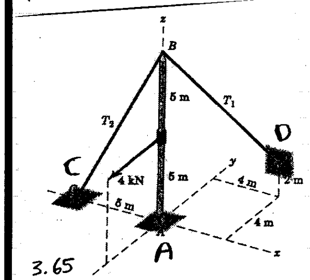
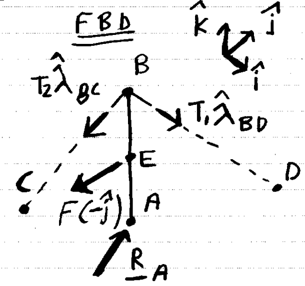
TAM 202, FALL 2000
HW3 due 9/17/02
"SOLUTIONS" by Andy Ruina

3.65 $\{T_1 = ?\}$
Mer. & Kraige



FBD



$F = 4\,kN$

$\hat{\lambda}_{BD}$ = unit vector pointing from B to D

$= \underline{r}_{BD}/|\underline{r}_{BD}|$ [IRONY: Unit vectors have no units.]

$= (4\hat{i} + 4\hat{j} - 8\hat{k})/\sqrt{4^2+4^2+(-8)^2}$

$= \frac{4}{\sqrt{96}}\hat{i} + \frac{4}{\sqrt{96}}\hat{j} - \frac{8}{\sqrt{96}}\hat{k}$

$\hat{\lambda}_{BC}$ = unit vector from B to C (no need to work out)

$\sum M_{axis\,AC} = 0$ [the key equation!]

$\Rightarrow \left(\sum \underline{M}_{/A}\right)\cdot$(any vector in $\pm$AC direc.) $= 0$

$\Rightarrow \left(\sum \underline{M}_{/A}\right)\cdot\hat{i} = 0$

$\Rightarrow \left[\underline{r}_{AE}\times(-F\hat{j}) + \underline{r}_{AB}\times(T_1\hat{\lambda}_{BD})\right]\cdot\hat{i} = 0$

(note line of action of T_2 intersects axis AC, so it drops out. Like wise for the reaction at A.)

$\left[(5\cancel{m}\hat{k})\times(-F\hat{j}) + (10\cancel{m}\hat{k})\times T_1\left(\frac{4}{\sqrt{96}}\hat{i} + \frac{4}{\sqrt{96}}\hat{j} - \frac{8}{\sqrt{96}}\hat{k}\right)\right]\cdot\hat{i} = 0$

#3.65 (cont'd)

$\Rightarrow \left[5F\hat{i} + \frac{10T_1}{\sqrt{96}}(4\hat{j} - 4\hat{i})\right]\cdot\hat{i} = 0$

$\Rightarrow 5F = \frac{40}{\sqrt{96}}T_1$

$\Rightarrow T_1 = \frac{5\sqrt{96}}{40}F$

$= \frac{\sqrt{16\cdot 6}}{8}(4\,kN)$

$\boxed{T_1 = 2\sqrt{6}\,kN \approx 4.9\ kN}$ 3.65

Comments on 3.65

*) One could instead assemble 6 eqs:

$\sum \underline{F} = 0$ (3 eqs)
$\sum \underline{M}_{/A} = 0$ (3 eqs)
6 scalar equations

↑ A or any other pt.

in 5 unknowns: T_1, T_2 and 3 comps. of $\underline{R}_A$ (R_{Ax}, R_{Ay}, R_{Az}).

For this problem one of these equations is just $0=0$. That is, none of the forces contribute to ΣM_{Az} (no moments about z axis through A). You could solve the 5 eqs. in 5 unknowns and get T_1 as well as T_2, R_{Ax}, R_{Ay} & R_{Az}.

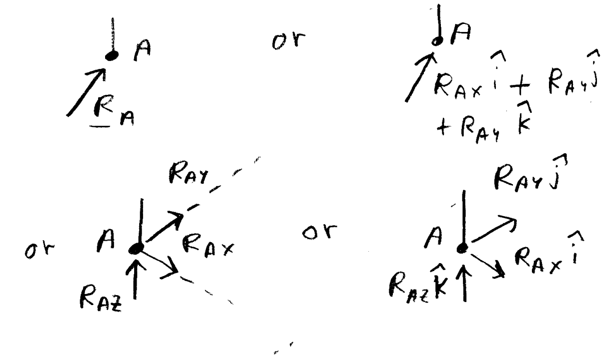
Although you had 5 eqs. in 5 unknowns, if you were alert you might have noticed that one of them, $\Sigma M_{AX} = 0$, had just one unknown, namely T_1. So you could solve it w/out ever solving the other 4 eqs.

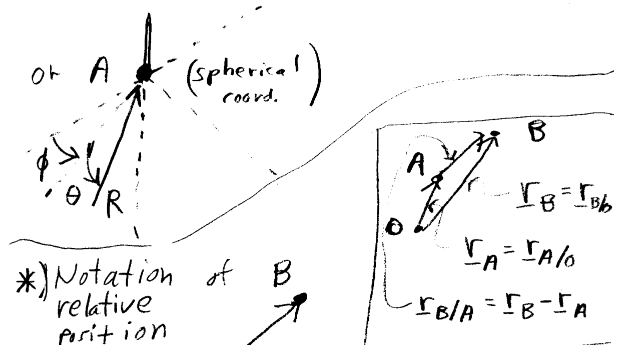
That one eqn. is equivalent to $\Sigma M_{AC} = 0$, the solution presented.

3.65 comments (cont'd)

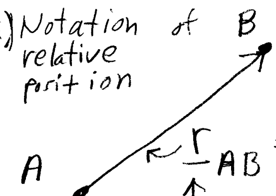
*) There are various ways to show forces on FBDs. In all cases the force should be fully defined in terms of knowns and unknowns. All of the following would be o.k. for the reaction at A

partial FBDs





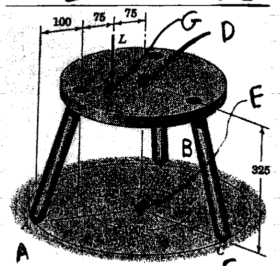
*) Notation of relative position



$\underline{r}_{AB} = \underline{r}_{B/A}$
↑ position of B relative to A
vector from A to B

3.69

$R_{Az} = ?,\quad R_{Bz} = ?,\quad R_{Cz} = ?$



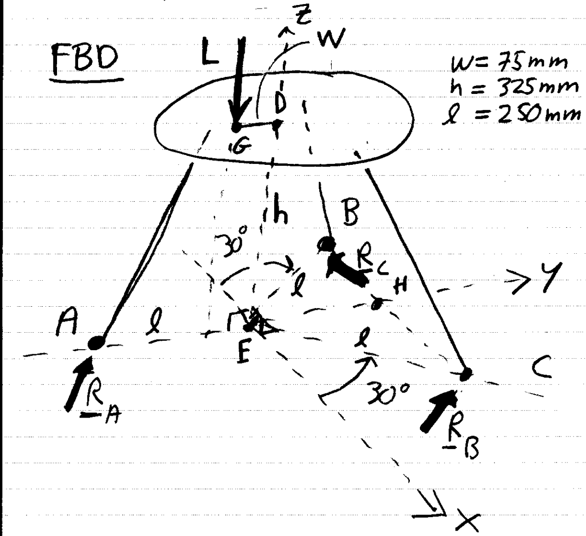
Dimensions in millimeters

Problem 3/69

Neglect weight.
No comment on friction at A, B, C!
So leave it in & see if we can get an answer.

FBD

$w = 75\,mm$
$h = 325\,mm$
$\ell = 250\,mm$



One FBD in 3D ⇒ 6 scalar eqs.
($\Sigma \underline{F} = \underline{0}$, $\Sigma \underline{M}_{/E} = \underline{0}$)
or any other pt.

We have 9 unknowns (3 comps each for $\underline{R}_A$, $\underline{R}_B$, $\underline{R}_C$ because their directions are unknown).

⇒ Problem is "statically indeterminate"! ⇒ We can't find all the unknowns. Can we find the one's we are asked for?

yes!

3.69 cont'd

$\Sigma M_{BC} = 0 \qquad (1)$

sum of moments about axis BC.

$\underline{R}_B$ & $\underline{R}_C$ go through axis and don't contribute. Likewise for R_{Ax} & R_{Ay}. So eqn. (1) is one eqn. in one unknown.

$(1) \Rightarrow \left[\underline{r}_{HG} \times (L(-\hat{k})) + \underline{r}_{HA} \times (R_{Az}\hat{k})\right] \cdot \hat{i} = 0$

Slide $-L\hat{k}$ down to x-y plane and all vectors are ⊥.

$\Rightarrow R_{Az}\,\ell(1 + \underbrace{\sin 30°}_{1/2}) = L(w + \ell\underbrace{\sin 30°}_{1/2})$

$\Rightarrow R_{Az} = \dfrac{w + \ell/2}{3\ell/2} L$

$= \dfrac{75 + 125}{3 \cdot 250/2} L$

$\boxed{R_{Az} = \frac{200}{375}L = \frac{8L}{15} \approx .53L}$

$\Sigma M_{AE} = 0 \qquad (2)$

$\Rightarrow R_{Cz} = R_{Bz}$ because they have the same lever arm $\ell\cos 30°$ about axis AE (the y axis).

We could find R_{Bz} by $\Sigma M_{AB} = 0$ but its easier to use (2) and

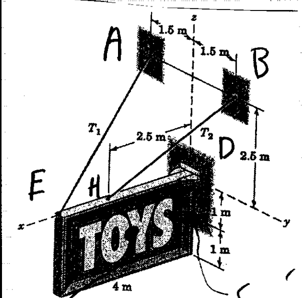
$\Sigma F_z = 0 \qquad [(\Sigma \underline{F}) \cdot \hat{k} = 0] \quad (3)$

$\Rightarrow -L + \frac{8}{15}L + 2R_{Cz} = 0$

$\Rightarrow \boxed{R_{Cz} = \frac{7}{30}L \approx .23L \Rightarrow R_{Bz} \approx .23L}$

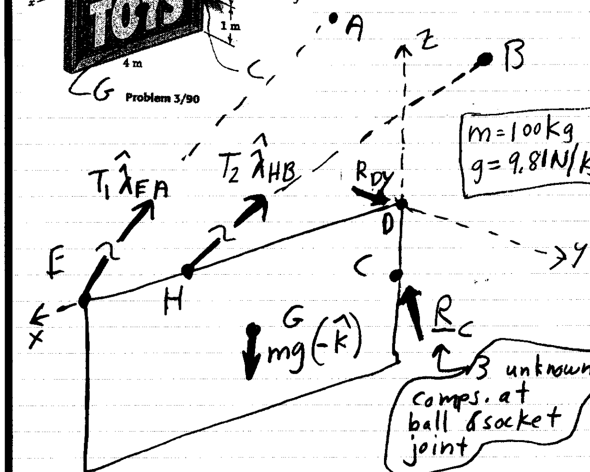
3.90

$T_1 = ?,\quad T_2 = ?,\quad R_{Dy} = ?$
$R_{Cx} = ?,\quad R_{Cy} = ?,\quad R_{Cz} = ?$



Problem 3/90

Method:
1) Draw FBD
2) Write Force & Moment Balance
3) Solve eqs.



$\Sigma \underline{F} = \underline{0} \qquad$ (3 eqs)
$\Sigma \underline{M}_{/C} = \underline{0} \qquad$ (3 eqs)
6 scalar equations

6 unknowns: T_1, T_2, R_{Dy}, R_{Cx}, R_{Cy}, R_{Cz} (don't be thrown off because the book answer only gives $R_C = |\underline{R}_C| = \sqrt{R_{Cx}^2 + R_{Cy}^2 + R_{Cz}^2}$).

Approach 1: Try to be clever.

For example $\Sigma M_{AB} = 0$ gives $\boxed{R_{Cx} = \frac{2}{3.5}mg}$

and $\Sigma M_{EH} = 0 \Rightarrow$ $\boxed{R_{cy} = 0}$

etc.

<u>Approach 2</u>: "Brute Force" (like end of lecture 9/9/02)

Write out 6 eqs. in 6 unkowns, Put in matrix form, and solve on computer.

<u>Some geometry first</u>:

$$\hat{\lambda}_{EA} = \frac{(-4\hat{i} - 1.5\hat{j} + 2.5\hat{k})}{\sqrt{4^2 + 1.5^2 + 2.5^2}}$$

$$\hat{\lambda}_{HB} = \frac{-2.5\hat{i} + 1.5\hat{j} + 2.5\hat{k}}{\sqrt{2.5^2 + 1.5^2 + 2.5^2}}$$

$\underline{r}_{DH} = 2.5\,m\,\hat{i}$

$\underline{r}_{DE} = 4\,m\,\hat{i}$

$\underline{r}_{DG} = 2m\hat{i} - 1m\hat{k}$, $\underline{r}_{DC} = -1m\,\hat{k}$

<u>Force Balance</u>

$\underline{0} = \Sigma \underline{F}$

$$\Rightarrow T_1 \hat{\lambda}_{EA} + T_2 \hat{\lambda}_{HB} + R_{Dy}\hat{j} + \underline{R}_C = mg\hat{k} \qquad (1)$$

we can take x, y, z comps (or dot w/ $\hat{i}, \hat{j}, \hat{k}$) to get 3 eqs. in 6 unknowns. Note eqn. (1) is re-arranged to put knowns on right & unknowns on left.

<u>Moment Balance</u>

$\underline{0} = \Sigma \underline{M}_{/D}$

$$\Rightarrow \underline{r}_{DE} \times (T_1 \hat{\lambda}_{EA}) + \underline{r}_{DH} \times (T_2 \hat{\lambda}_{HB})$$

$$+ \underline{r}_{DC} \times \underline{R}_C = \underline{r}_{DG} \times (mg\hat{k}) \qquad (2)$$

After carrying out cross products we can also break (2) into comps. to get 3 more eqs. for the same 6 unknowns. Fortunately the cross products are pretty sparse.

(2) $\Rightarrow$

$$4\hat{i} \times (\lambda_{EAx}\hat{i} + \lambda_{EAy}\hat{j} + \lambda_{EAz}\hat{k})T_1$$
$$+ 2.5\hat{i} \times (\lambda_{HBx}\hat{i} + \lambda_{HBy}\hat{j} + \lambda_{HBz}\hat{k})T_2$$
$$+ -\hat{k} \times (R_{cx}\hat{i} + R_{cy}\hat{j} + R_{cz}\hat{k})$$
$$= (2\hat{i} - \hat{k}) \times (mg\hat{k})$$

$$\Rightarrow (4\lambda_{EAy}\hat{k} - 4\lambda_{EAz}\hat{j})T_1$$
$$+ (2.5\lambda_{HBy}\hat{k} - 2.5\lambda_{HBz}\hat{j})T_2$$
$$- R_{cx}\hat{j} + R_{cy}\hat{i} = -2mg\hat{j} \qquad (2^*)$$

Now break (1) into comps, break (2*) into comps, & write out all 6 eqs. in an organized way.

$$\lambda_{EAx}T_1 + \lambda_{HBx}T_2 + R_{cx} \overset{(3)}{=} 0$$

$$\lambda_{EAy}T_1 + \lambda_{HBy}T_2 + R_{cy} + R_{Dy} \overset{(4)}{=} 0$$

$$\lambda_{EAz}T_1 + \lambda_{HBz}T_2 + R_{cz} \overset{(5)}{=} mg$$

$$R_{cy} \overset{(6)}{=} 0$$

$$-4\lambda_{EAz}T_1 - 2.5\lambda_{HBz}T_2 - R_{cx} \overset{(7)}{=} -2mg$$

$$4\lambda_{EAy}T_1 + 2.5\lambda_{HBy}T_2 \overset{(8)}{=} 0$$

Now, eqs 3-8 (the comps. of force & moment balance) are 6 eqs in 6 unknowns.

One could fudge through, but

lets continue in "Brute Force" style. Eqns. 3-8 can be written in matrix form as

$$[A][X] = [Y]$$

with

$$[A] = \begin{array}{c} (3) \\ (4) \\ (5) \\ (6) \\ (7) \\ (8) \\ \\ \end{array}
\begin{array}{c}
\begin{bmatrix}
\lambda_{EAx} & \lambda_{HBx} & 1 & 0 & 0 & 0 \\
\lambda_{EAy} & \lambda_{HBy} & 0 & 1 & 0 & 1 \\
\lambda_{EAz} & \lambda_{HBz} & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
-4\lambda_{EAz} & -2.5\lambda_{HBz} & -1 & 0 & 0 & 0 \\
4\lambda_{EAy} & 2.5\lambda_{HBy} & 0 & 0 & 0 & 0
\end{bmatrix} \\
\begin{array}{cccccc} T_1 & T_2 & R_{cx} & R_{cy} & R_{cz} & R_{Dy} \end{array}
\end{array}$$

$$[X] = \begin{bmatrix} T_1 \\ T_2 \\ R_{cx} \\ R_{cy} \\ R_{cz} \\ R_{Dy} \end{bmatrix} \qquad [Y] = \begin{bmatrix} 0 \\ 0 \\ mg \\ 0 \\ -2mg \\ 0 \end{bmatrix}$$

We can now feed this to a calculator or computer to get a soln.

The following soln. uses MATLAB. The key is the backslash (\) command. Given a matrix A & a column vector Y

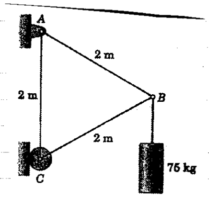
$$X = A \backslash Y$$

finds the col. vector X that solves $AX = Y$ (caveats in Math 294!)

```
% Meriam and Kraige 3.90
% "Solution" by Andy Ruina on 9/11/02
% This is more expansive than need be.

% Using consistent units (meters, kg, Newtons),
% so they don't appear in the calculations below.

m = 100;    g = 9.81;    %let's keep the numbers more round

% Geometry first,  position vectors:
rEA =  [-4   -1.5  2.5];  %position of A relative to E
rHB =  [-2.5  1.5  2.5];  %position of B relative to H

% unit vectors:
lamEA = rEA/norm(rEA); % a unit vector in EA direction
lamHB = rHB/norm(rHB); % a unit vector in HB direction

% define the components of the unit vectors:
lamEAx = lamEA(1); lamEAy = lamEA(2); lamEAz = lamEA(3);
lamHBx = lamHB(1); lamHBy = lamHB(2); lamHBz = lamHB(3);

% set up the matrix A (called [A] in  solution handwork):
A = [  lamEAx        lamHBx      1   0   0   0
       lamEAy        lamHBy      0   1   0   1
       lamEAz        lamHBz      0   0   1   0
         0             0         0   1   0   0
      -4*lamEAz   -2.5*lamHBz   -1   0   0   0
       4*lamEAy    2.5*lamHBy    0   0   0   0];

% Now set up the right hand side, [y]:
y =  [ 0   0 m*g   0  -2*m*g    0]'; % note: ' means transpose

% Now solve the equations. Use the great backslash command:
% All the trouble above, the hand vector calculations and
% the computer work, was to use this one line of code:
x = A\y;

% Now unpack the [x] column vector to our variables:
T1   = x(1)
T2   = x(2)
RCx  = x(3)
RCy  = x(4)
RCz  = x(5)
RDy  = x(6)
RC = norm([RCx RCy RCz])

%End of m file
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
T1 =   346.84 N
T2 =   430.58 N

RCx =  560.57 N
RCy =    0    N
RCz =  525.54 N

RDy =  -63.06 N

RC =   768.39 N
```

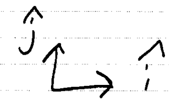
This is the output from the .m file above. Edited in a text editor to save space.

Answers to 3.90

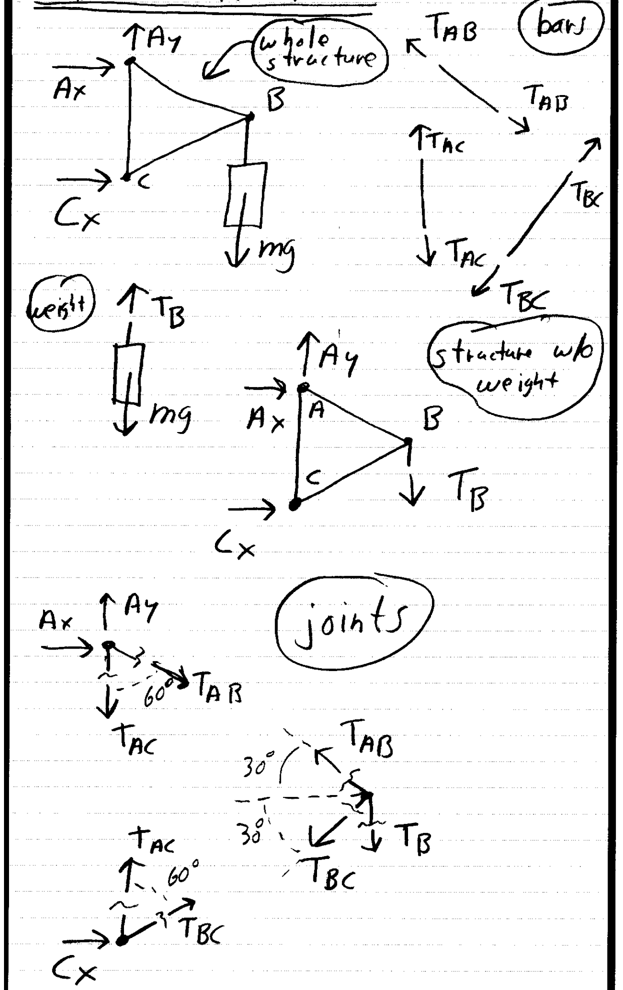


Problem 4/1

Find all tensions.



FBDs GALORE:



FBD of Weight, $\sum F_y = 0 \Rightarrow T_B = mg$.

FBD of joint B

$\sum \underline{F} = \underline{0} \Rightarrow$

(*)

$$\{ T_{AB}(-\cos 30^\circ \hat{i} + \sin 30^\circ \hat{j}) + T_{BC}(-\cos 30^\circ \hat{i} - \sin 30^\circ \hat{j}) - mg\hat{j} = \underline{0} \}$$

$\{\ \}\cdot\hat{i}$ (that is, take x-comp of force balance eqn.)

$$\Rightarrow T_{AB} = -T_{BC} \quad (1)$$

$\{\ \}\cdot\hat{j}$ (take y comp. of *)

$$\Rightarrow T_{AB}\sin(30^\circ) - T_{BC}\sin(30^\circ) - mg = 0 \quad (2)$$

$$(1) \,\&\, (2) \Rightarrow T_{AB}\left(\tfrac{1}{2}\right) + T_{AB}\,\tfrac{1}{2} = mg$$

$$\Rightarrow \boxed{T_{AB} = mg = 75\,kg \cdot 9.81\,N/kg,\quad T_{AB} \approx 736\,N}$$

$$\boxed{T_{BC} = -T_{AB} \approx -736N}$$

"Tension in BC is −736N, comp. is 736N."

FBD of joint C, $\Sigma F_y = (\Sigma \underline{F}) \cdot \hat{j} = 0$

$\Rightarrow \quad T_{AC} + T_{BC} \cos 60^\circ = 0$

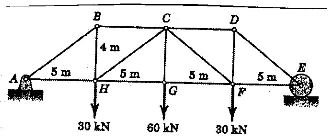
$\Rightarrow \quad T_{AC} = -T_{BC} \cos 60^\circ$

$= -(-736N) \frac{\sqrt{3}}{2}$

$\boxed{T_{AC} \approx 637N}$

"tension in AC is 637N"

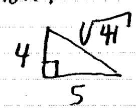
4.7 Find all "bar forces".



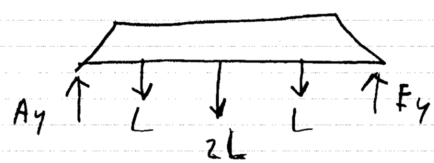
Problem 4/7

$L = 30kN$

note:



FBD of structure

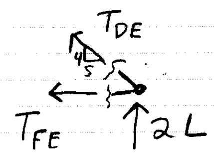


$\Sigma F_y = 0$ & symmetry

$\Rightarrow A_y = E_y = 2L$

Now we start at joint E & work our way down the structure.

joint E:



$\Sigma F_y = 0 \Rightarrow 2L + T_{DE} \frac{4}{\sqrt{41}} = 0$

$\boxed{T_{DE} = \frac{-\sqrt{41}\, L}{2} = \frac{-\sqrt{41}\, 30kN}{2} \approx -96\,kN}$

also T_{AB} by symmetry

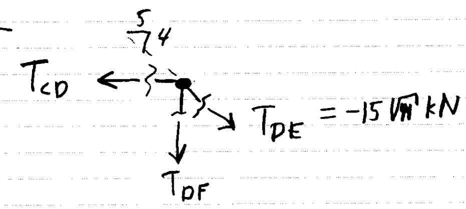
("tension in DE is $-96\,kN$, compression is $96\,kN$")

$\Sigma F_x = 0 \Rightarrow -T_{DE} \frac{5}{\sqrt{41}} - T_{FE} = 0$

$\Rightarrow \boxed{T_{FE} = \frac{-5\,T_{DE}}{\sqrt{41}} = \frac{150}{2}kN = 75\,kN}$

$(= T_{AH}$ by symmetry

joint D



$\Sigma F_x = 0 \Rightarrow -T_{CD} + T_{DE} \frac{5}{\sqrt{41}} = 0$

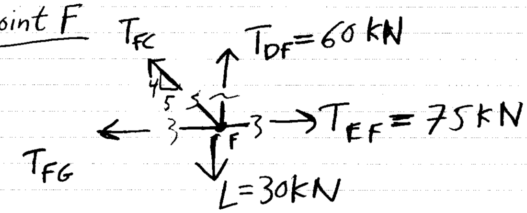
$\boxed{T_{CD} = \frac{5}{\sqrt{41}}\left(-15\sqrt{41}\,kN\right) = -75\,kN}$

Symmetry $\Rightarrow T_{BC} = -75kN$

$\Sigma F_y = 0 \Rightarrow -T_{DF} - T_{DE} \frac{4}{\sqrt{41}} = 0$

$\Rightarrow \boxed{T_{DF} = -\frac{4}{\sqrt{41}}\left(-15\sqrt{41}\,kN\right) = 60\,kN \quad (T_{BH} = 60kN)}$

joint F



$\Sigma F_y = 0 \Rightarrow$

$T_{FC} \frac{4}{\sqrt{41}} + T_{DF} - L = 0$

$\boxed{T_{FC} = \frac{\sqrt{41}}{4}\left[-60\,kN + 30\,kN\right] \approx -48\,kN \quad (T_{CH} \approx -48kN)}$

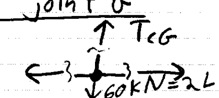
$\Sigma F_x = 0 \Rightarrow$

$T_{EF} - T_{FC} \frac{5}{\sqrt{41}} - T_{FG} = 0$

$\Rightarrow \boxed{T_{FG} = T_{EF} - \frac{5}{\sqrt{41}} T_{FC} = 75kN - \frac{5}{\sqrt{41}}(-48\,kN) \approx 112.5\,kN}$

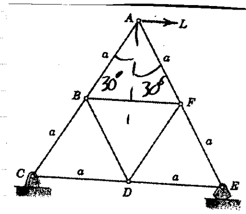
(symmetry $\Rightarrow T_{HG} \approx 112.5kN$)

Finally joint G



$\Sigma F_y = 0 \Rightarrow \boxed{T_{CG} = 60kN} = 2L$

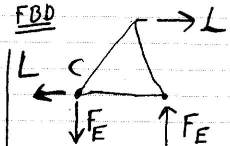
#4.15



Problem 4/15

Find all bar forces.

FBD



$\sum M_{/C} = 0 \Rightarrow F_E = \frac{\sqrt{3}}{2} L$

joint A

$\sum F_y = 0 \Rightarrow T_{AF} = T_{AB}$

$\sum F_x = 0$

T_{AB}, T_{AF} $\Rightarrow$

$-\frac{T_{AB}}{2} + \frac{T_{AF}}{2} + L = 0$

$\Rightarrow \boxed{T_{AF} = -L,\ T_{AB} = L}$

joint E

T_{EF}, T_{DE}, $\frac{\sqrt{3}}{2} L$

$\sum F_y = 0 \Rightarrow T_{EF} \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} L = 0$

$\Rightarrow \boxed{T_{EF} = -L}$

$\sum F_x = 0 \Rightarrow \boxed{T_{DE} = \frac{L}{2}}$

joint F

$-L$, F, T_{BF}, $-L$, T_{FD}

$\sum F_x = 0, \sum F_y = 0 \Rightarrow$

$\boxed{T_{BF} = T_{FD} = 0}$

joint D

T_{DB}, $T_{DF} = 0$, $T_{DE} = \frac{L}{2}$, T_{CD}

$\Rightarrow \boxed{T_{DB} = 0}$ (zero force member)

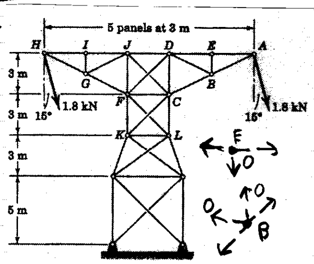
$\Rightarrow \boxed{T_{CD} = L/2}$

joint B

$T_{AB} = L$, 0, 0, T_{BC} $\Rightarrow \boxed{T_{BC} = L}$

4.27

!



E: 0, 0

B: 0, 0

joint E

$\Rightarrow T_{EB} = 0$

joint B

$\Rightarrow \boxed{T_{DB} = 0}$

"Zero force members"

That's all folks.