"SOLUTIONS"

Need to get 4 probs 100% correct,

TAM 202

Homework-based exam

Tues. Dec. 10, 2002

10 AM - 4 PM.

This exam is for students who think they can do all the homework but fear getting a grade of less than C- in this class.

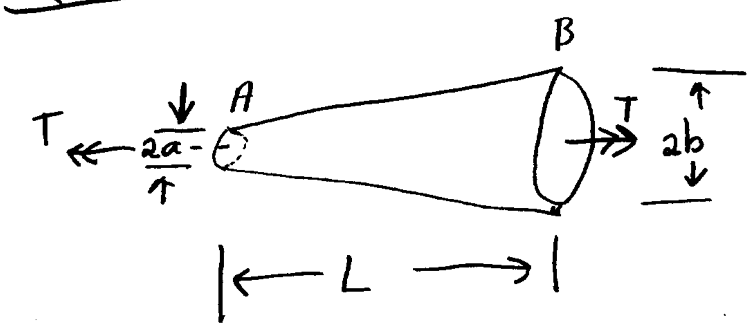
If you are not in this group please do not turn in the exam for grading.

Thank you.
– Andy Ruina

{Note! Choice between 2 & 2b
(2 was technically illegal for this exam, due Dec. 10)}

1. (based on G3.4-10)

Name: RUINA



Solid shaft
Conical shape.

In terms of a, b, L, G & T find the rotation of end B relative to A.

$$\phi_{B/A} = \int d\phi = \int_0^L \frac{T}{JG}\,dx$$

$J = \pi r^4/2 \;(= I_p)$

This is the key.

$r = a + \frac{x}{L}(b-a)$

$$= \frac{2T}{\pi G}\int_0^L \frac{dx}{\left(a + \frac{x}{L}(b-a)\right)^4}$$

from here down its just math !!!

let $r = a + \frac{x}{L}(b-a)$, Limits: $x=0 \Rightarrow r=a$, $x=L \Rightarrow r=b$

$dr = \frac{b-a}{L}\,dx \Rightarrow dx = \frac{L}{b-a}\,dr$

$$= \frac{2T}{\pi G}\int_a^b \frac{1}{r^4}\,\frac{L\,dr}{(b-a)} = \frac{2TL}{\pi G(b-a)}\left(\frac{-r^{-3}}{3}\right)\Bigg|_a^b = \underbrace{\frac{2TL}{3G(b-a)\pi}\left[\frac{1}{a^3} - \frac{1}{b^3}\right]}$$

perfectly good final answer

$$\phi_{B/A} = \frac{2TL}{3G\pi}\,\frac{b^3 - a^3}{a^3 b^3\,(b-a)} = \frac{2TL}{3G\pi}\,\frac{a^3\left((b/a)^3 - 1\right)}{a^3 b^3\, a\,(b/a - 1)}$$

$$\boxed{\phi = \frac{2TL}{3\pi G a b^3}\left[\left(\frac{b}{a}\right)^2 + \left(\frac{b}{a}\right) + 1\right]}$$

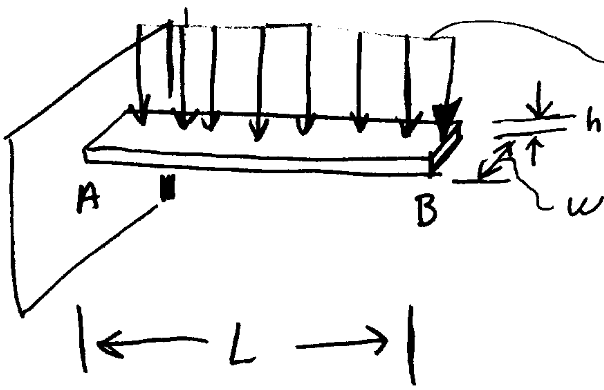
a little simpler in some ways

check units $[\phi] = [\,]$

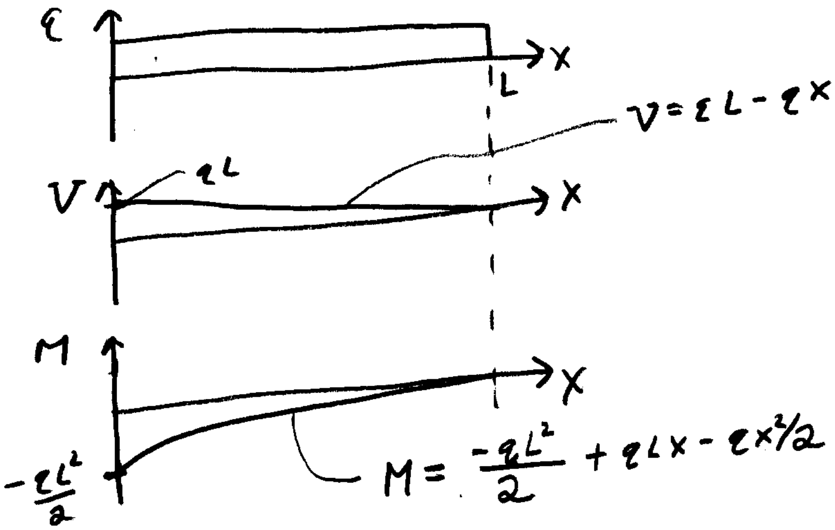
$[\text{right side}] = \frac{[FL][L]}{[F/L^2][L]^4}[\,] = [\,]$

units on right side = units on left side

2. (based on 6.9.3-6) Name: RUINA



Given that the deflection at the end B is δ what is E? (answer in terms of L, w, h, q & δ).



$$EI\,u''(x) = M(x)$$
$$= q\left[\frac{-L^2}{2} + Lx - \frac{x^2}{2}\right]$$

$$EI\,u'(x) = q\left[\frac{-L^2}{2}x + \frac{Lx^2}{2} - \frac{x^3}{6}\right] + C$$

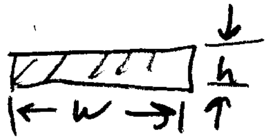
$$u'(0) = 0 \Rightarrow C = 0$$

$$EI\,u(x) = q\left[\frac{-L^2x^2}{4} + \frac{Lx^3}{6} - \frac{x^4}{24}\right]$$

$$\boxed{\delta = -u(L)}$$

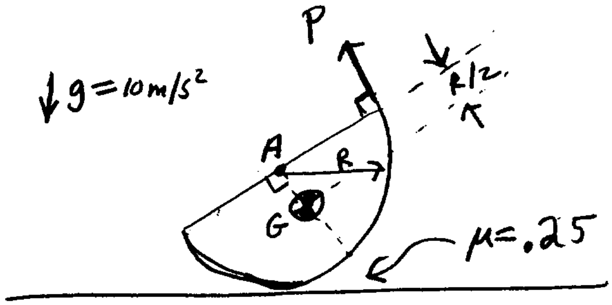
$$-EI\delta = EI\,u(L) = qL^4\left[\frac{-1}{4} + \frac{1}{6} - \frac{1}{24}\right]$$
$$= qL^4\,(-3/24)$$
$$= -qL^4/8$$

$$I = \frac{wh^3}{12}$$



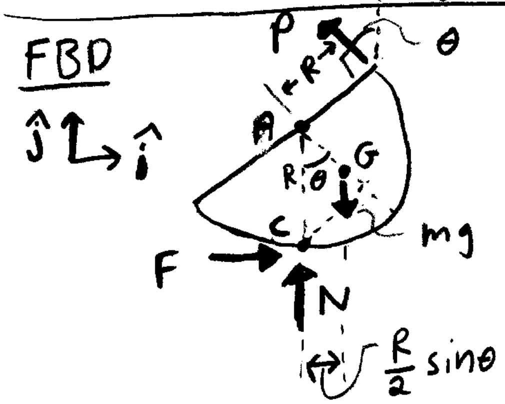
$$\Rightarrow \boxed{E = \frac{qL^4}{8I\delta}} \boxed{= \frac{3qL^4}{2wh^3\delta}}$$

3. (based on Meriam 6.131) Name: RUINA



$R = 1m$
$R/2 = 0.5m$
$m = 10\ kg$

Mass is distributed in a half cylinder so that distance $AG = R/2$ (this simplifies the problem slightly).
What is biggest P possible with no slip?



$$\sum M_{/C} = 0 \Rightarrow (PR + PR\sin\theta) - mgR\sin\theta/2 = 0$$

$$P = \frac{mg\ \sin\theta}{2(1+\sin\theta)} \quad (1)$$

$$\sum F_x = 0 \Rightarrow F - P\sin\theta = 0 \Rightarrow F = P\sin\theta \quad (2)$$

$$\sum F_y = 0 \Rightarrow N - mg + P\cos\theta = 0$$

$$\Rightarrow N = mg - P\cos\theta \quad (3)$$

$$(1),(2),(3) \Rightarrow \frac{F}{N} = \frac{P\sin\theta}{mg - P\cos\theta} = \frac{\sin\theta\ \sin\theta/2(1+\sin\theta)}{1 + \sin\theta\cos\theta/2(1+\sin\theta)}$$

$$\frac{F}{N} = \frac{\sin^2\theta}{2 + 2\sin\theta + \sin\theta\cos\theta}$$

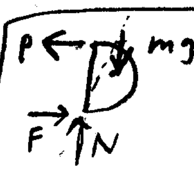
Critical case:

$$\frac{F}{N} = \mu \Rightarrow \frac{1}{4} = \frac{\sin^2\theta}{2 + 2\sin\theta + \sin\theta\cos\theta}$$

$$2 + 2\sin\theta + \sin\theta\cos\theta - 4\sin^2\theta = 0$$

root finding on calculator $\Rightarrow \theta = \pi/2$

$\sin\theta = 1$
$\cos\theta = 0$

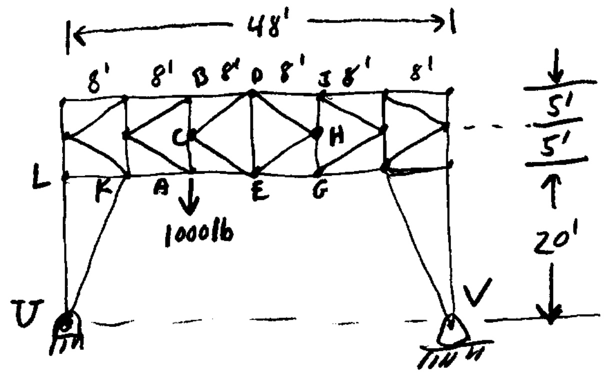


$$P = mg\frac{\sin\theta}{2(1+\sin\theta)}$$

$$\Rightarrow P = \frac{mg}{4}$$

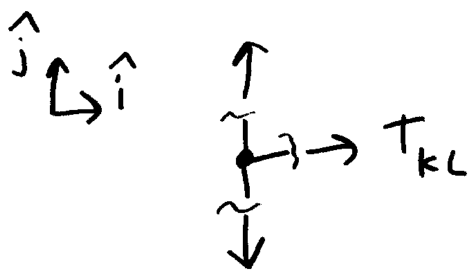
$$\boxed{P = 25N}$$

4. (based on M 4.49) Name: Ruina



What is T_{KL} (tension in bar KL)?

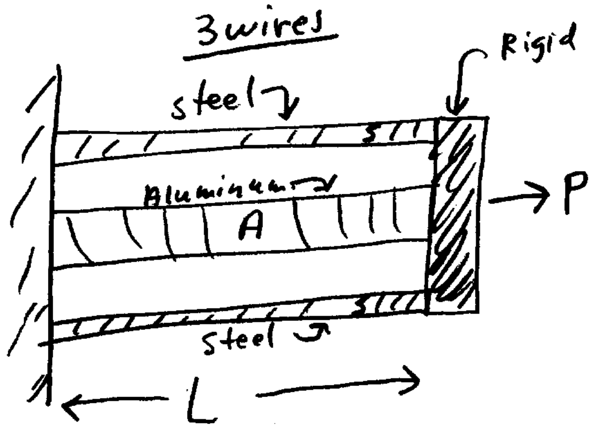
FBD of joint L

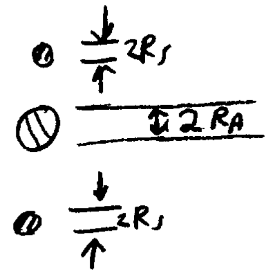


$\sum F_x = 0$

$\Rightarrow \boxed{T_{KL} = 0}$

5. (based on G 2.5-5) Name: RUINA





Stress-free at room temperature. Then temp. raised & P applied.

Given:

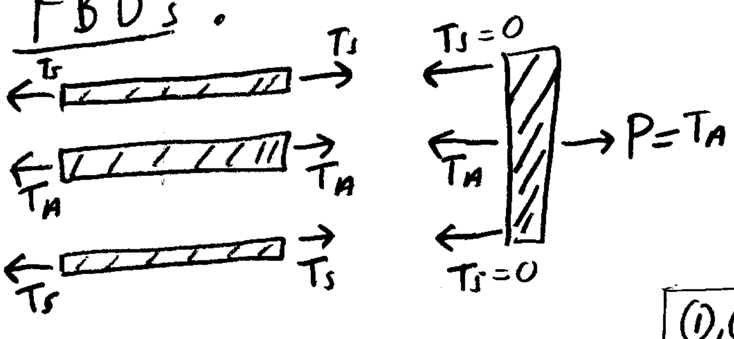
$\pi R_s^2 = 1 \text{ in}^2$ $\quad \alpha_s = 5\times10^{-6}/°F$ $\quad E_s = 30\times10^6 \text{ lb/in}^2$

$\pi R_A^2 = 2 \text{ in}^2$ $\quad \alpha_A = 10\times10^{-6}/°F$ $\quad E_A = 10\times10^6 \text{ lb/in}^2$

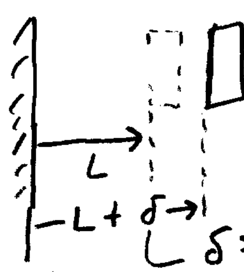
$\Delta T = 100°F$

Find P so that the load carried by the steel = 0.

FBDs:



Geometry



$\delta = \delta_s = \delta_A$

Given: $T_s = 0$

$$\delta_s = \frac{\overset{0}{\cancel{T_s}} L}{A_s E_s} + \alpha_s(\Delta T)L \quad (1)$$

$$\parallel$$

$$\delta_A = \frac{T_A L}{A_A E_A} + \alpha_A(\Delta T)L \quad (2)$$

$(1),(2) \Rightarrow \alpha_s(\Delta T)L = \dfrac{PL}{A_A E_A} + \alpha_A(\Delta T)L$

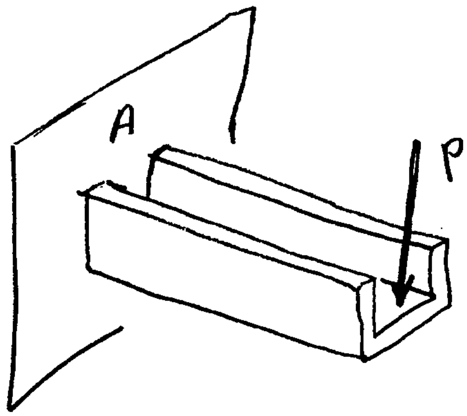
$\Rightarrow P = (\alpha_s - \alpha_A)(\Delta T) A_A E_A$

$= (5\cdot10^{-6}/°F)(100°F)(2 \text{ in}^2)(10\cdot10^6 \text{ lb/in}^2)$

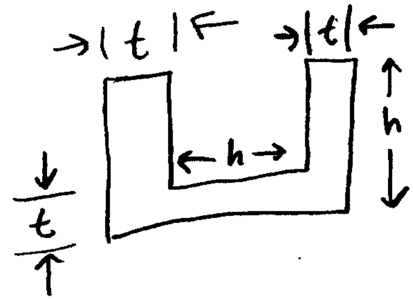
$\boxed{P = -10{,}000 \text{ lb}}$ (need to push in, not pull out)

2b (based on 5.6-16) Name: RUINA

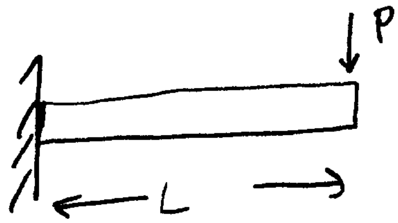
perspective



cross section



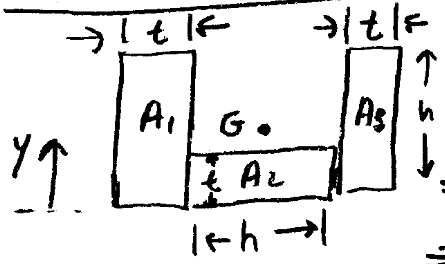
side



$L = 1$ m

$h = 10$ cm
$t = 1$ cm
$E = 30\times10^6$ lb/in^2

What is the ratio of the max tension stress to the max compression stress in this beam?

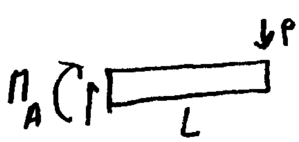


Find y_G, pos. of neutral axis.

$$y_G(A_1+A_2+A_3) = y_1A_1 + y_2A_2 + y_3A_3$$

$$\Rightarrow y_G(3ht) = \frac{h}{2}ht + \frac{t}{2}ht + \frac{h}{2}ht$$

$$\Rightarrow y_G = (2h+t)(1/2)/3 = \frac{2h+t}{6} = \frac{21}{6}\text{cm} = \frac{7}{2}\text{cm}$$

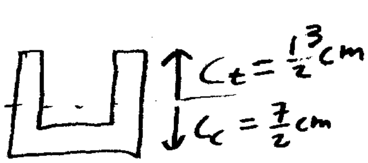


$\Rightarrow M_A = -PL$

Max tension $= Mc_t/I$

Max Comp $= Mc_c/I$

$\boxed{\text{ratio} = \frac{13}{7} \approx 1.86}$



$$\frac{\sigma_{t,max}}{\sigma_{c,max}} = \frac{13}{7}$$