V\ Q_{max}}{I\ t_{min}}$ is along the N.A

because: $t_{min} = t_w = 7.4\ mm$ there

$Q_{max} = \int_0^c y\,dA$ maximum there

$I_x = 70.8 \times 10^6\ mm^4$ is found already

On the N.A:



Q is for the shaded area:

$Q = A_1 \bar{X}_1 + A_2 \bar{X}_2$

$A_1 = (t_f)(b_f)$ $\quad \bar{X}_1 = \frac{d}{2} - \frac{t_f}{2}$

$A_2 = (t_w)(\frac{d}{2} - t_f)$

$\bar{X}_2 = (\frac{d}{2} - t_f)/2 = \frac{d}{4} - \frac{t_f}{2}$

$t_f = 11.0\ mm$, $t_w = 7.4\ m$

$d = 247\ mm$, $b_f = 202\ mm$

$\Rightarrow$

$Q = (11.0\ mm)(202\ mm)(\frac{247\ mm}{2} - \frac{11.0}{2}\ mm) +$

$(7.4\ mm)(\frac{247\ mm}{2} - 11.0\ mm)(\frac{247}{4} - \frac{11.0}{2})\ mm$

$= 0.31 \times 10^{-3}\ m$

Cont. 5.14 (b)

$$\Rightarrow \tau_{max} = \frac{VQ}{It}$$

$$= \frac{(8\times10^3)N\,(0.31\times10^{-3}\ m)}{(70.8\times10^6\times10^{-12})\,m^4\,(7.4\times10^{-3})\,m}$$

$$= 4.73 \quad MPa$$

$$\boxed{\tau_{max}^{(2)} = 4.73\ MPa}$$

Comparing $\tau_m^{(1)}$ and $\tau_m^{(2)}$:

$$\tau_m^{(1)} < \tau_m^{(2)}$$

This's because:

$$\tau_m^{(1)} = \frac{V}{A_{web}} = \frac{V}{(t_w)(d)}$$

$$\tau_m^{(2)} = \frac{VQ}{It} = \frac{V}{\left(\frac{It}{Q}\right)}$$

Plugging the value into them:

$$(t_w)(d) > \left(\frac{It}{Q}\right)$$

$$\Rightarrow \tau_m^{(1)} < \tau_m^{(2)}$$

5.58.

Given: AB is $0.4''\times12''$ rectangular

$\theta = 40°$

Find: $\tau_a, \tau_b, \tau_c, \sigma_a, \sigma_b, \sigma_c$

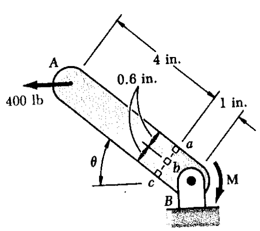
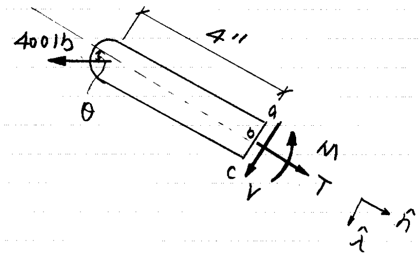


Fig. P5.58

Cont. 5.58

Soln: need to get the internal forces on a-b-c section at first

- Cut at a-b-c and draw the FBD of the left part:



$(\hat{n})\cdot(\Sigma\vec{F}) = 0 \Rightarrow$

$$T - (400\ lb)\cos\theta = 0$$

$$\Rightarrow T = (400\ lb)\cos(40°)$$

$$= 306.4 \quad lb$$

$(\hat{\lambda})\cdot(\Sigma\vec{F}) = 0 \Rightarrow$

$$V + (400\ lb)\sin\theta = 0$$

$$\Rightarrow V = -(400\ lb)\sin(40°)$$

$$= -257.1 \quad lb$$

$\circlearrowleft \Sigma M_{a-b-c} = 0 \Rightarrow$

$$M + (400\ lb)\sin\theta\,(4\ in) = 0$$

$$\Rightarrow M = -(400\ lb)\sin 40°\,(4\ in)$$

$$= 1028.5 \quad lb\cdot in$$

- So, we have:

$T = 306.4$ lb

$V = -257.1$ lb on section a, b, c

$M = 1028.5$ lb·in

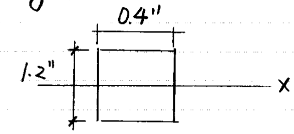
$$\Rightarrow \sigma = \sigma^{(M)} + \sigma^{(T)}$$

$$\tau = \frac{VQ}{It} = \tau^{(v)}$$

$$\sigma^{(M)} = -\frac{My}{I}, \quad \sigma^{(T)} = \frac{T}{A} = \frac{306.4\ lb}{(0.4'')(1.2'')} = 638.3\ psi$$

Cont. 5.58

- Solving for the common variable I

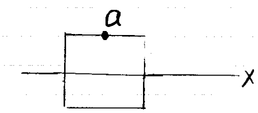


$$I_x = \frac{1}{12}(0.4'')(1.2'')^3$$

$$= 0.0576 \quad in^4$$

Now, we can solve for $\sigma_a, \tau_a, \sigma_b, \tau_b, \sigma_c, \tau_c$

(a) σ_a, τ_a



- $\sigma_a = \sigma_a^{(M)} + \sigma_a^{(T)}$

$\sigma_a^{(T)} = 638.3$ psi (solved before)

$\sigma_a^{(M)} = -\frac{My}{I}$ $y = 0.6''$

$$= -\frac{(1028.5\ lb\cdot in)(0.6\ in)}{0.0576\ in^4}$$

$$= 10.713 \quad ksi$$

$$\Rightarrow \sigma_a = 10713\ psi + 638.3\ psi$$

$$= 11.35 \quad ksi$$

- $\tau_a = \frac{VQ_a}{It}$ need to get Q_a

$Q_a = 0$ because the area above point a = 0

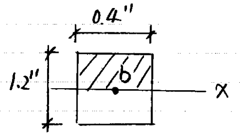
$$\Rightarrow Q_a = A\bar{y} = 0$$

$$\boxed{\begin{aligned}\sigma_a &= 11.35\ ksi\\ \tau_a &= 0\end{aligned}}$$

(b) σ_b τ_b

• σ_b

$$\sigma_b = -\frac{M y_b}{I} + (638 \text{ psi})$$



$y_b = 0$. point b is on the NA (x)

$\Rightarrow$ $\sigma_b = 638$ psi

• $\tau_b = \dfrac{VQ_b}{It}$

Q_b is about the shaded area above b :

$$Q_b = A\bar{y} = (0.4")(0.6")\left(\frac{0.6"}{2}\right)$$

$$= 0.072 \text{ in}^3$$

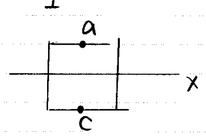
$$\Rightarrow \tau_b = \frac{(-257.1 \text{ lb})(0.072 \text{ in}^3)}{(0.0576 \text{ in}^4)(0.4 \text{ in})}$$

$$= -803 \text{ psi}$$

$$\boxed{\begin{array}{l}\sigma_b = 638 \text{ psi} \\ \tau_b = -803 \text{ psi}\end{array}}$$

(c). τ_c, σ_c

• $\sigma_c = -\dfrac{M y_c}{I} + (638 \text{ psi})$



Point c is symmetric about point a :

$\Rightarrow \sigma_c^{(M)} = -\sigma_a^{(M)} = -10731$ psi

$\Rightarrow \sigma_c = (-10731 + 638)$ psi

$= -10.07$ ksi

Cont. 5.58 (c)

$\tau_c = \tau_a = 0$

$$\boxed{\begin{array}{l}\sigma_c = -10.07 \text{ ksi} \\ \tau_c = 0\end{array}}$$

5.66.

Given : $d_1 = 1.90$ in, $d_2 = 1.61$ in

Find : σ, τ at (a) point H, (b) point K.

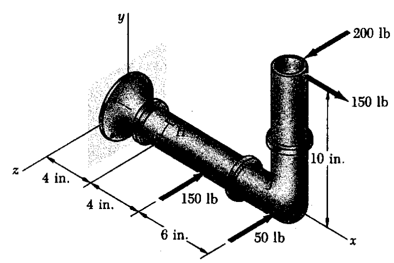
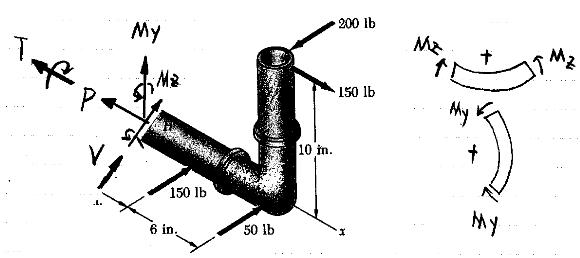


Fig. P5.66

Soln :

• Solving for internal forces on H-k section at first.

Cut at H-k and draw the FBD of the right part :



$(\hat{i}) \cdot (\Sigma \vec{F}) = 0 \Rightarrow$

$P - 150 \text{ lb} = 0 \Rightarrow P = 150$ lb

$V = 0$

Cont. 5.66

$+\circlearrowleft \Sigma M_x^{H\text{-}K} = 0 \Rightarrow$

$T - (200 \text{ lb})(10") = 0$

$\Rightarrow T = 2000$ lb·in

$+\circlearrowleft \Sigma M_y^{H\text{-}K} = 0$

$M_y - (200 \text{ lb})(10 \text{ in}) + (150 \text{ lb})(4 \text{ in}) + (50 \text{ lb})(10") = 0$

$\Rightarrow M_y = 900$ lb·in

$+\circlearrowleft \Sigma M_z^{H\text{-}K} = 0$

$M_z + (150 \text{ lb})(10 \text{ in}) = 0 \Rightarrow$

$M_z = -1500$ lb·in

Thus, we have on section H-k :

$P = 150$ lb
$T = 2000$ lb·in (Torsion)
$M_y = 900$ lb·in (Bending around y)
$M_z = -1500$ lb·in (〃 〃 z)

• The superposion method for σ, τ.

$\sigma = \sigma^{(P)} + \sigma^{(M_y)} + \sigma^{(M_z)}$ where :

$$\sigma^{(P)} = \frac{P}{A} = \frac{150 \text{ lb}}{\pi\left(\left(\frac{d_1}{2}\right)^2 - \left(\frac{d_2}{2}\right)^2\right)}$$

$$= \frac{150 \text{ lb}}{\pi[(0.95")^2 - (0.805")^2]}$$

$$= 187 \text{ psi}$$

$\sigma^{(M_y)} = -\dfrac{Mz}{I_y}$ (caused by M_y)

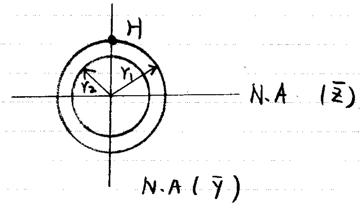
$\sigma^{(M_z)} = -\dfrac{MY}{I_z}$ (caused by M_z)

$\tau = \tau^{(T)} = \dfrac{TC}{J}$ (caused by Torsion M_x)

Cont. 5.66.

Now we can solve for (a) and (b)

(1) part (a) at point H



$$I_{\bar{z}} = \frac{1}{4}\pi(r_1^4 - r_2^4)$$
$$= \frac{1}{4}\pi\left[\left(\frac{1.90''}{2}\right)^4 - \left(\frac{1.61''}{2}\right)^4\right]$$
$$= 0.30989 \quad in^4$$

$I_{\bar{y}} = I_{\bar{z}} = 0.30989$ (symmetric)

• For the normal stress:

$$\sigma_H = \sigma_H^{(P)} + \sigma_H^{(M_y)} + \sigma_H^{(M_z)}$$

$\sigma_H^{(M_y)} = -\frac{M_y z}{I_{\bar{y}}} = 0$ (because $z=0$ at H)

$\sigma_H^{(M_z)} = -\frac{M_z y}{I_z}$, now

$y = r_1 = 0.95''$ at point H

$$\Rightarrow \sigma_H^{(M_z)} = -\frac{(-1500\ lb \cdot in)(0.95'')}{0.30989\ in^4}$$
$$= 4598 \quad psi$$

$$\Rightarrow \sigma_H = \sigma_H^{(P)} + \sigma_H^{(M_z)}$$
$$= (187 + 4598)\ psi$$
$$= 4785 \quad psi$$

• For the shearing stress

$$\tau_H = \frac{TC_H}{J}$$

Cont. 5.66

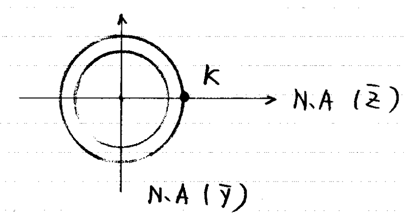
$$J = \frac{1}{2}\pi(r_1^4 - r_2^4) = 2I$$
$$= 0.61978 \quad in^4$$

$C_H = r_1 = 0.95''$

$$\Rightarrow \tau_H = \frac{T\,C_H}{J} = \frac{(2000\ lb\cdot in)(0.95'')}{0.61978\ in^4}$$
$$= 3065 \quad psi$$

$$\boxed{\begin{aligned}\sigma_H &= 4785\ psi\\ \tau_H &= 3065\ psi\end{aligned}}$$

(2) part b, σ, τ at point K



• For the normal stress

$$\sigma_K = \sigma_K^{(P)} + \sigma_K^{(M_{\bar{y}})} + \sigma_K^{(M_z)}$$

$\sigma_K^{(M_y)} = -\frac{M_y z_K}{I_y}$ $\quad z_K = r_1 = 0.95''$

$$= \frac{-(900\ lb\cdot in)(0.95'')}{0.30989}$$
$$= -2759 \quad psi$$

$\sigma_K^{(M_z)} = -\frac{M_z y_K}{I_z}$ $\quad y_K = 0$ for K

$$= 0$$

$$\Rightarrow \sigma_K = (187 - 2759)\ psi$$
$$= -2572 \quad psi$$

Cont. 5.66 (b)

• For the shearing stress:

$$\tau_K = \frac{T\,C_K}{J} = \frac{T\,r_1}{J} = \tau_H$$
$$= 3065 \quad psi$$

$$\boxed{\begin{aligned}\sigma_K &= -2572\ psi\\ \tau_K &= 3065\ psi\end{aligned}}$$