4.2, 4.10, 4.20, 4.26, 4.46.

4.2. Find the stress at (a) point A. (b) point B

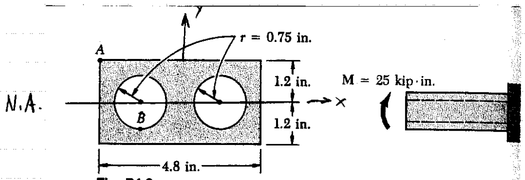


Fig. P4.2

Solution:

$\sigma_z = -\frac{My}{I_x}$, so we need to get I for the beam

From symmetry, the neutral axis is in the middle of the rectangular.

$I_x = \sum_1^3 I_i$. where $I_1$ is for the rectangular area, $I_2, I_3$ are for the circle.

$I_1 = \frac{1}{12}(bh^3)$

$I_2 = I_3 = \frac{1}{4}\pi r^4$

$\Rightarrow$

$I_x = I_1 - I_2 - I_3$ (because the two holes are taken out of the cross area)

$= \frac{1}{12}bh^3 - 2(\frac{1}{4}\pi r^4)$

$= \frac{1}{12}(4.8in)(2.4in)^3 - 2(\frac{1}{4}\pi(0.75in)^4)$

$= 5.03\ in^4$

(a). For point A: $y = 1.2$ in $\Rightarrow$

$\sigma_z = -\frac{(25\ kip\cdot in)(1.2\ in)}{5.03\ in^4}$

$= -5.96$ ksi

(b) For point B: $y = -0.75$ in $\Rightarrow$

$\sigma_z = -\frac{(25\ kip\cdot in)(-0.75\ in)}{5.03\ in^4}$

$= 3.73$ ksi

$$\boxed{\begin{array}{l} A:\ \sigma = -5.96\ ksi \\ B:\ \sigma = 3.73\ ksi \end{array}}$$



4.10. Find the maximum tensile and compressive stress in portion BC.

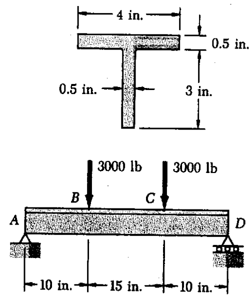
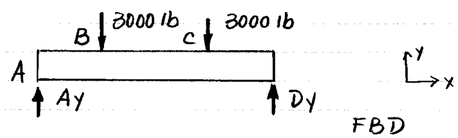


Fig. P4.10

Soln:

The stress can be found from $\sigma = -\frac{My}{I}$

• Finding the moment in BC, first find reactions

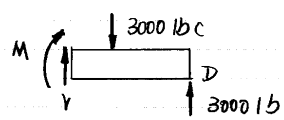



$\curvearrowleft \Sigma M_A = 0 \Rightarrow$

$D_y(35 in) - 3000 lb(10 in) - 3000 lb(25 in) = 0$

$\Rightarrow D_y = 3000$ lb

Cutting at any point between B and C, drawing the right cut:




$\Sigma F_y = 0 \Rightarrow V = 0$

$\curvearrowleft \Sigma M_D = 0 \Rightarrow (3000\ lb)(10\ in) - M = 0$

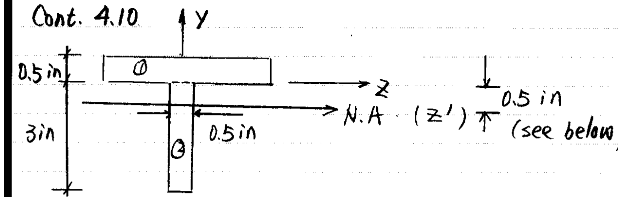
$\Rightarrow M = 30000\ lb\cdot in$.

• Finding I

Set up the original coords as follows:



Cont. 4.10




Breaking T area to area ① and area ②

$\Rightarrow$

N.A. $\bar{y}$ will be: $\bar{y}$ is y coords of N.A

$\bar{y} = \frac{\Sigma A_i \bar{y}_i}{\Sigma A_i} = \frac{A_1\bar{y}_1 + A_2\bar{y}_2}{A_1 + A_2}$

$= \frac{(4in)(0.5in)(0.25in) + (0.5in)(3in)(-1.5in)}{(4in)(0.5in) + (0.5in)(3in)}$

$= -0.5$ in

$\Rightarrow I_{z'} = \Sigma(I_{z_i'} + A_i d_i^2)$

$= (I_{z_1'} + A_1 d_1^2) + (I_{z_2'} + A_2 d_2^2)$

$d_1 = 0.5\ in + 0.25\ in = 0.75\ in$

$d_2 = 1.5\ in - 0.5\ in = 1\ in$

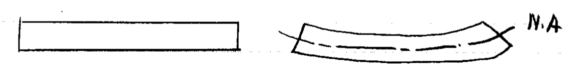
$\Rightarrow I_{z'} = \{\frac{1}{12}(4in)(0.5in)^3 + (4in)(0.5in)(0.75in)^2\}$

$+ \{\frac{1}{12}(0.5in)(3in)^3 + (0.5in)(3in)(1in)^2\}$

$= 3.79\ in^4$

• Finding the position for the maximum tensile and compressive stress

$M = 30000\ lb\cdot in \Rightarrow$ it's a smiling beam




$\Rightarrow$ upper part (above the N.A) under compression
lower part (below the N.A) under tension

$y_{tension} = -2.5$ in

$y_{compression} = 1$ in

$\{\sigma = -\frac{My}{I}\}$

$\Downarrow$

• Finding the maximum tensile and compressive stress

$\sigma_{max}^{(1)} = -\frac{(30000\ lb\cdot in)(+1 in)}{3.79\ in^4} = -7.1$ ksi

$\sigma_{max}^{(2)} = -\frac{(30000\ lb\cdot in)(-2.5 in)}{3.79\ in^4} = 19.78$ ksi

Cont. 4.10

$$\boxed{\begin{array}{l} \text{max tensile stress} = 19.78 \text{ ksi} \\ \text{max compr. stress} = -7.1 \text{ ksi} \end{array}}$$

4.20.

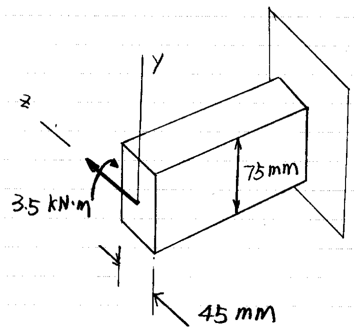
Part I: (Using #5 in book, not required)

Given: $E = 200\,GPa$, $M = 3.5\,kN\cdot m$

Find: $\rho$, maximum stress when

(a) M is around horizontal axis

(b) M is around vertical axis




Soln:

(a) $\begin{cases} \sigma = -\dfrac{MY}{I_z} \\ \rho = \dfrac{EI_z}{M} \end{cases}$ we can get:

$$I_z = \frac{1}{12} b h^3 = \frac{1}{12}(45mm)(75mm)^3$$

$$= 1.58 \times 10^{-6}\ m^4$$

Maximum stress:

$$\sigma_m^{(1)} = -\frac{(3.5\,kN\cdot m)(-75\,mm/2)}{1.58\times 10^{-6}\ m^4}$$

$$= -\frac{(3.5\times 10^3 N\cdot m)(0.0375\ m)}{1.58\times 10^{-6}\ m^4}$$

$$= 83.0\ MPa$$

Curverture:

$$\rho = \frac{EI_z}{M} = \frac{(200\,GPa)(1.58\times 10^{-6}\,m^4)}{3.5\,kN\cdot m}$$

$$= 90.3\ m$$

$$\boxed{\begin{array}{l} \sigma_m = 83.0\ MPa \\ \rho = 90.3\,m \end{array}}$$



Cont. 4.20

(b): $I_y = \frac{1}{12}(75mm)(45mm)^3$

$$= 0.57 \times 10^{-6}\ m$$

$$\sigma_m = -\frac{(3.5\,kN\cdot m)(0.0225\,m)}{0.57\times 10^{-6}\,m}$$

$$= 138.3\ MPa$$

When M is rotated about Y axis:

$$\sigma = -\frac{Mz}{I_y}$$

$$\rho = \frac{EI_y}{M} = \frac{(200\times 10^9\,Pa)(0.57\times 10^{-6}\,m)}{3.5\,kN\cdot m}$$

$$= 32.5\ m$$

$$\boxed{\begin{array}{l} \sigma_m = 138.3\ MPa \\ \rho = 32.5\ m \end{array}}$$

Part II: Solving for part I with wood material

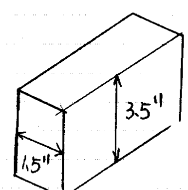
$E = 2\times 10^6$ psi,

$M = 2000$ lb·in

$A = b\times h = 1.5'' \times 3.5''$

(a). It's similar to part I, different in parameters.

$\begin{cases} \sigma_m = -\dfrac{MY}{I_z} \\ \rho = \dfrac{EI_z}{M} \end{cases}$




$$I_z = \frac{1}{12}(1.5'')(3.5'')^3 = 5.36\ in^4$$

$\Rightarrow$

$$\sigma_m = -\frac{(2000\ lb\cdot in)(-1.75\ in)}{5.36\ in^4}$$

$$= 653\ psi$$

$$\rho = \frac{(2\times 10^6 psi)(5.36\ in^4)}{2000\ \ lb\cdot in} = 5.36\times 10^3\ in$$



Cont. Part II. (a)

$$\boxed{\begin{array}{l} \sigma_m = 653\ psi \\ \rho = 5.36\times 10^3\ in \end{array}}$$

(b). $I_y = \frac{1}{12}(3.5'')(1.5'')^3 = 0.98\ in^4$

$$\sigma_m = -\frac{(2000\ lb\cdot in)(0.75'')}{0.98\ in^4}$$

$$= 1.53\ ksi$$

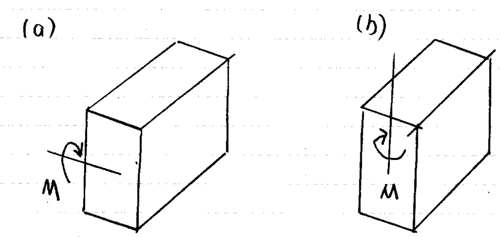
$$\rho = \frac{(2\times 10^6\ psi)(0.98\ in^4)}{2000\ lb\cdot in}$$

$$= 0.98\times 10^3\ in$$

$$\boxed{\begin{array}{l} \sigma_m = 1.53\ ksi \\ \rho = 0.98\times 10^3\ in \end{array}}$$

Conclusion:

Compare $\sigma_m$ and $\rho$ for (a), (b) in part II:




The (a) is 4 times as less curved as (b).
and (a) is twice as less stress as (b).

4.26. The rectangular cross is sewed from a circular section.

Find: (a) $\frac{d}{b}$ to get the smallest $\sigma_m$

(b) $\frac{d}{b}$ to get the maximum $\rho$.

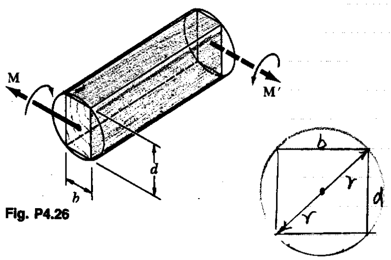


Fig. P4.26

Soln:

(a): We know that:

$\sigma_m = + \frac{MC}{I}$ C is the farest distance to the N.A

$\Rightarrow \sigma_m = \frac{M(\frac{d}{2})}{\frac{1}{12} b d^3} = \frac{6M}{bd^2}$

$= 6M \frac{1}{b(4R^2 - b^2)}$

$= \frac{6M}{F(b)}$

where

$F(b) = b(4R^2 - b^2)$

R: radius for the circle

Min ($\sigma_m$) $\Longleftrightarrow$ when y is max :

$\frac{dF}{db} = 4R^2 - 3b^2 = 0$

$\Rightarrow b^2 = \frac{4R^2}{3}$

$\Rightarrow d^2 = 4R^2 - \frac{4}{3}R^2 = \frac{8}{3}R^2$

$\Rightarrow \boxed{\frac{d}{b} = \sqrt{2}}$ to minimize max $\sigma$



Cont. 4.26

(b) $\rho = \frac{EI}{M}$

$= \frac{E}{M}\left(\frac{1}{12} bd^3\right)$

$= \frac{1}{12}\frac{E}{M} F(b)$

where

$F(b) = bd^3 = b(4R^2 - b^2)^{\frac{3}{2}}$

$\rho$ max $\Longleftrightarrow$ when y is max :

$\frac{dF}{db} = (4R^2 - b^2)^{\frac{3}{2}} + b(4R^2 - b^2)^{\frac{1}{2}}(-2b)\left(\frac{3}{2}\right)$

$= (4R^2 - b^2)^{\frac{1}{2}}(4R^2 - b^2 - 3b^2)$

$= 0$

$4R^2 - b^2 \neq 0$ $(b < 2R)$

$\Rightarrow$

$4R^2 - 4b^2 = 0$

$\Rightarrow b^2 = R^2$

$\Rightarrow d^2 = 4R^2 - b^2 = 3R^2$

$\Rightarrow \boxed{\frac{d}{b} = \sqrt{3}}$ to min. curvature or max. stiffness

4.46.

Given: $E_c = 20\,GPa$, $E_s = 200\,GPa$

$\sigma_{all}^c = 10\,MPa$, $\sigma_{all}^s = 150\,MPa$

Find: the largest moment can be applied

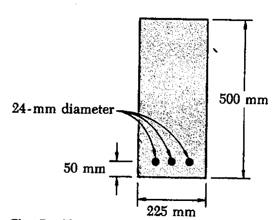


Fig. P4.46



Cont. 4.46.

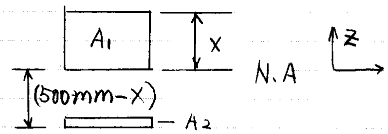
We know: $\sigma_m = -\frac{MC}{I}$ and we have

$\sigma_m^s < \sigma_{all}^s$, $\sigma_m^c \leq \sigma_{all}^c$

Each of the maximum stress give the limitation for $M_m^s$, $M_m^c$ and we need to choose the smaller one.

- Finding I at first.

(1). N.A. let it be x mm far away from the top.




} concrete has $\sigma > 0$ here so we assume $\sigma = 0$

$\Rightarrow A_1 = (225\ mm)\,x$

$= 225x\ mm$

$A_2 = nA_s = \left(\frac{E_s}{E_c}\right) A_s$

$= \left(\frac{200\,GPa}{20\,GPa}\right) 3\left(\frac{\pi}{4} 24^2 mm\right)$

$= 13572\ mm^2$

] we replace steel with concrete, multiplying area by modulus ratio.

$\bar{z}_1 = \frac{x}{2}$

$\bar{z}_2 = -(500\,mm - x - 50\,mm)$

$= -(450\,mm - x)$

$\Rightarrow \bar{z}_3 = \frac{A_1\bar{z}_1 + A_2\bar{z}_2}{A_1 + A_2} = 0$

because now the original position for x is along the N.A. $\bar{y}$.

$\Rightarrow A_1\bar{z}_1 + A_2\bar{z}_2 = 0$

$\Rightarrow$

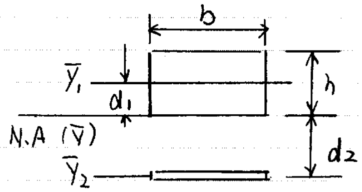
$(225x)\left(\frac{x}{2}\right) + (13572)(x - 450) = 0$

$\Rightarrow x = 180.36\ mm$

(2). I

$I = \sum_1^2 (I_{\bar{y}_i} + A_i d_i^2)$

Cont. 4.46




For the concrete part:

$$I_{\bar{y}}^{(1)} = I_{\bar{y}_1} + A_1 d_1^2$$

$$= \frac{bh^3}{12} + (bh)\left(\frac{h}{2}\right)^2$$

$$= \frac{bh^3}{3}$$

For the steel part which is below $\bar{y}$:

$$I_{\bar{y}}^{(2)} = I_{\bar{y}_2} + A_2 d_2^2$$

$$I_{\bar{y}_2} = b(500\text{ mm} - h - d_2) \approx 0$$

$$A_2 = 13572 \text{ mm}^2$$

$$d_2 = 500\text{ mm} - h - 50\text{ mm}$$
$$= 450\text{ mm} - h$$

$$\Rightarrow I_{\bar{y}}^{(2)} = (13572\text{ mm}^2)(450\text{mm} - h)^2$$

$$h = 180.36 \text{ mm}$$

$$I_{\bar{y}} = I_{\bar{y}}^{(1)} + I_{\bar{y}}^{(2)}$$

$$= \frac{1}{3}(225\text{mm})(180.36)^3 + (13572\text{ mm}^2)(450 - 180.36)^2 \text{ mm}^2$$

$$= 1426.8 \times 10^{-6} \text{ m}^4$$

- Finding M

For concrete: $\sigma_{all} = 10$ MPa, $C = x = 0.181$ m

$$M = \frac{\sigma I}{c} = (10\text{ MPa})\frac{1426.8 \times 10^{-6}\text{m}^4}{0.181\text{ m}}$$

$$= 79.1 \text{ kN}\cdot\text{m}$$

For steel: $\sigma_{all} = 150$ MPa
$c = 450\text{ mm} - x$
$= 0.26964$ m



Cont. 4.46

$$M = \frac{\sigma_{steel}}{n} \cdot \frac{I}{c}$$

$$= \frac{150\text{ MPa}}{10} \cdot \frac{1426.8 \times 10^{-6}\text{ m}^4}{0.26964\text{ m}}$$

$$= 79.4 \text{ kN}\cdot\text{m}$$

We choose the smaller on:

$$\boxed{M = 79.1 \text{ kN}\cdot\text{m}}$$