"SOLUTIONS"

Your Name: RUINA, BURNS

Section day & time: ALL

TA name & section #: ____________

ENGRD 202 Final Exam

Tuesday Dec. 17, 2002, 9:00 AM — 11:30 AM

This version last edited December 15, 2002.

5 problems, 100 points, and 150 minutes (no over time).

Please follow these directions to ease grading and to maximize your score.

a) No calculators, books or notes allowed besides the one-sided formula sheet which is being passed out with this exam. A blank page for tentative scrap work is provided at the back. Ask for extra scrap paper if you need it.

b) Full credit if

- →free body diagrams← are drawn whenever force or moment balance is used;
- correct vector notation is used, when appropriate;
- ↑→ any dimensions, coordinates, variables and base vectors that you add are clearly defined;
- ± all signs and directions are well-defined with sketches and/or words;
- →| reasonable justification, enough to distinguish an informed answer from a guess, is given;
- you clearly state any reasonable assumptions if a problem seems **poorly defined**;
- work is I.) neat,
  II.) clear, and
  III.) well organized;

- your answers are TIDILY REDUCED (Don't leave simplifiable algebraic expressions.);
- □ your answers are $\boxed{\text{boxed}}$ in; and
- ≫ unless otherwise stated, you will get full credit for, instead of doing a calculation, presenting Matlab code that would generate the desired answer. To ease grading and save space, your Matlab code can use shortcut notation like "R_{Ax} = 18" instead of, say, "RAX = 18".

c) Substantial partial credit if your answer is in terms of well defined variables and you have not substituted in the numerical values. Substantial partial credit if you reduce the problem to a clearly defined set of equations to solve.

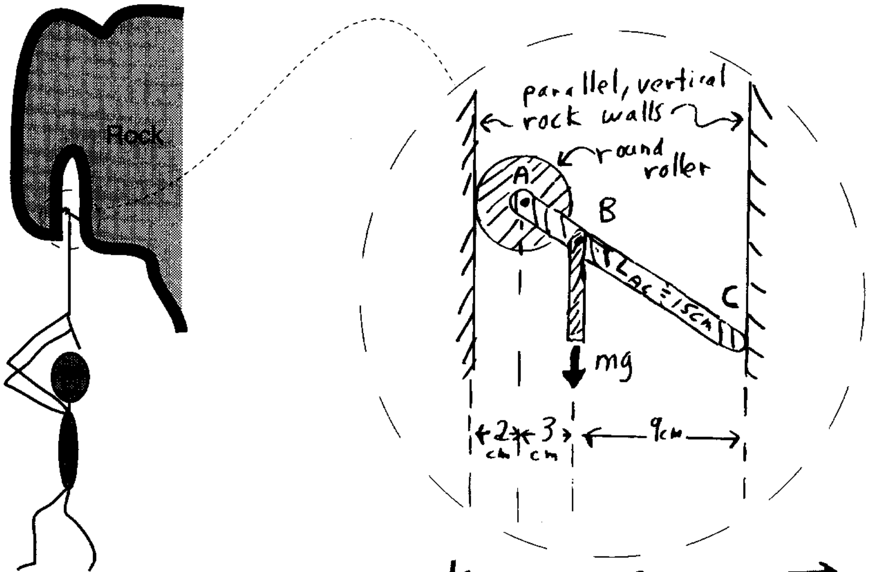
hopefully!

Prob. 6 was not given.
Included w/ solns as a free bonus!

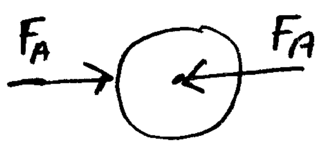
Problem 1:	20 /20
Problem 2:	20 /20
Problem 3:	20 /20
Problem 4:	20 /20
Problem 5:	20 /20
TOTAL:	100 **/100**

1) **(20 pts)** A candidate rock-climbing device consists of a roller (radius 2 cm) frictionlessly pinned at A to diagonal-member AC. The length of AC from point A to the wall-contact point at C is L_{AC}=15 cm. The climber (m = 60 kg) hangs from a rope connected to AC by a pin at B. B is on the line AC and located as shown in the figure. If needed, assume g =10 m/s^2.

What is the minimum coefficient of friction μ at C that is needed to hold up the climber?



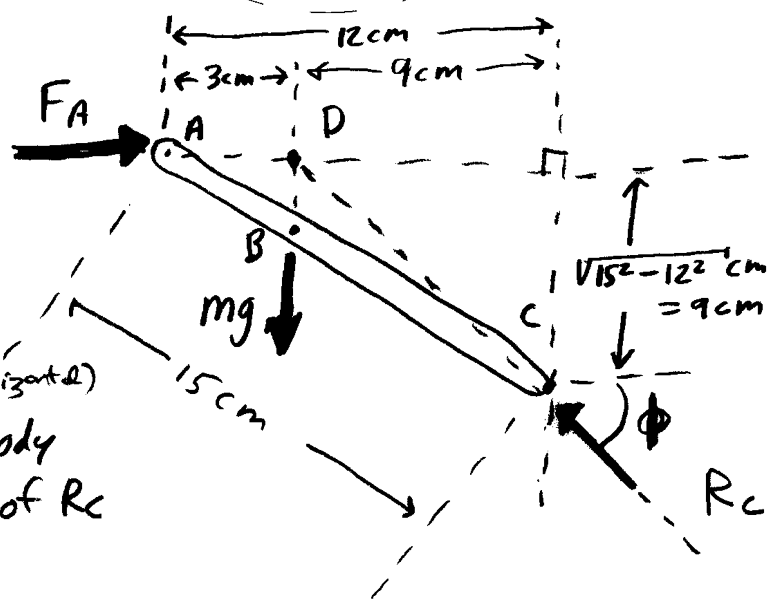
FBDs:



roller

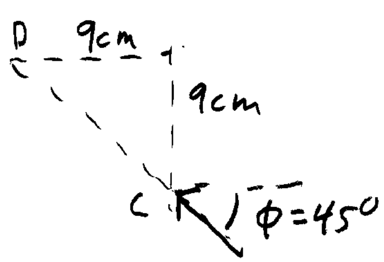
A "2-force" body

(so regardless of friction, wall force is horizontal)

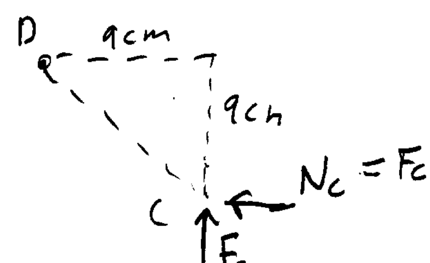


AC is a "three-force" body

($\sum M_{/D}=0 \Rightarrow$ line of action of R_c passes through D)



$\Rightarrow$



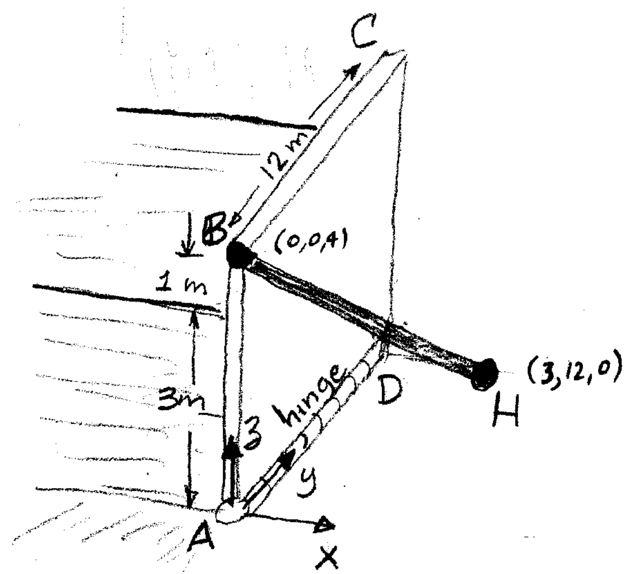
$F_c = N_c \Rightarrow \boxed{\boxed{\mu \geq 1}}$ is needed

2) **(20 pts).** A 4-meter-high "door" holds back a stream (mass density ρ) that is 3 meters deep and 12 meters wide. The door is hinged along its bottom and is propped up by a thin rod BH (cross-sectional area A) that goes from a ball joint at H [located at (3, 12, 0)] to another ball joint at the upper left corner B [at (0, 0, 4)] of the door.

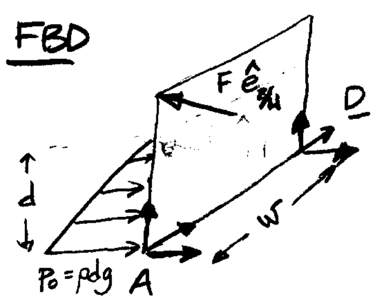
a) **(5 pts).** Draw a <u>free body diagram</u> of the door. Ignore the masses of the door and the rod.

b) **(12 pts).** Compute the <u>axial force</u> in the rod BH. Choose $\rho = 10^3$ kg/m^3, $g = 10$ m/s^2.

c) **(3 pts).** Express the <u>change in length</u> of B[redacted]in terms of any or all of A, ρ, g, E, G. Is the rod lengthened or shortened?



ⓐ Let's consider the fluid depth as d (=3m) and the width of the stream w(=12m).



The hydrostatic pressure $P_0 = \rho g d$ acts across the entire width of the door and so is equivalent to a loading $q = \rho g d w$

To compute reactions, the hydrostatic load can be considered to act at the centroid

$L = \sqrt{3^2+4^2+12^2} = 13$

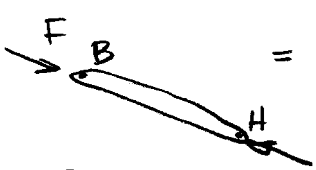
$\boxed{\hat{e}_{B/H} = -\frac{3}{13}\hat{i} - \frac{12}{13}\hat{j} + \frac{4}{13}\hat{k}}$

$\frac{1}{2}(\rho g w d^2)$ $d/3$

[None of forces acting at hinge appear. Only rod force & hydrostatic

ⓑ Statics requires $\sum M_{hinge} = \sum \underline{M}_{/A} \cdot \hat{j} = 0$

$$\sum M_{A_y} = \frac{1}{2}\rho g w d^2\left(\frac{d}{3}\right) + \underbrace{\left[(d+1)\hat{k} \times F\hat{e}\right]\cdot\hat{j}}_{(r \times F\hat{e})\cdot\hat{j}} = \frac{\rho g w d^3}{6} - (d+1)F\frac{3}{\sqrt{3^2+4^2+12^2}}$$



$$= \frac{\rho g (12)3^3}{\cancel{6}\,2} - \frac{F(4)(3)}{13} = 0 \Rightarrow 9\cancel{(6)}\rho g \cdot \frac{13}{\cancel{12}\,2} = F = \frac{117}{2}\rho g \ m^3$$

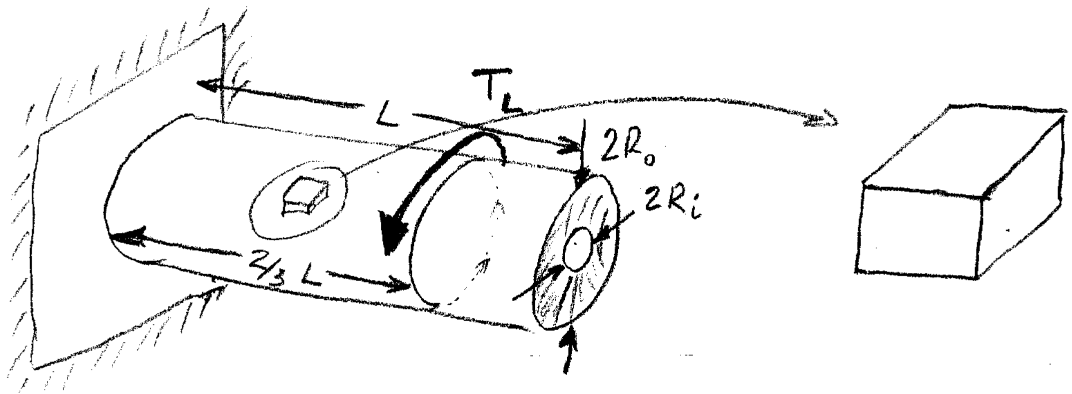
$\therefore F = 58.5 \ 10\cdot10^3$

$\boxed{= 5.85\times10^5 \text{ N}}$

ⓒ $\delta = -\frac{FL}{AE} = -\frac{117(m^3)\rho g L}{[redacted]AE}$

It's a compressive force $\therefore$ shorter

3) (20 pts). A hollow circular tube (outer radius R_o and inner radius R_i) of total length L is rigidly attached to the left-hand wall (Assume you also know the mass M, shear modulus G, Young's modulus E, and Poisson ratio ν.). Neglect gravity throughout. A person grasps the tube two-thirds of the way from the left end, at that point applying an axial torque T_L (torsion not bending).



a) **(2 pts).** Sketch a free body diagram of the tube when it is twisted with torque T_L.

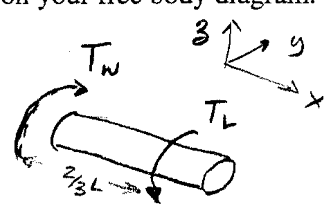
b) **(4 pts).** Compute the maximum shear stress in the tube and show (or describe) the locations at which it occurs.

c) **(5 pts).** On a sketch draw all the stresses (normal and shear) that act on an exterior surface element between the hand and the wall. That is, on a cube (or element; see figure) representing a small piece of material at the outside edge of the tube, draw all stresses that act.

d) **(4 pts).** Taking into account that elements may have innumerable different orientations, what is the maximum tension on any surface in the tube? Where does it occur?

e) **(5 pts).** The person gets tired. So, after worrying about the issues above, the right end is rigidly glued to another wall on the right side and then the torque from the hands is released. Draw a free body diagram of the tube after it has been released by the hands (but is still attached to the walls on both ends). Calculate the value of any torques that appear on your free body diagram.

a)



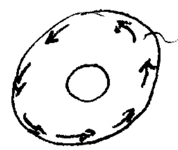
$\Sigma M_x = 0 \Rightarrow \boxed{T_W = T_L}$

Prob #3 (SOLN) FALL 2002 FINAL

(b)

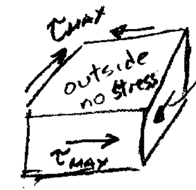
$$\tau = \frac{T\rho}{I_p}$$ where I_p = polar moment of inertia $= \frac{\pi}{2}(R_o^4 - R_i^4)$

$$\boxed{\tau_{MAX} = \frac{2 T_L R_o}{\pi (R_o^4 - R_i^4)}}$$

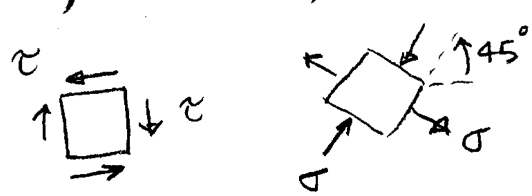


τ_{MAX} occurs at the largest radius (i.e., at outside radius). Since T acts throughout the section between the wall and application point, τ_{MAX} true too

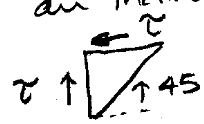
(c) We look at an exterior element



(d) Even though, as seen in c) above, the exterior element is in pure shear, if rotated, the element will experience a normal stress



By considering stress on an inclined surface

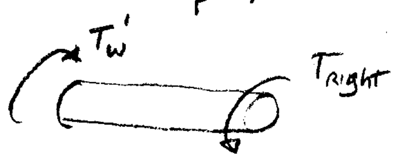


we find $\boxed{\sigma = \tau}$

This max σ occurs with max τ (i.e., on outer elements all the way between the wall and hand) but now on elements at 45° to axial direction

(e) When the original torque is applied, the Right-hand end is rotated through

$$\phi = \frac{T_L(\frac{2}{3}L)}{I_p G}$$. This same angular displacement occurs in the final configuration



$\Sigma M_x = 0 \quad \therefore \quad T_R = T_W'$. This torque causes same angular displacement

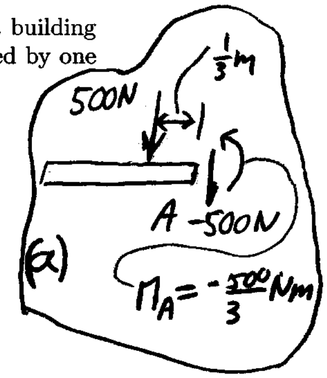
$$\frac{T_L(\frac{2}{3}L)}{I_p G} = \frac{T_R L}{I_p G} \implies \boxed{T_R = \frac{2}{3} T_L}$$

4) **(20 pts)** A snow loaded bus-stop awning (shown partially cut away at right) on the side of a building is supported by horizontal, cantilevered, inverted-T beams. The loading that is carried by one beam is as shown below.

a) Find the reaction force and couple at the wall at A (the force and moment acting on one beam from the wall).

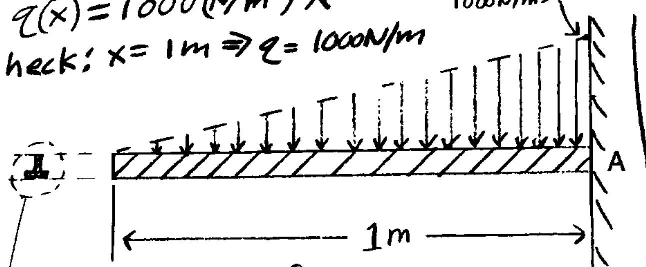
b) Draw shear and bending moment diagrams on the axes below. Clearly mark the values of V and M at $x = 0$, $x = 0.5$m, and $x = 1$m.

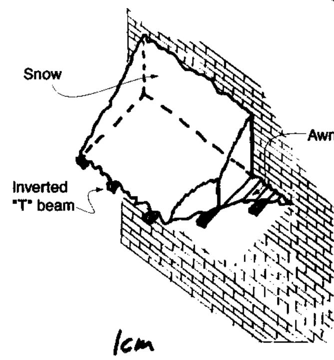
c) On the $\sigma - y$ axes below, graph the tension stress in the beam at A (but just far enough from the wall that end-effects can be neglected) as a function of vertical position in the beam. Clearly show where $\sigma = 0$ and the values of σ at the top and bottom of the beam.



$q(x) = 1000\,(N/m^2)\,x$

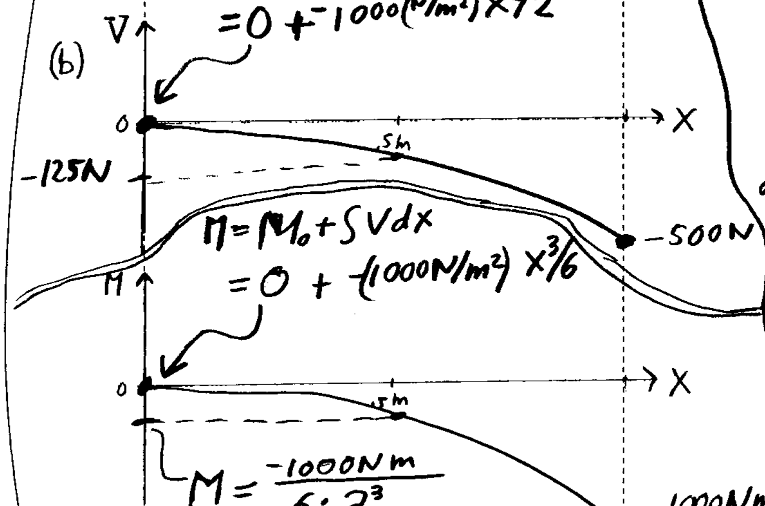
check: $x = 1m \Rightarrow q = 1000N/m$





$V = V_0 \mp \int q\,dx$
$= 0 + -1000(N/m^2)\,x^2/2$

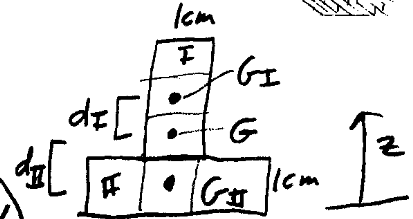
$M = M_0 + \int V\,dx$
$= 0 + -(1000N/m^2)\,x^3/6$



$M = \frac{-1000Nm}{6 \cdot 2^3} = \frac{-125}{6}Nm$

$\frac{-1000Nm}{6}$

$M = \frac{-500}{3}Nm$



$\bar{z} = (z_I A_I + z_{II} A_{II}) / (A_I + A_{II})$

$= \frac{.5cm \cdot 3cm^3 + 2.5cm\ 3cm^2}{6cm^2}$

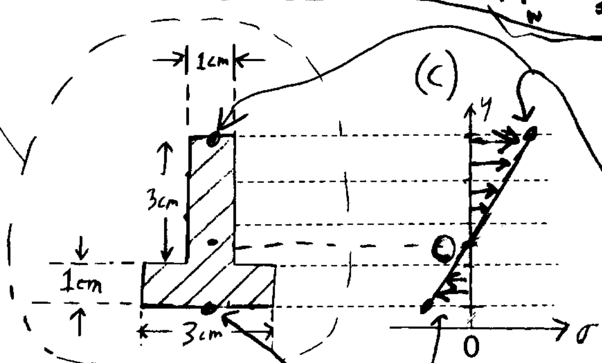
$= 1.5cm$

$\Rightarrow d_I = d_{II} = 1cm$

$I = (I_1 + d_1^2 A_1) + (I_2 + d_2^2 A_2)$

$= \left[\left(\frac{3^3}{12} + 3\right) + \left(\frac{3}{12} + 3\right)\right] cm^4$

$= \left(6 + \frac{10}{4}\right) cm^4 = \frac{17}{2} cm^4$



$\sigma = \frac{-My}{I}$

$= \frac{+500Nm}{3 \cdot \frac{17}{2} cm^4} \left(\frac{100cm}{m}\right)^4 \cdot \begin{cases} .025m \text{ (top)} \\ -.015m \text{ (bottm)} \end{cases}$

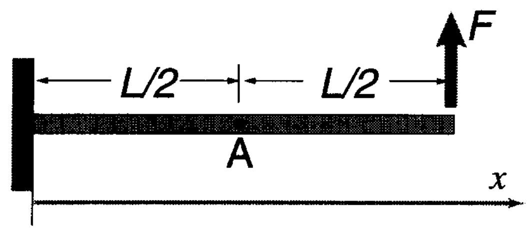
$= \boxed{\frac{\cancel{4}25}{51} \cdot 10^8 N/m^2}$

$\boxed{\frac{-15}{51} \cdot 10^8 N/m^2}$

5a) (10 pts) The uniform cantilever beam shown below is built into a rigid wall at its left end and has a vertical force F at its right end. Assume the load is small enough that the beam is linear elastic. Neglect gravity. You can take as given that

$$V(x) = -F \qquad \text{and} \qquad M(x) = F \cdot (L - x).$$

Find the deflection of the midpoint A of the beam in terms of some or all of $F, L, E,$ and I.



$$EI v'' = M(x)$$
$$= FL - Fx$$

$$\Rightarrow EI v' = EI v'(0) + FLx - Fx^2/2$$

$v'(0) = 0$ Built into wall

$$= FLx - Fx^2/2$$

$$\Rightarrow EI v = EI v(0) + FLx^2/2 - Fx^3/6$$

$v(0) = 0$ Built into wall

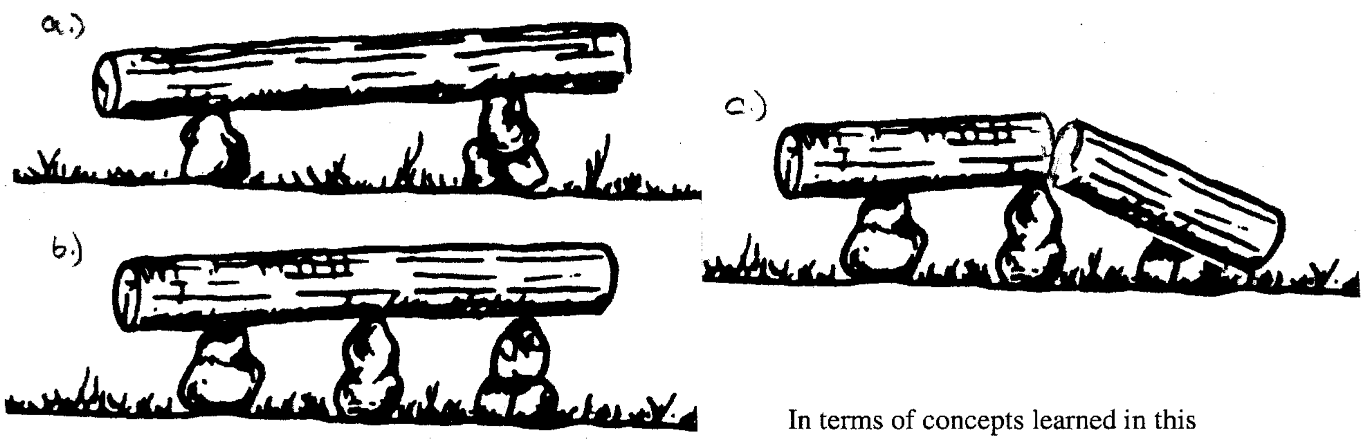
$$= FLx^2/2 - Fx^3/6$$

$$EI v(L/2) = FL\left(\tfrac{L}{2}\right)^2/2 - F\left(\tfrac{L}{2}\right)^3/6$$
$$= FL^3\left(\tfrac{1}{8} - \tfrac{1}{48}\right)$$
$$= \frac{5FL^3}{48}$$

$\left(1/8 = \frac{6}{48}\right)$

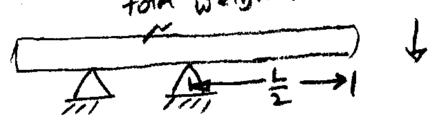
$$\boxed{v\left(\tfrac{L}{2}\right) = \frac{5FL^3}{48EI}}$$

5b) (10 pts). In *Dialogues Concerning Two New Sciences* Galileo recounts the following experience. Marble columns destined for a cathedral in Rome were stored lying on their sides. Because the ground was soft and muddy in spots, during construction of the cathedral the columns were held off the ground by a.) rock supports one-quarter of the way in from their ends. One or the other of the supports sometimes settled a bit into the mud but the columns generally remained on their supports, whether they were on mud or firm rock. Since marble was precious, the Renaissance designers became cautious and began b.) to prop up the columns in their centers as well [see Figure]. Shortly thereafter, some columns were found to c.) have fractured right above the central support. *Hint*: Consider the column's loading when an end support sinks in the mud.



In terms of concepts learned in this course, why might columns more readily break when they have three supports rather than two? Some simple calculations (or bending moment diagrams) are required.

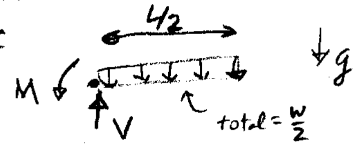
When an end sinks in the mud, the column is supported like



distance to CG of RH half

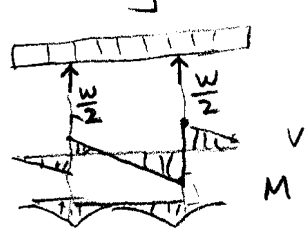
Essentially the central support takes all the load (the column's weight W)

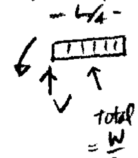
We have for RH half



$\boxed{M_{MAX} = \left(\frac{W}{2}\right)\left(\frac{L}{4}\right) = \frac{WL}{8}}$

Previously the bending moment was max at the supports (which are at $\frac{L}{4}$ in from end)





$\boxed{M_{max} = \frac{W}{4} \cdot \frac{L}{8} = \frac{WL}{32}}$

Max bending stress with "3 supports" = $\boxed{4 \times \text{ bigger}}$

6) **Everyone knows that steel is stronger and stiffer than balsa wood.** For student projects structures are often built of balsa wood rather than with a stronger engineering material such as steel. A student team makes a solid square-cross-section cantilever beam (clamped at one end, loaded transversely at the other) and finds that its failure load is 100 lb. They then plan to take an **equal length** and **equal mass** square cross section steel beam and load it.

not given

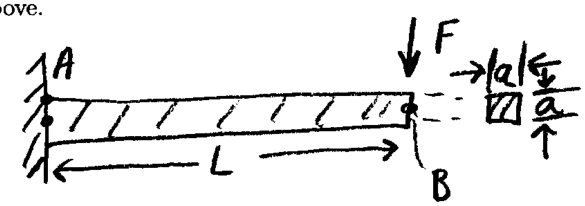
Some approximately-correct assumptions about the material properties that you can use:
- The failure stress of mild steel in tension is 100 times that of balsa wood.
- The "Young's modulus" (the elastic modulus in tension) of steel is 100 times that of balsa.
- The density (mass per unit volume) of steel is 36 times that of balsa wood.

not given

a) What do you predict as the failure load of the steel beam.
Make the standard strength-of-materials assumptions for both beams.

b) Which beam is stiffer? What is the ratio of the stiffnesses?
(stiffness = force/deflection)

c) In words, give a qualitative explanation for the solutions you found, or should have found, above.



$$\left.\begin{array}{l} V_{ol} = L a^2 \\ m = \rho V_{ol} \end{array}\right] \Rightarrow a = \sqrt{\frac{m}{\rho L}}$$

At failure:

$$\sigma_y = \sigma_A = \frac{-My}{I} = \frac{(F \cdot L)(a/2)}{a^4/12} \Rightarrow F_{max} = \frac{\sigma_y a^3}{6L} = \frac{\sigma_y \left(\frac{m}{\rho}\right)^{3/2}}{6L^{5/2}}$$

$$\Rightarrow F_{max} = \frac{\sigma_y m^{3/2}}{6L^{5/2}} \rho^{-3/2}$$

$$\boxed{\begin{array}{l} m_b = m_s \\ L_b = L_s \end{array}}$$

$$\Rightarrow \frac{F_{max}\ \text{steel}}{F_{max}\ \text{balsa}} = \frac{\sigma_{ys}}{\sigma_{yb}} \left(\frac{\rho_b}{\rho_s}\right)^{3/2} = (100)\left(\frac{1}{36}\right)^{3/2} = \frac{100}{216} \Rightarrow \boxed{216\,\text{lb}}$$

$\Rightarrow$ balsa can carry 2.16 what steel can carry!

$$\frac{\text{Stiffness balsa}}{\text{stiffess steel}} = \frac{E_b I_b / L^3}{E_s I_s / L^3} = \left(\frac{E_b}{E_s}\right)\left(\frac{a_b}{a_s}\right)^4 = \frac{E_b}{E_s}\left(\frac{\rho_s}{\rho_b}\right)^2$$

$$= \frac{1}{100}(36)^2 \approx \frac{1500}{100} = 15$$

$\boxed{\text{Balsa is 15 times as stiff as steel}}$

c) Even though $\sigma_{ys} >> \sigma_{yb}$ and $E_s >> E_b$ and even $\frac{\sigma_{ys}}{\rho_s} > \frac{\sigma_{yb}}{\rho_b}$ and $\frac{E_s}{\rho_s} > \frac{E_b}{\rho_b}$, bending (stiffness & strength) depend on I and because balsa is less dense it has much bigger I for given m.