ENGINEERING 202

LABORATORY MANUAL

STATICS AND STRENGTH OF MATERIALS

(READ BEFORE ATTENDING LAB.)

FALL 2002 , SPRING 2003

This manual has evolved over the years. Contributors in the past two decades include: Kenneth Bhalla, Jason Cortell, Jill Evensizer, Richard Lance, Jamie Manos, Dan Mittler, Francis Moon, Leigh Phoenix, James Rice, Andy Ruina, and Alan Zehnder.

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THEORETICAL AND APPLIED MECHANICS 202
STATICS AND STRENGTH OF MATERIALS

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INTRODUCTION

PURPOSE

These laboratories are designed to complement the lectures, text, and homework. They should help you gain a physical feel for some of the basic concepts in statics and strengths of solids: force, stress, deflection, strain, yield, failure and buckling. You will also get exposure to equipment which you may use in the future. Some mathematics from MATH 192 and 293 will be used, helping you make the tie between mathematics and physical reality that is essential to most engineering. The labs may come either before or after you cover the relevant material in lecture. Thus, they can be either a motivation for the lecture material or an application of what you have learned, depending on the timing.

Lab groups are small enough (1 or 2 people) that you can get direct experience with the instruments and equipment. Both you and your lab partner should learn how to do all aspects of the lab. The laboratory teaching assistants will have scheduled office hours in the laboratories so that you can return to use the facilities independently, or ask questions. These hours will be posted on the door of the laboratory. You may also ask the lab TA about other course material if time allows.

CONTENT

There are four labs and one assignment which require write-ups:

0) Error Analysis (handed in with the Lab 1 report)
1) Truss
2) Tension Test
3) Beam
4) Compression

It is essential that you read through the lab (especially the procedure section) and answer the pre-lab questions *before* coming to lab. The reading for Lab 1 is quite long and may look a bit intimidating, but the rest are short. In addition to the regular labs, there are some demonstrations set up for you to experiment with, if enough time is available. Some of the labs may also include extra topics for you to research, or ask you to develop a small experiment of your own.

LOCATION

The laboratories are in Thurston 101A.

SCHEDULING

Each of the four labs is taught for two weeks. You will be scheduled to attend lab during one week of the two. The meeting dates for your laboratory section will be posted in the hallway around the corner from room 101A Thurston, during the first week of classes. In general, you will have a lab once every three weeks, but be aware that this may vary due to exam and break schedules. (Summer session lab schedules will differ.) See the Secretary in Kimball 212 if you have problems with your lab schedule. You'll need to get her approval for any changes, so that the lab sections do not become overly full. Turning in a course change form to the registrar is not enough.

TURNING IN YOUR LAB REPORT

Lab reports are due one week from the day you performed the lab, at 8:00 in the morning, unless your TA specifies another time.

Turn in reports to the boxes in the Don Conway room on the first floor of Thurston Hall. Be sure to put your report in the correct box corresponding to the TA in charge of your <u>lab</u> section.

MISSED LAB AND LATE REPORT POLICY

All make-up labs must be arranged with your TA and the Secretary in 212 Kimball Hall. If you know in advance that you'll be gone, you should sign up with her at least one week <u>prior</u> to the scheduled lab. This gives you a better chance to sign up for a convenient time, and there's no point penalty. If you miss your lab without arranging a make-up lab in advance, you should still try to arrange a new time. However, a one-point lab report penalty will be imposed, unless you were ill, etc. You should arrange to make up a missed lab as soon as possible, since the lab setups are changed after a lab is finished. In special circumstances, labs may be made up at the end of the semester; sign up with the Secretary in 212 Kimball Hall and your TA.

If you show up for lab after it is under way, your lab instructor may ask you to leave, and to perform the lab another time. The testing machines used in ENGRD 202 are potentially dangerous and quite expensive, and should not be used without proper training. Note also that answers to pre-lab questions are due at the beginning of lab, and will not be accepted for credit later.

Reports turned in late will be marked down 2 points, and 4 points if they are more than a week late. Maximum late penalty is 4 points. Late reports may be handed in until one week after the last regularly scheduled lab of the semester.

Exceptions to the above policies may be made in the case of documented illness or other emergency. Talk to your lab TA about late reports; see your TA and the Secretary in 212 Kimball about making up a lab.

PRE-LAB QUESTIONS

Each lab has pre-lab questions which should be answered *before* you come to lab. These questions encourage you to read through the laboratory procedure prior to attending the lab, and gain an understanding of what will be done during the laboratory period.

Because of this policy your answers must be turned in at the start of the lab if you want to receive credit for them.

ACADEMIC INTEGRITY

Your pre-lab answers and lab reports should be in your own words, based on your own understanding and your own calculations. You are encouraged to discuss the material with other students, friends, TAs, or faculty. Any help you receive from such discussion must be acknowledged on the cover of your lab report, including the name of the person or persons and the exact nature of the help. Violations of this policy will be reported to the academic integrity board.

You may, however, do a joint report with your lab partner (turn in one report for two people). Both partners get the same grade.

When you are through in the lab, you must have your TA sign one of your data sheets. This sheet must include the name of your lab partner, if you had one, and the time and date the lab was performed. The TA will not sign this sheet until your work station is clean and all equipment is accounted for. No lab reports will be accepted without this signed sheet.

CREDIT AND GRADING

Each lab is graded from 0-15. At the end of the semester the grades will be rescaled so that the average grade given by each TA is the same. This grade will be given to your recitation TA.

Grading:

You must attend the lab and turn in a report to get credit for the lab.

+2 points for pre-lab questions.
+5 points for attendance, competent in-lab performance, and a minimally passable report.
+5 points for clear, neat report that is mostly correct.
+3 points for perfectly correct report.
+? points for observations that go beyond the direct questions.
-? points for penalties (see the missed lab and late report policy).

PROBLEMS AND COMPLAINTS

1) Your teaching assistant. The lab TA's job is to help you. See your TA if you have problems with the pre-lab questions, lab, or lab report. In-lab office hours will be available if you need to redo some part of the lab, or want to collect additional data (For this you can see any of the lab TAs.)

2) Dan Mittler, 218 Kimball (5-9172), dm68@cornell.edu. See Dan if you have problems with equipment operation. You may also arrange with him to redo part of a lab, or perform additional lab work.

3) Andy Ruina, 309 Kimball (5-7108), Ruina@cornell.edu. See Professor Ruina about problems with laboratory content or policy.

4) Secretary , 212 Kimball (5-5062), sm138@cornell.edu. See her if you have problems with your lab schedule, or need to make up a missed lab (read the policy, above).

LABORATORY NOTES

A rule of laboratory work is to keep a neat, complete record of what has been done, why it was done, how it was done, and what the result was.

The success or failure of an experiment in a research laboratory often depends critically upon the record made of the experiment. The outcome of a poorly documented experiment becomes a matter of personal recollection, which is not reliable enough to serve as a basis for further work, especially by someone else. You should take copious notes. If in doubt, WRITE IT DOWN. One can ignore what is written, but one can not resurrect that which was never recorded. Similarly, NEVER erase in your lab notes. If an erroneous reading was made, strike it out with a single line and record the new data. You may later decide that it was not in error.

All lab notes, in their original form, must be submitted with your report.

THE LAB REPORT

Your laboratory report should be neatly printed or typed. Do not crowd your writing. Make sure there is room for comments by your TA. The report should communicate clearly and convincingly what was demonstrated or suggested by the lab work. Your TA is looking for evidence of thought and understanding on your part. Your logic and methods are as important as results or "correct" answers. It is essential that you provide information and calculations which indicate how you arrived at your conclusions. It is permissible (and a good idea if you want a very good grade) to discuss observations and material relevant to the lab which are not specifically asked about in the questions.

Format:

The lab reports should contain the following material in the order specified.

I) *Cover page:* A plain sheet, firmly attached to the rest of the report with staples or another binder. The cover should contain the following (with appropriate substitutions for the words in quotes):

"NAME OF THE LAB"
TAM 202
By:
"Your name and your signature (both partners if a joint report)"
Performed:
"Date"
Performed with:
"Name of person(s) with whom you performed the lab"
Discussed lab with:
"Names of people with whom you discussed the lab, and nature of the discussions"
TA:
"Laboratory Teaching Assistant's name"
TA signed the data on page:
"Page #"

II) *Procedure*: (1/2 page maximum) This section should be included only when you deviate from the procedure specified in the lab manual. This section will be needed when there are problems with malfunctioning equipment, or if you develop your own procedure.

III) *Answers to questions:* Concisely answer the questions that are asked and number them as they are numbered in the lab manual. Include any necessary plots, data or calculations. Your answers should be self-contained and presented in an orderly fashion (*i.e.*, the reader of the report should not have to refer back to the questions that are asked, nor should he or she have to hunt through the report to find your answers). While many questions require that you perform calculations, written explanations of what you are doing and sketches can be very helpful. Show all calculations that you perform in arriving at your answers. If you are performing repetitive calculations, you need show only one sample calculation.

IV) *Observations and conclusions:* If you did anything or observed anything in the lab which was not covered in your answers to questions, this is the place to discuss it. This is optional.

V) *Supplemental procedures and questions:* If there are multiple parts to the lab, repeat sections II, III, and IV for each topic.

VI) *Mistakes and suggestions:* This is an optional section. Point out errors in any of the documentation or oral information you were given. Make suggestions for changes in the lab procedure, instructions, content, etc. Please put this section on a separate page, so that it may be kept by the TA for future reference.

VII) *Appendix:* Append ALL notes and records taken in the laboratory (including data sheets signed by your TA). If you have used an $x - y$ plot or data table in your previous answers to questions you need not include it here again.

DATA ANALYSIS AND PRESENTATION

[Read this section carefully. See Lab 0 for further comments.]

I) *Significant figures:* When reporting numerical data, an appropriate number of significant figures should always be used. Large numbers should be written in scientific notation, so that the number of significant figures is not ambiguous.

The numbers 3.840, 0.003840, and 3.840×10^5 each have four significant figures.

When multiplying two numbers together, the general rule of thumb is to write the answer using the same number of digits as the multipliers. When the multipliers have different numbers of significant digits the smallest is used. Thus $0.3526 \times 1.2 = 0.42$ (not 0.42312). This same method should be used for division.

Addition is different. Consider the example: $0.2056 + 14.25 + 576.1 = 593.1$. An answer of 593.1276 is not appropriate because the last three digits (.0276) add nothing to the accuracy of the results, since one of the numbers being added (576.1) is accurate only to tenths. Subtraction should be done in a similar manner.

II) *Percentage difference calculation:* Percentage difference calculations can be used to quantify how well experimental results agree with theoretical or expected values. Rather than writing "the experimental results agree very well with the theoretical calculations," this phrase can be changed to make a quantifiable statement; "the experimental results are within 5 percent of the theoretical calculations." Percentage difference is calculated as:

100% * (Value being compared -Reference value)/(Reference value)

Formal error analysis should only be used if it is necessary to make a point, but for full credit your answers should include some discussion of the type and relative importance of errors in your data.

III) *Units:* The dimensions of all physical quantities should be clearly presented in all calculations, tables and graphs.

IV) *Graphs:* Figure i.1 is an example of how your graphs should appear in lab reports. The following is a checklist of the items your graph should include.

- Use graph paper or computer.
- Curves should be drawn with rulers or french curves (not sketched).
- Graph title.
- Both axes should be titled, with the appropriate units listed in parentheses.
- Numerical values on the axes should be at reasonable intervals and scales be chosen so that all of the data points can be displayed on the graphs.
- On graphs with more than one curve a legend should be used to identify the curve. Data points can be enclosed by some symbol (*i.e.* circle, rectangle, etc.) to distinguish different data sets.
- The independent variable should be placed on the horizontal axis.
- All labels, symbols, etc... should be neat and readable.
- When plotting with a computer, the considerations of labeling axes, etc., still apply.

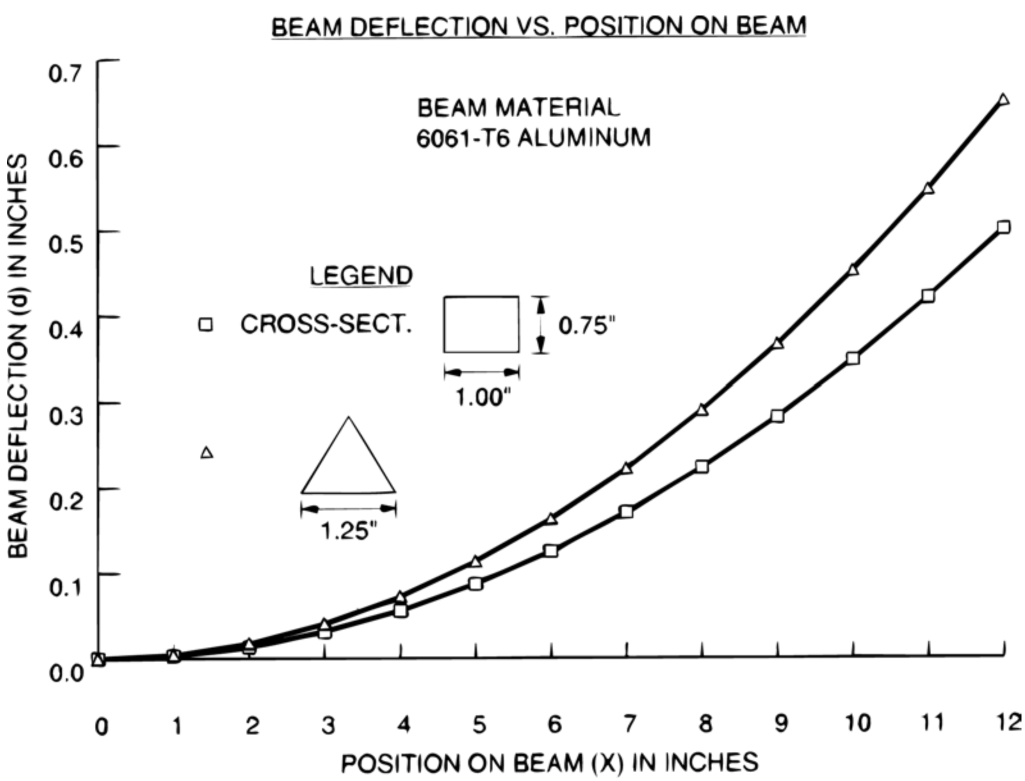

FIGURE i.1

V) *Linear, semi-log and log-log plot interpretation:*

- Linear: a straight-line plot on linear graph paper indicates a relationship of the general form $y = mx + b$, where m is the slope and b is the y-axis intercept. Choose two points along the line $(x_1, y_1), (x_2, y_2)$, preferably well separated. Then

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{Since } y_1 = mx_1 + b,$$

$$b = y_1 - mx_1$$

Sometimes it is useful to plot a function of x and/or y, instead of plotting x and y directly. For example, if y^2 is proportional to x, you could plot y^2 vs. x, and obtain a straight line. The slope of the line then gives you the constant of proportionality. If you don't have (or don't want to use) semi-log or log-log graph paper, you can plot ln y vs. x or ln y vs. ln x, respectively. (Logs to other bases will also work.) This is often very useful with computer-generated graphs. The procedure to obtain an equation from the graph is similar to the one described above. For a graph of ln y vs. ln x which forms a straight line,

$$\ln y = m(\ln x) + b.$$

Find two well-separated points on the line, and write

$$m = \frac{(\ln y)_2 - (\ln y)_1}{(\ln x)_2 - (\ln x)_1}$$

$$\text{since } (\ln y)_1 = m(\ln x)_1 + b,$$

$$b = (\ln y)_1 - m(\ln x)_1.$$

Then

$$e^{\ln y} = e^{m(\ln x)+b} = e^b e^{\ln(x^m)},$$

and therefore

$$y = e^b x^m,$$

a power-law relationship (e^b is just a constant).

- Semi-log: a straight-line plot on semi-log paper indicates an exponential relationship of the general form $y = ae^{cx}$. Choose two points along the line, as above, reading the values from the graph paper scales. Then

$$c = \frac{\ln(y_2) - \ln(y_1)}{x_2 - x_1}.$$

Since $y_1 = ae^{cx_1}$,

$$a = \frac{y_1}{e^{cx_1}}$$

If you want to plot your data on a computer, use a log scale for the y axis, or plot $\ln y$ vs. x with linear scales, as described earlier.

- Log-log: a straight-line plot on log-log paper indicates a power-law relationship of the general form $y = ax^n$. Choose two points along the line, as above, reading the values from the graph paper scales. Then

$$n = \frac{\ln(y_2) - \ln(y_1)}{\ln(x_2) - \ln(x_1)}.$$

Since $y_1 = ax_1^n$,

$$a = \frac{y_1}{x_1^n}$$

If you want to plot your data on a computer, use log scales for both axes, or plot $\ln y$ vs. $\ln x$ with linear scales, as described earlier.

LABORATORY 0
ERROR ANALYSIS

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This laboratory assignment is to be done during the first few weeks of the semester, before you do Laboratory I, and turned in along with your report for Laboratory I.

INTRODUCTION

The collection of data is an important part of all laboratory work, and interpreting the data is the major part of a laboratory report. Laboratory 0 presents a brief overview of techniques and concepts needed to estimate and analyze the errors inherent in experimental work. Although formal error analysis is optional for laboratory reports in this class, you will need to be familiar with its basic principles while writing your reports. Concepts you should be familiar with (and apply when needed) include accuracy and precision, systematic and random error, error propagation, absolute and relative error, and significant figures. If you would like a more detailed discussion of error analysis, a good reference is *An Introduction to Error Analysis* by John R. Taylor.

EXPERIMENTAL ERROR

Error (or *uncertainty*) is defined as the difference between a measured or estimated value for a quantity and its true value, and is inherent in all measurements. Knowledge of the type and degree of error likely to be present is essential if data are to be used wisely, whether the data being considered were measured personally or merely read from manufacturer's data sheets for a material or component. In medical research, biology, and the social sciences, the plan for the data acquisition and analysis is the heart of the experiment. Engineers also need to be careful; although some engineering measurements have been made with fantastic accuracy (e.g., the speed of light is 299,792,458 $\pm$ 1 m/sec.), for most an error of less than 1 percent is considered good, and for a few one must use advanced experimental design and analysis techniques to get any useful data at all. Making measurements and analyzing them is a key part of the engineering process, from the initial characterization of materials and components needed for a design, to testing of prototypes, to quality control during manufacture, to operation and maintenance of the final product.

Reported experimental results should always include a realistic estimate of their error, either explicitly, as plus/minus an error value, or implicitly, using the appropriate number of significant figures. Furthermore, you need to include the reasoning and calculations that went into your error estimate, if it is to be plausible to others. An explicit estimate of the error may be given either as a measurement plus/minus an *absolute* error, in the units of the measurement; or as a *fractional* or *relative* error, expressed as plus/minus a fraction or percentage of the measurement. The advantage of the fractional error format is that it gives an idea of the relative importance of the error. A 10-gram error is a tiny 0.0125% of the weight of an 80-kg man, but is 33.3% of the weight of a 30-g mouse.

Errors may be divided roughly into two categories: *Systematic* error in a measurement is a consistent and repeatable bias or offset from the true value. This is typically the result of miscalibration of the test equipment, or problems with the experimental procedure. On the other hand, variations between successive measurements made under apparently identical experimental conditions are called *random* errors. Random variations can occur in either the physical quantity being measured, the measurement process, or both. We will outline statistical procedures for handling this type of error.

In reporting experimental results, a distinction should be made between "accuracy" and "precision." *Accuracy* is a measure of how close the measured value is to the true value. A highly accurate measurement has a very small error associated with it. Note that in experimental work the true value is often not known, and thus what is reported must be an estimated accuracy or error. *Precision* is a measure of the repeatability and resolution of a measurement -- the smallest change in the measured quantity that can be detected reliably. Highly precise experimental equipment can consistently measure very small differences in a physical quantity. Note that a highly precise measurement may, nevertheless, be quite inaccurate. High precision in a measurement is a necessary but insufficient condition for high accuracy.

EXAMPLE: a typical 3 1/2 digit voltmeter can measure voltages between –199.9 volts and 199.9 volts, when set to its 200 volt range. Suppose that in measuring a voltage an engineer obtains an average reading of 47.1 volts, and that the right-most digit flickers up and down between 0, 1, and 2 in an apparently random way. Thus, the precision is approximately ± 0.1 volts in absolute terms. The fractional or percentage precision is ± 0.1 volts/ 47.1 volts, or $\pm 0.2\%$. The precision can be estimated from the measurements obtained; the accuracy must be found by comparison to an accepted voltage standard. Looking at the manufacturer's specification sheet, which includes the results of such a comparison, the engineer finds that the rated accuracy of the voltmeter is $\pm 0.2\%$. Thus, the accuracy is an order of magnitude worse than the precision. (The accuracy of a voltmeter is only as good as its resistors and other components. Resistor values drift as the resistors get older, and they vary with temperature.) In terms of absolute error, the measurement should be reported as 47.1 $\pm$(0.2)(47.1) volts, or 47.1 $\pm$ 1 volts (not $\pm$ 0.1 volts, the estimated precision).

MINIMIZING SYSTEMATIC ERROR

Systematic error can be difficult to identify and correct. Given a particular experimental procedure and setup, it doesn't matter how many times you repeat and average your measurements; the error remains unchanged. No statistical analysis of the data set will eliminate a systematic error, or even alert you to its presence. Systematic error can be located and minimized with careful analysis and design of the test conditions and procedure; by comparing your results to other results obtained independently, using different equipment or techniques; or by trying out an experimental procedure on a known reference value, and adjusting the procedure until the desired result is obtained (this is called *calibration*). A few items to consider:

a) What are the characteristics of your test equipment, and of the item you are testing? Under what conditions will the instrument distort or change the physical quantity you are trying to measure? For example, a voltmeter seems straightforward enough. You hook it up to two points in a circuit and it gives you the voltage between them. Under conditions of very low current or high voltage, however, the voltmeter itself becomes a significant part of the circuit, and the measured voltage may be significantly altered. Similarly, a large temperature probe touched to a small object may significantly affect its temperature, and distort the reading.

b) Check that any equations or computer programs you are using to process data behave in the way you expect. Sometimes it is wise to try a program out on a set of values for which the correct results are known in advance, much like the calibration of equipment described below.

c) It is unusual to make a direct measurement of the quantity you are interested in. Most often, you will be making measurements of a related physical quantity, often several times removed, and at each stage some kind of assumption must be made about the relationship between the data you obtain and the quantity you are actually trying to measure. Sometimes this is a straightforward conversion process; other cases may be more subtle. For example, gluing on a strain gauge is a common way to measure the strain (amount of stretch) in a machine part. However, a typical strain gauge gives the average strain along one axis in one particular small area. If it is installed at an angle to the actual strain, or if there is significant strain along more than one axis, the reading from the gauge can be misleading unless properly interpreted.

d) *Calibration:* Sometimes systematic error can be tracked down by comparing the results of your experiment to someone else's results, or to results from a theoretical model. However, it may not be clear which of the sets of data is accurate. Calibration, when feasible, is the most reliable way to reduce systematic errors. To calibrate your experimental procedure, you perform it upon a reference quantity for which the correct result is already known. When possible, calibrate the whole apparatus and procedure in one test, on a *known* quantity similar in size and type to your *unknown* quantities.

   EXAMPLE: Suppose that you want to calibrate a standard mechanical bathroom scale to be as accurate as possible. It has lines marked on the dial every two pounds, and a small knob for zero adjustment. You can get a fairly good idea of its precision by stepping on and off of it several times , and looking at the variation between measurements. If the measured weight varies between 149 and 151 pounds, for example, the precision is about one pound. The accuracy cannot be any better than this, but it can certainly be worse, particularly if the scale has not been calibrated recently. If the scale is linear, a plot of the actual weight vs. the weight as measured by the scale (the calibration curve) will be a straight line, and can be determined by establishing two calibration points. For convenience, the first reference weight is usually zero, though it need not be.

   The first step in calibrating the scale, therefore, is to adjust the scale to read zero when there is nothing on it. The second step is to see what it reads with a known weight on it. This second calibration point should be as far from the first as feasible, to establish an accurate calibration curve. This known weight could be obtained by weighing yourself on a scale known to be highly accurate (in a doctor's office, for example), and then immediately weighing yourself on the bathroom scale. Suppose that the true weight is known to be 160 pounds, and the scale reading averages 150 pounds. Some instruments have a *range* adjustment to correct this error, but bathroom scales generally don't. Instead, you would note that the true weight is 6.7% higher than what the scale reads, and the calibration would be complete. If the scale read 75 pounds, you'd know that the true weight was 80 pounds, and so forth.

   If the scale was not linear, you would have to use many different calibration weights to produce a well-defined calibration curve. Not surprisingly, engineers use linear measurement equipment whenever possible. Even if the scale were somewhat nonlinear, you could still get good accuracy in the region of your weight with only two calibration points. For example, if you weigh 160 pounds, you could calibrate the scale at 155 and 165 pounds, and be reasonably certain that the slope of the calibration curve would not change significantly over a 10 lb. interval. Far outside that interval, though, the scale could be quite inaccurate.

MINIMIZING RANDOM ERROR

In contrast to systematic error, random error can usually be estimated and minimized through statistical analysis of repeated measurements. Many books have been written on statistics and data analysis; for the labs in this class, however, a short summary should be adequate. Make repeated measurements, and find the sample average $(\bar{x})$ and standard deviation (S_x) (most scientific calculators will do this for you):

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

$$S_x = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

where N is the sample size (i.e., the number of measurements). Although there is no single accepted standard, one commonly used way of reporting error, given a sample standard deviation of S_x, is to write $\bar{x} \pm 2S_x$. Assuming a normal distribution in the measurements and a sample size of ten or more, this implies that there is a 95% probability that the true value lies within the upper and lower error limits. For example, if you measured the height of a random sample of 25 Cornell students and found $\bar{x}$ of 5'7" and S_x of 4", you would report a student height of 5'7" ± 8". (In this case, this is the height we would expect if we measured another Cornell student at random, since there is no single true height.)

We can go on to estimate how close $\bar{x}$, our sample mean, is to the true or *population* mean. The population mean is what we would get by taking and averaging a huge number of measurements; for example, averaging the heights of all the students at Cornell. We would expect our average of a sample of 25 student heights to be closer to the true average, in general, than any single height measurement would be. However, there is still uncertainty in $\bar{x}$, and we can estimate this by calculating $S_{\bar{x}}$, the *standard deviation of the mean:*

$$S_{\bar{x}} = \frac{S_{\bar{x}}}{\sqrt{N}}$$

For the average student height, $S_{\bar{x}} = 4/\sqrt{25} = 0.8$ inches, and you could report the average Cornell student height as 5'7" ± 2 $S_{\bar{x}}$", or 5'7" ± 1.6", with 95 percent confidence. In other words, if we repeated our 25-student sampling procedure 20 times, we would expect our error limits to include the true mean 19 times out of the 20 samples, and miss it completely in one case out of 20.

Two pitfalls should be kept in mind when using the standard deviation of the mean: First, although you can make $S_{\bar{x}}$ arbitrarily small with enough repetitions of a measurement, remember that it is the precision you are improving. The overall accuracy will be good only if the systematic error present is similarly small. Second, note that $S_{\bar{x}}$ refers to the error in measuring the average; whether this is useful depends on the situation. If you are interested in finding out how tall Cornell students are, compared to Stanford students, it makes sense to compare the averages and use $S_{\bar{x}}$. If you are designing a staircase with a low{hanging beam, however, and want to ensure that most students will not bump their heads, you need to use $\bar{x}$ and S_x, which properly characterize the uncertainty in the height of the Cornell students, rather than the uncertainty in the average.

PROPAGATION OF ERRORS

In many cases our final results from an experiment will not be directly measured, but will be some function of one or more other measured quantities. For example, if we want to measure the density of a rectangular block, we might measure the length, height, width, and mass of the block, and then calculate density according to the equation

$$p = \frac{M}{(L \times H \times W)}$$

Each of the measured quantities has an error associated with it -- $\pm \Delta M, \pm \Delta L, \pm \Delta H, \text{and} \pm \Delta W$ -- and these errors will be carried through in some way to the error in our answer, $p \pm \Delta p$.

Writing the equation above in a more general form, we have:

$$p = f(M, L, H, W).$$

The change in p for a small error in (e.g.) M is approximated by

$$\Delta p \approx \frac{\partial p}{\partial M} \Delta M$$

where $\frac{\partial p}{\partial M}$ is the partial derivative of p with respect to M . In the worst-case scenario, all of the individual errors would act together to maximize the error in p . In this case, the total error would be given by

$$\Delta p \approx \left| \frac{\partial p}{\partial M} \Delta M \right| + \left| \frac{\partial p}{\partial L} \Delta L \right| + \left| \frac{\partial p}{\partial H} \Delta H \right| + \left| \frac{\partial p}{\partial W} \Delta W \right|$$

If the individual errors are independent of each other (i.e., if the size of one error is not related in any way to the size of the others), some of the errors in $M, L, H, \text{and} W$ will cancel each other, and the error in p will be smaller than shown above. For independent errors, statistical analysis shows that a good estimate for the error in p is given by

$$\Delta p \approx \sqrt{\left(\frac{\partial p}{\partial M} \Delta M \right)^2 + \left(\frac{\partial p}{\partial L} \Delta L \right)^2 + \left(\frac{\partial p}{\partial H} \Delta H \right)^2 + \left(\frac{\partial p}{\partial W} \Delta W \right)^2}$$

Differentiating the density formula, we obtain the following partial derivatives:

$$\frac{\partial p}{\partial M} = \frac{1}{LHW} \qquad \frac{\partial p}{\partial L} = \frac{-M}{L^2 HW} \qquad \frac{\partial p}{\partial H} = \frac{-M}{LH^2 W} \qquad \frac{\partial p}{\partial W} = \frac{-M}{LHW^2}$$

Substituting these into the formula for Δp,

$$\Delta p \approx \sqrt{\left(\frac{\Delta M^2}{L^2 H^2 W^2} \right) + \left(\frac{M^2 \Delta L^2}{L^4 H^2 W^2} \right) + \left(\frac{M^2 \Delta H^2}{L^4 H^4 W^2} \right) + \left(\frac{M^2 \Delta W^2}{L^4 H^2 W^4} \right)}$$

Dividing by $p = \frac{M}{LHW}$ to obtain the fractional or relative error,

$$\frac{\Delta p}{p} \approx \sqrt{\left(\frac{\Delta M}{M} \right)^2 + \left(\frac{\Delta L}{L} \right)^2 + \left(\frac{\Delta H}{H} \right)^2 + \left(\frac{\Delta W}{W} \right)^2}$$

This gives us quite a simple relationship between the fractional error in the density and the fractional errors in $M, L, H, \text{and} W$. It may be useful to note that, in the equation above, a large error in one quantity will drown out the errors in the other quantities, and they may safely be ignored. For example, if the error in the height is $\pm$ 10% and the error in the other measurements is $\pm$ 1%, the error in the density is $\pm$ 10.15%, only 0.15% higher than the error in the height alone.

ERROR PROPAGATION IN SOME SIMPLE FORMULAE

The partial derivative technique used above will work on almost any equation. Below are example results for some common types of formula. In the cases below, Q is the result; c and d are constants, assumed to have zero error; and $u, v,$ and w are the measured quantities. For convenience, fractional errors will be written as e; for example, e_Q equals the absolute error ΔQ divided by Q. The formulas given for the case in which the errors are not independent of each other represent the worst-case scenario in which each individual error contributes maximally to the total error. Use these equations if large systematic error is present in the test equipment, affecting more than one parameter in your equation. Use the independent-error equations if random errors are the major source of uncertainty.

For example, suppose that you are measuring the volume of a brick, and use the same ruler to measure height, width, and length. If the ruler has shrunk by 2% (a very cold day!), each of the three individual measurements will be off by 2% in the same direction (too high). The error in the volume will be 6%, as found from the dependent-error equation, and not $\sqrt{2^2 + 2^2 + 2^2} = 3.5\%$.

Suppose instead that the ruler used is of the correct length, and that random errors creep in while reading it. Then you might read it either a little high or a little low for each of the three measurements. On average, some of the error will be cancelled out, and it makes sense to use the independent-error equations.

A) Addition and subtraction: *Example:* $Q = d(cu + v - w)$

with independent errors

$$\Delta Q \approx d\sqrt{(c\Delta u)^2 + \Delta v^2 + \Delta w^2}$$

with dependent errors

$$\Delta Q \approx d(|c\Delta u| + |\Delta v| + |\Delta w|)$$

I.e., absolute errors add during addition and subtraction.

B) Multiplication and division: *Example:* $Q = cuv/w$

with independent errors

$$e_Q = \sqrt{e_u^2 + e_v^2 + e_w^2}$$

with dependent errors

$$e_Q = |e_u| + |e_v| + |e_w|$$

I.e., fractional errors add during multiplication and division; multiplying by a constant does not affect the fractional error.

C) Exponents: *Example:* $Q = e^{cu}$

$$e_Q = c\Delta u$$

Note that e_Q is a fractional error, while Δu is an absolute error.

D) Powers: *Example:* $Q = u^c v^2$

with independent errors

$$e_Q \approx \sqrt{(ce_u)^2 + (2e_v)^2}$$

with dependent errors

$$e_Q \approx |ce_u| + |2e_v|$$

SIGNIFICANT FIGURES

If the error in a measurement can only be estimated to the nearest order of magnitude, it may be most convenient to include the error implicitly, by writing only the appropriate number of significant figures in the reported result. The right-most significant figure in a result should be the only one containing any uncertainty. Or, if you want to be quite conservative, it should instead be the last digit known with certainty. For example, if the raw measurement (or calculator display) is 1.70784, and the four digits on the right (0784) are not known exactly, you should either write your result to three significant figures, as 1.70, or to two significant figures, as 1.7.

The number of significant figures gives a rough approximation of the fractional error in the measurement. The unit size of the least significant digit is an indication of the absolute error. In the first example above, the unit size of the least significant digit is 0.01; this corresponds to the reported absolute error. The reported fractional error, then, is 0.01/1.70 = 0.6%. When collecting data and performing calculations, it's a good idea to use a few more digits than are actually significant, to avoid losing information unnecessarily. All reported results, however, should either show explicit error estimates, or be rounded to the proper number of significant figures.

As discussed in the lab manual introduction, when multiplying two numbers together, the general rule of thumb is to write the answer using the same number of significant figures as the multipliers. When the multipliers have different numbers of significant figures, the smallest is used. Put another way, the fractional error in the product will be of the same order of magnitude as the largest of the fractional errors in the numbers you started with. Thus 0.3526 x 1.2 = 0.42 (not 0.42312). This same method should be used for division.

Addition is different. Consider the example: 0.2056 + 14.25 + 576.1 = 593.1. An answer of 593.1276 is not appropriate because the last three digits (.0276) add nothing to the accuracy of the results, since one of the numbers being added (576.1) is accurate only to tenths. Here the absolute error in the sum will be of the same order of magnitude as the largest of the absolute errors in the original numbers. Subtraction works similarly.

Rules for counting significant figures in a number:

a) The leftmost non-zero digit is the first significant figure.

b) If there is no decimal point, the rightmost non-zero digit is the last significant figure.

c) If there is a decimal point, the rightmost digit is significant, zero or not.

d) Any digits between significant figures are also significant.

QUESTIONS

1) Calculate the volume of the bottom of the cylindrical can in Fig. 0.1. The outside measures $18.00 \pm .04$ cm, the inside measures $17.7 \pm .07$ cm, and the diameter measures 8 ± 0.1 cm. Assume the errors are independent. Write down your result and its error, using:
   a) Absolute error.
   b) Fractional error (as a percentage).
   c) Significant figures.

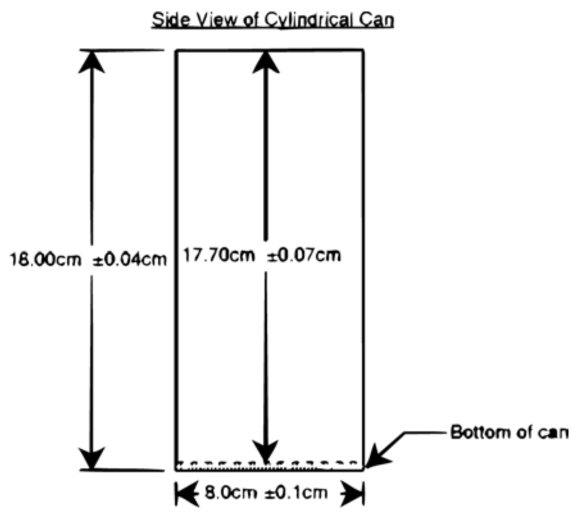


FIGURE 0.1

2) Students in T&AM 203 (Dynamics) can choose between two different procedures when measuring the period of an oscillating weight on a spring. In Procedure A, a timer is started and stopped electronically, as the weight repeatedly passes through a light beam; the error in starting and stopping the timer is negligible. The timer reads in units of 1 millisecond, but is not crystal-controlled, and typically runs 2 percent fast or slow.

   In Procedure B a regular hand-held digital stopwatch is used, which reads in units of 0.01 seconds, or 10 ms. Being crystal-controlled, the stopwatch runs at the correct speed, and is not significantly fast or slow. However, since it is manually operated, an error of about $\pm$ 0.1 second is introduced once as the stopwatch is started and again when it is stopped (assume these errors are independent).

   Assume that the period of oscillation of the weight is exactly two seconds.

   a) If Procedures A and B are each used to measure one oscillation of the weight, what are their respective precisions? Their accuracies?
   b) If Procedure A is used to measure 10 oscillations of the weight, and the resulting ten measured periods are averaged, what are the precision and accuracy of the average?
   c) If Procedure B is used to measure 10 oscillations of the weight, and the resulting ten measured periods are averaged, what are the precision and accuracy of the average?
   d) Suppose that the stopwatch from Procedure B is used, but somewhat differently: the stopwatch is started, ten oscillations of the weight are counted, and then the

stopwatch is stopped. The resulting time measurement is divided by ten. Find the accuracy and precision, and compare them to the previous values.

e) Which procedure has the largest systematic error?

3) A carpenter laying out a 40 foot wall with studs every 16 inches can't find his long measuring tape, so he carefully cuts a scrap board to a length of 16 ± 0.05 inches, and starting from one end, repeatedly lays it down along the floor to mark out the locations for the studs. Assuming that there are no other sources of error in the marking process, estimate the error in the position of the final stud.

4) Ultrasonic range finders bounce high-frequency pulses of sound off objects, and time the echoes to determine distance. One popular model has a range of 0.3 to 10 meters, and (at an air temperature of 20ºC) gives an echo time of 0.0017 seconds at a distance of 0.3 meters, and 0.0583 seconds at 10 meters. Based on the way it works, do you think the device is linear? (Assume uniform air properties.) If possible, calibrate the range finder by finding a formula relating echo time to distance. Optional: What else could you measure with this device?

LABORATORY I
ANALYSIS OF A TRUSS

Revised: July 2002

PRELAB QUESTIONS

(Due at the beginning of your first lab period)
Read through the laboratory instructions and the introduction section and then answer these questions:

1) Define tension stress, σ , and elongation strain, ε .
2) Will the testing machine be used in tension or compression while loading the small truss?
3) How large are the loads that will be placed upon the small truss?
4) What data will you be recording during the small truss experiment?
5) What should be the setting of the speed switch when loading a specimen?
6) Given the cross sectional area *A* and elastic modulus *E* of the bars making up the truss and the hanging weight experiment, derive the equation relating the force to the measured strain, i.e., find $f(\varepsilon)$ that makes this equation true:

$$F_{meas} = f(\varepsilon)$$

7) From the laws of statics, calculate the forces in the bars of the truss for an applied load P. Your calculations must show at least two free body diagrams (FBD). One FBD should be of joint A, and the other of joint B. See Figure I.1a. You will need these equations to perform the lab.

INTRODUCTION

Trusses are structures composed of long thin rods fastened together at their ends. They are used because of their light weight, their ability to support a large variety of load systems, and because they are easily assembled. Trusses are easy to study in the laboratory, and when suitably idealized they are relatively simple to analyze. The goal of such analysis in practice is to determine whether a structure is safe for some planned or possible loading.

In this lab you will learn some of the basic ideas about trusses by determining the forces in two welded truss structures, for a specific load and support systems, by two methods:

a) experimentally, and
b) using basic laws of static mechanics applied to an idealization of the structure.

APPARATUS

I. *Simple Truss*

1) The structure is composed of one symmetric, plane, welded aluminum truss. It is loaded at the midpoint and supported at the ends. (Figure I.1a).
2) Mechanical testing machine with digital load readout.
3) Digital strain indicator (in yellow boxes).
4) Wire strain gauges (attached to the truss bars). (Figure I.1a).

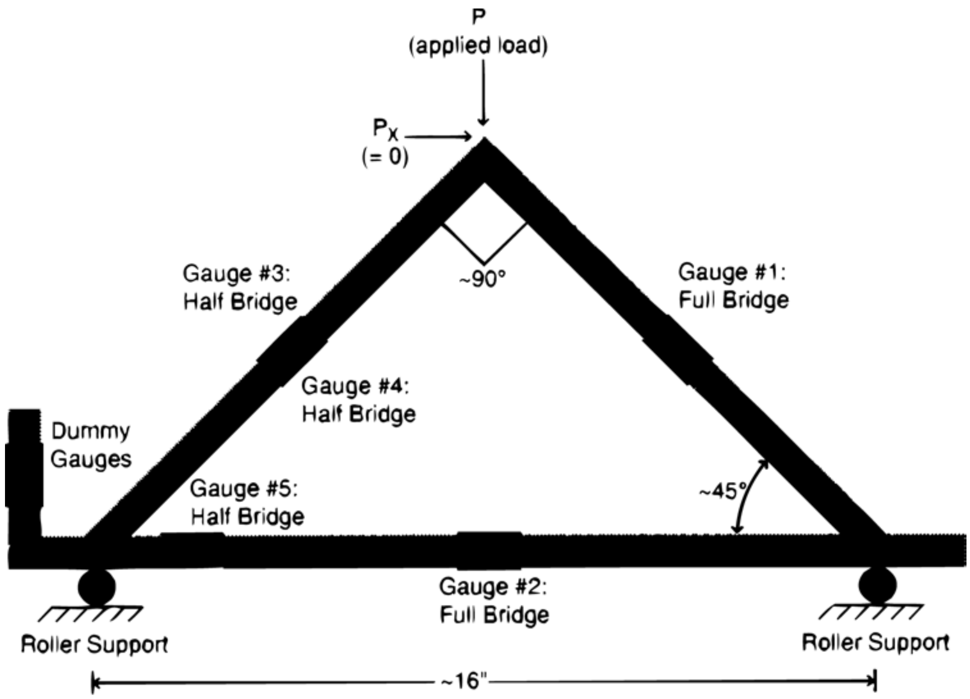


Figure I.1a: Side view of the small truss

II. Hanging Weight

1) A 100 lb. weight hanging from a bar identical to the truss bars.
2) Four strain gauges (two active, two dummies)
3) Digital strain indicator.

III. Complex truss (Figure I.1b)

1) Two joined planar trusses to be loaded with your weight.
2) Strain gauges on eight bars, four per side. Gauge locations are given in Fig. 1.1b, and also on the truss itself.
3) Digital strain indicator.

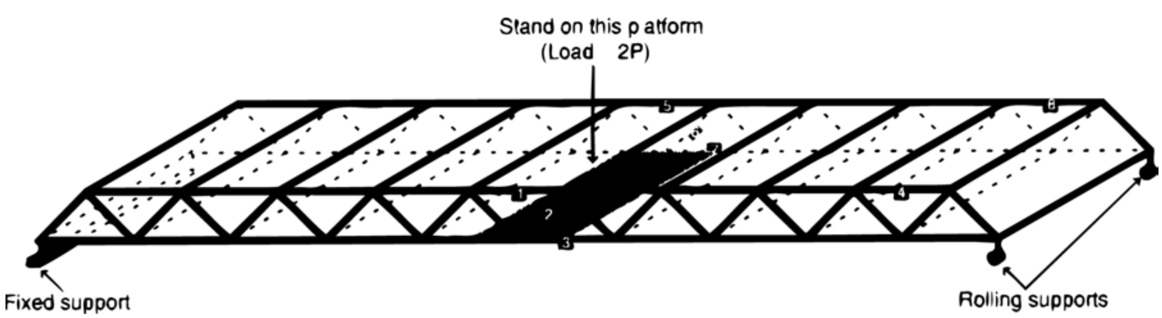


Figure I.1b: View of complex truss

MECHANICS OF TRUSSES

For this experiment, the truss should be idealized as a set of tension or compression members. The welded connections between the members will be assumed to be pinned. Also, the roller supports are assumed to be frictionless. Without such simplifications, the problem of force analysis of the truss is beyond the content of this course. The laws of statics require that the sum of the forces on the truss, or on any part of the truss, must vanish:

$$\sum \underline{F} = \underline{0}$$

Also, the sum of the moments on the truss, or any part of the truss, must vanish:

$$\sum \underline{M} = \underline{0}$$

These equations of statics may be applied to a free body diagram of each pin connection, each bar, or any other section of the truss one may choose. For a determinate truss, of which figures I.1a and I.1b are examples (with the pinned joint assumption), the equations of statics are sufficient to determine all of the bar forces. The methods for doing this are given in the Beer and Johnston *Statics* text in sections 6.1-6.4 and 6.7. Briefly summarized:

1) Between the joint A and the joint B (see Figure I.1a) is the bar AB. This bar has a force applied at each end from the joints. The pinned connections are assumed to allow free rotation so no couple (moment) is transmitted to the bar. The free body diagram of bar AB is shown in Fig. I.2.

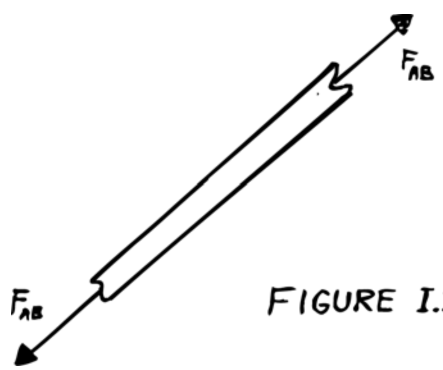


FIGURE I.2

F_{AB} is the tension in bar AB. It is the force pulling equally on both ends of the bar. If the tension is negative the bar is said to be in compression.

2) A free body diagram of the entire truss can be used to find the reaction forces. For example, the force and moment equilibrium imply that the reaction forces R_1 and R_2 in the structure in Fig. I.3 are $R_1 = R_2 = P/2 = 0$, and $P_x = \underline{0}$.

3) The joints: The forces applied to the joints come from the applied loads and from the bar forces. For example, the joint A in the truss for this lab is acted on by the reaction force R_1 as well as the two bar forces F_{AB} and F_{AC} . A free body diagram of joint A is shown in Fig. I.4.

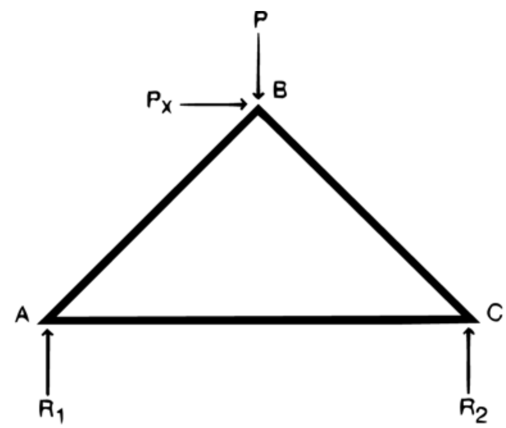


Figure I.3

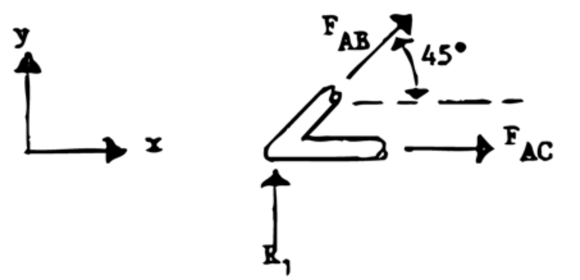


FIGURE I.4

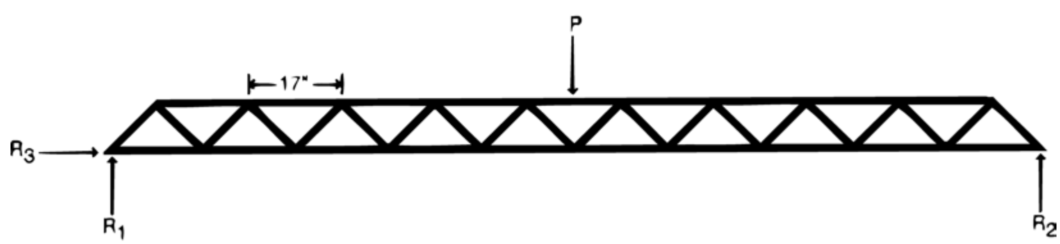


Figure I.5

The equations of vertical and horizontal force balance then determine the bar forces F_{AB} and F_{AC} in terms of the reaction force R_1. Note that F_{AB} will turn out to be a negative number (assuming R_1 is positive) and thus bar AB is in compression. The same method applied sequentially to the other joints will determine the other bar forces.

Example: Calculate the member forces for the floor truss shown in Figure I.5.

Step 1:

Check the truss for static determinacy by using the equation:

$$\#\,\text{degrees of freedom} = \#\,\text{constraints, OR} : 2j = m + r$$

where:
j is the number of joints. (each joint has 2 degrees of freedom for a planar truss)
m is the number of members. (one constraint is imposed by each bar)
r is the number of reaction constraints. (number of reaction forces)

so: 2 * 21 = 39+3, or 42 = 42. Equality in this formula means that the truss is statically determinate and the method below can be used. If $2j > m + r$ the truss is quite possibly rigid but cannot be analyzed using statics alone (analysis awaits upper level courses).

Step 2:

Calculate the reactions at the support points.
Because the truss is symmetrical about the centerline, inspection shows that $R_1 = R_2 = P/2$ and $R_3 = 0$. These reactions can also be calculated using the laws of statics on a free body diagram of the whole structure ($\sum Moments = \sum F_x = \sum F_y = 0$).

Step 3:

Assume that all members are in tension. Free body diagrams (FBDs) of all members will ALWAYS be drawn as shown in Fig. I.6.

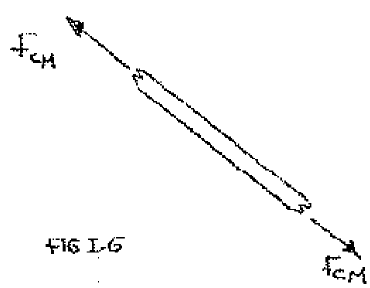


FIG I-6

Once numerical values have been calculated for F the following is true:
if $F > 0$ the member is in tension.
if $F < 0$ the member is in compression.
if $F = 0$ the member is a zero force member.

Step 4:

Draw FBDs for each joint and calculate the member forces. Remember that Newton's third law implies that the forces acting on the member are equal and opposite to the forces acting on the joints. Since the truss is symmetrical about the centerline, the member forces on the right half are identical to those on the left half. An analysis of some of the bar forces is shown in Fig. I.7.

MECHANICS OF LINEAR ELASTIC BARS

The bar forces can also be *measured* (indirectly) in the laboratory with the use of strain gauges. The basic mechanics will be summarized below but may also be read in the Beer and Johnston text *Mechanics of Materials* (sections 1.1, 1.2, 2.1, 2.2 and 2.5).

The free body diagram of a bar, with and without a load F, is shown in Fig. I.8. The load F is associated with a change of the length of the bar from L to L + ΔL (note that ΔL will be too small to be visible to your unaided eye in this experiment). The cross sectional area of the bar is A.

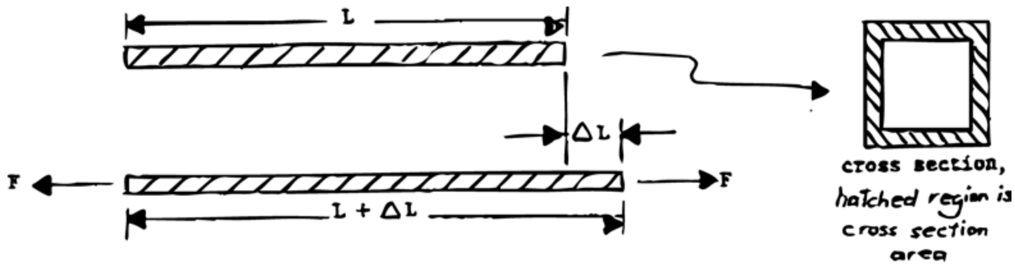


Figure I.8

The tension stress σ is defined as the force per unit area that is carried by the bar,

stress = force/area;
$\sigma = F/A$; $A = 0.3125\ in^2$ in the lab truss

The tension strain ε in the bar is defined as the increase in length per unit length (strictly speaking, this is "engineering" strain),

strain = (increase in length)/(initial length)
$\varepsilon = \Delta L/L$

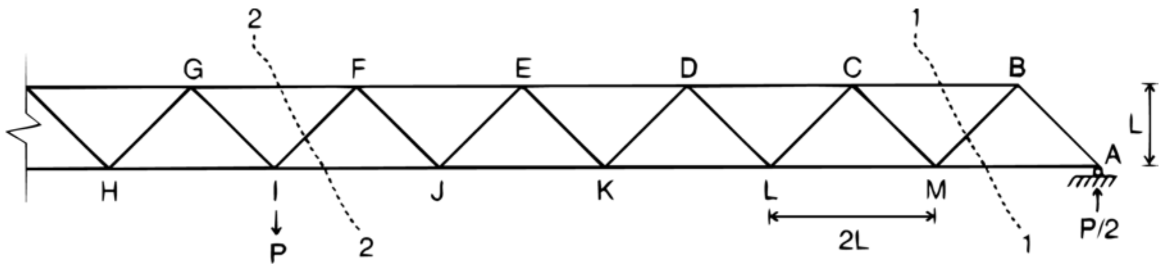
The relation between stress (load) and strain (deformation) is a material property. For metals under small loads the relation is linear (stress is proportional to strain) and elastic (the relation between stress and strain is the same for unloading as it is for loading). This linear elastic relationship between stress and strain is known as Hooke's Law. For a uniaxial load, as found in a truss member, load is proportional to deformation:

$\sigma = E\,\varepsilon$, $E = 1.0\times10^{7}\ lb/in^{2}$ for the aluminum in the lab truss.

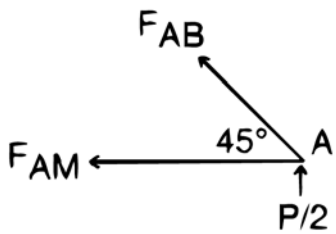
The proportionality constant *E* is known as Young's Modulus and is given above for aluminum, the truss material.

The above relations together imply that:

Bar force = (Young's modulus) × (tension strain) × (cross sectional area)
$F = E\,\varepsilon\,A$.



Joint A

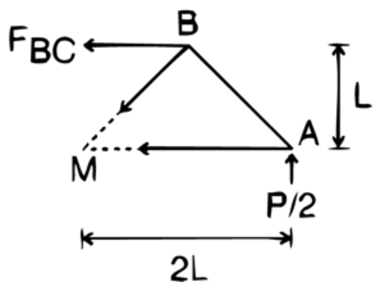


$$\sum F_y = 0$$

$$\frac{\sqrt{2}}{2} F_{AB} + P/2 = 0$$

$$\Rightarrow F_{AB} = -\frac{\sqrt{2}}{2} P$$

Section 1-1

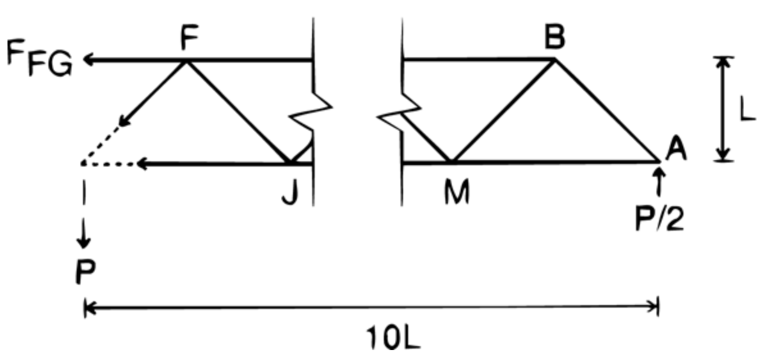


$$\sum M_M = 0$$

$$F_{BC}(L) + (P/2)(2L) = 0$$

$$\Rightarrow F_{BC} = -P$$

Section 2-2



$$\sum M_I = 0$$

$$F_{FG}(L) + (P/2)(10L) = 0$$

$$\Rightarrow F_{FG} = -5P$$

FIGURE I.7

That is, if the strain ε could be measured, then, knowing the cross sectional area A and the modulus *E*, the bar force can also be measured. The electrical resistance strain gauge, a small device glued to the bar, is used to measure strain. In this lab you will use strain gauges as an indirect method of measuring the bar forces.

STRAIN GAUGES

All of the experiments in the TAM 202 laboratory make use of strain gauges, at least indirectly. In the truss experiment you will be interested in measuring strain in the bars of a structure. However, in all of the experiments you will also be measuring force. The force measurement is accomplished with a “load cell”. The load cell is essentially just a piece of metal with strain gauges attached (all hidden from view).

What is a strain gauge? Strain ε is a measure of deformation. It is the fractional change in length of a material element. A strain gauge is a device that measures strain (deformation). The most common strain gauge is the Electrical Resistance Strain Gauge (ERSG). An ERSG is a small electrical resistor that is closely bonded to the material whose deformation (strain) is to be measured. As the base material (aluminum bars in the truss lab) deforms the ERSG also deforms. The ERSG has the property that its electrical resistance increases when it is stretched and decreases when it is contracted. The deformation (strain) is measured by measuring the (very small) change of electrical resistance.

The basic circuit when there is only one active strain gauge used to measure the change in resistance is pictured in Fig. I.9. A voltage V_o is applied to the four resistor circuit. The four resistors are: the strain gauge attached to the sample, a strain gauge attached to the “dummy” sample, and two resistors on the sides of a balancing potentiometer. The voltage ΔV is measured across the so-called “Wheatstone resistor bridge”. The ERSG resistance is measured by reading the voltage ΔV .

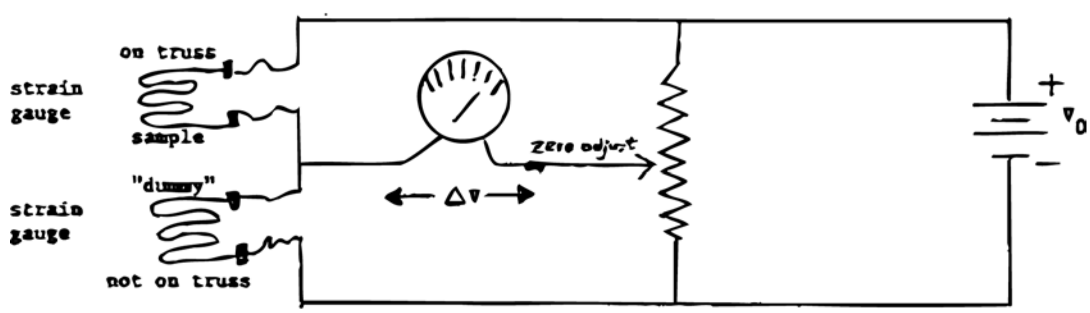


Figure I.9

The strain gauge attached to the dummy sample is for temperature compensation. The ERSG electrical resistance depends on temperature, and the aluminum on which it is mounted expands or contracts when the temperature changes. In this lab you want to measure deformation caused by load, not by random temperature changes in the room. Effects due to temperature are cancelled by using a dummy sample. If both strain gauges change their resistance by the same amount (due, say, to temperature changes in the room) the bridge balance is unaffected. Thus no temperature induced strains (deformations) are detected. The circuit shown above is a “quarter” bridge since only one of the four resistors in the Wheatstone circuit is an active strain gauge. Gages 3, 4 and 5 are quarter bridges. However, the dummy gauges make the circuit look like a half bridge to the electronic circuit the gauges are connected to. Gauges 1 and 2 are half bridges, that become full bridges when the dummy gauges are considered. They each have two active gauges, so the resulting strain display must be divided by two.

The indicator box you will be using has amplifiers inside that are already roughly calibrated for the strain gauges you will be using. That is, in a ERSG the change of resistance is roughly proportional to the strain in the gauge (which, if the gluing is good, is the same as the strain in the underlying material). The proportionality constant is called the “gauge factor”. ERSGs of different types have different gauge factors. The gauge factor for the gauges you will be using is preset on the indicator box. The digital output is labeled directly in units of strain (ε = *fractional* change of length is measured in inches per inch).

Summarizing, in the truss lab you measure the bar forces by measuring the strain in the bars. The strain ε is proportional to the stress σ, for small strains anyway. The stress times the cross sectional area is the bar force. Thus the bar force is proportional to the strain, as explained before.

Digital Strain Indicator

When the bridge is not balanced the voltage ΔV will not be zero. This will cause the strain to be digitally displayed. The units of the strain displayed are ε *in/in* (or equivalently, “microstrain” in 1.0×10^{-6} *in/in*).

Zero-Load Reading

Even with no load applied to the truss there may be a non-zero strain reading. That is, what you want to think of as zero strain will correspond to a non-zero reading. This is due to such things as resistance changes when the strain gauge was glued, etc. To make sense of your strain readings when a load is applied, you must subtract the zero-load readings (that is, the strain readings when no load is applied) .

TESTING MACHINE

For reasons of ethics and pride one tries to design things that will not break too easily. How better to test ones success at such a design than to see how easily something breaks? This is the basic motivation for use of a mechanical testing machine. Put a “structure” in the machine and break it. Measure the load when it breaks. Or, as in this lab, measure the load as the structure deforms without breaking.

The testing machines we use are relatively inexpensive (about 20,000 dollars each) tension/ compression (stretching/squeezing) machines designed to exert and measure forces as great as 20,000 lbs. (30,000 lbs. for one of the machines) The forces are induced through movement of the crosshead and measured by a ‘load’ cell, which is built into the machine.

In a mechanical test one gives instructions to the testing machine to make the cross head move. As the crosshead moves the structure to be tested, which is mounted in the machine, deforms. The force required to cause this deformation is measured with the load cell. As the structure deforms one may make observations (“It's really cracking now!”) and measurements of other quantities besides load (e.g. the crosshead displacement or the strain). In the first part of this lab you will learn how the machine works. You will then use the machine to *temporarily* deform (*and not break*) the truss.

PROCEDURE (Parts III, IV and V can be done in any order).
CAUTION: DO NOT LOAD THE TRUSS WITH A FORCE OF MORE THAN 2000 LBS.

I. Basics

1) Read this set of instructions all the way through.
2) Write down your teaching assistant's name (learn to pronounce it) and his/her office hours.
3) Write down the name and phone number of your lab partner.
4) Ask your teaching assistant any general questions you have.
5) You are required to wear eye protection in the lab, whenever a testing machine is in use.
6) Data reduction is to be done as experiment is performed. This will make writing up lab reports easier in the long run.

II. Testing Machine Introduction

1) Pick a machine to use with your lab partner.
2) Look at the testing machine and all the controls that are on it.
3) Before pushing any buttons or switches notice the following items (see Figure I.10):
    a) The crosshead, the black horizontal piece on the testing machine.
    b) The load cell on the crosshead.
    c) The safety shut-off buttons on the top and bottom of the crosshead.
    d) The drive screws at the sides of the crosshead.
    e) Any items that could get hit if the crosshead moved up or down.
4) Locate the "STOP" (Figure I.11-item a) button on the control panel. Push it. The machine is capable of exerting very large forces (over 20,000 lbs) and doesn't care whether your hand or a delicate piece of testing equipment is in the way. At any time during the lab that anyone suspects the possibility of damage to anyone or anything push the "STOP" button without a second thought. Push it now again for practice. Both partners.
5) Turn power on (Figure I.11-item b).
6) Turn on the digital load readout (Figure I.11-item c).
7) Set "load/lock" switch under the load readout to "load" (Figure I.11-item d).
8) Find out where to push and pull on the crosshead in order to get a reading on the load readout. This is the load cell test fixture; for the truss lab, it is a cylindrical block of steel.
9) Check the number displayed on the load readout (Figure I.11-item k). Adjust the zero knob (Figure I.11-item e) until the display reads zero. Caution: do this only when the actual load is clearly zero; i.e., when neither the truss nor anything else is touching the test fixture. The digital readout should show the load in pounds; on machines 5and 6, check that the "English/Metric" switch (not shown) is set to "English."
10) Set "load/lock" switch to "lock".
11) Now push up and pull down on the test fixture. Notice that the maximum load is stored on the digital readout.
12) Set switch back to "load" position.
13) The "tension/compression" switch (Figure I.11-item f) determines whether the crosshead will move up (tension) or down (compression).

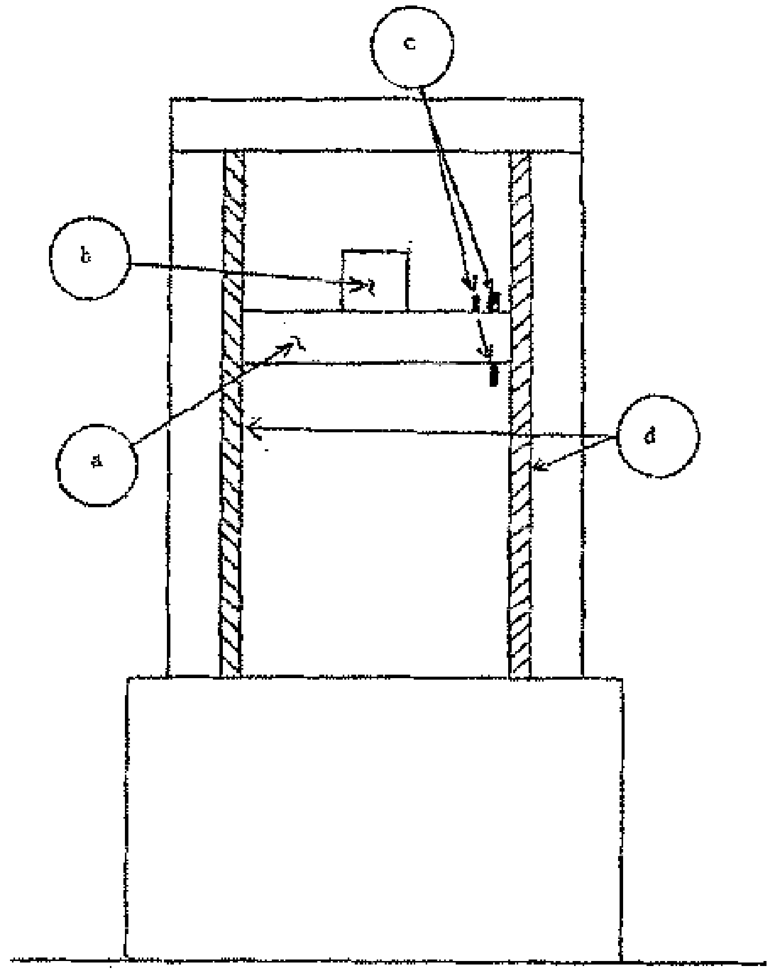


Figure I.10

Mechanical Testing Machine

a--Crosshead
b--Load Cell
c--Safety Shut-Off Buttons
d--Drive Screws

14) There are two ways to control the crosshead speed: The “low/high” switch (Figure I.5-item g) and the speed control knob (Figure I.11-item h). All loading of specimens should be done with the speed switch set to “low”. The high speed range is intended only for coarse positioning of the crosshead between tests.
15) Important! Check that no harm will be done by the crosshead moving up or down several inches. Ask your TA if you have any doubts. This is important.
16) Start the testing machine, by first setting the speed control knob to zero, and then pressing the “test” button (Figure I.11{item i). This prevents abrupt and unexpected movement of the crosshead.
17) Turn up the speed control knob and see its effect on the crosshead. Remember the stop button. Use it. Both partners.
18) Try various speeds, tension and compression, until you have a feel for what the machine does. Use the stop button freely. Do not switch from tension to compression or back unless the machine is stopped.

III. Hanging Weight Experiment

Figure I.12 shows three enlarged photographs of strain gauges. The gauge in Figure I.12.a is similar to the gauges used in this experiment.

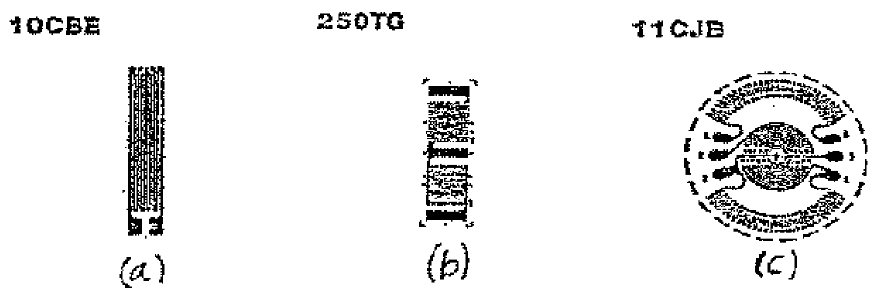


FIGURE I.12

Two methods can be used to calculate forces in the truss members. The first is to use the equation $F=E*\varepsilon*A$, where E and A are known values. ε is determined from the strain indicator reading. The strain indicator, however, only measures changes in resistance. Because there are a variety of strain gauges, and because all of these gauges have different resistances, it is necessary to determine a relationship between the gauges' resistance, and the engineering strain that the gauge is undergoing. The gauge factor determines this relationship. For this experiment the gauge factor specified by the manufacturer is 2.07. If the above method is used, then gauge calibration is not needed. (Essentially you must assume that the values you are given for E, A, and the gauge factor are correct.)

A more direct method of calculating the member forces is to determine a direct relationship between the member forces and the strain indicator readings. Figure I.13 shows the gauge calibration apparatus, which is essentially a rod in tension with two strain gauges attached in the longitudinal direction. The rod is identical to those used in the truss.

Gauges 1 and 2 are wired together and are used as a half-bridge. Since we use external balancing dummies the strain indicator should be set on “full-bridge”.

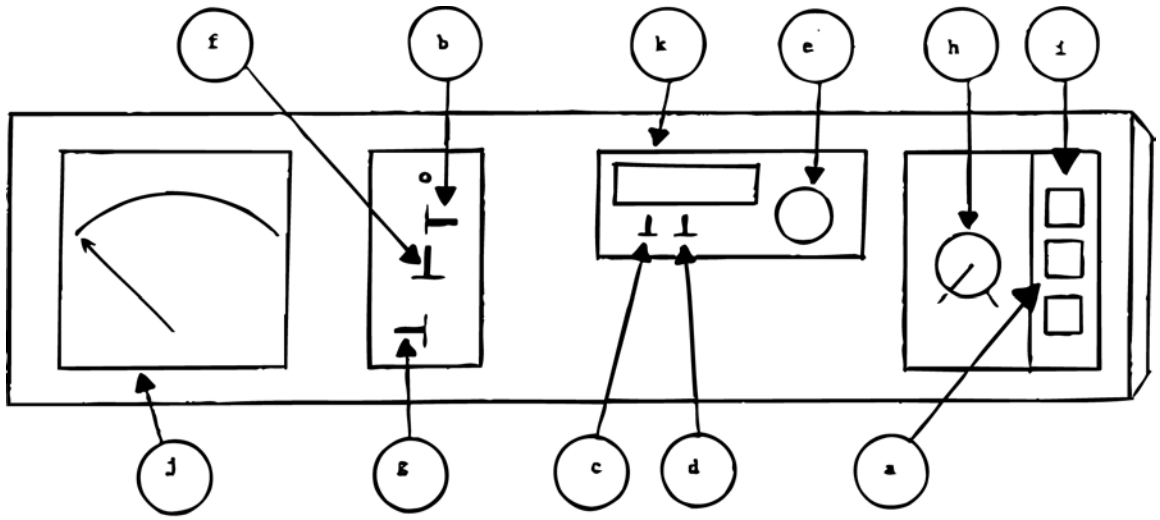


Figure I.11

Testing Machine Control Panel

a--Stop Button
b--Power Switch
c--Load Readout On/Off Switch
d--Load/Lock Switch
e--Load Readout Zero Knob
f--Tension/Compression Switch
g--Low/High Switch (speed)
h--Speed Control Knob
i--Test Button (starts motion)
j--Tachometer (measures speed)
k--Digital Load Readout

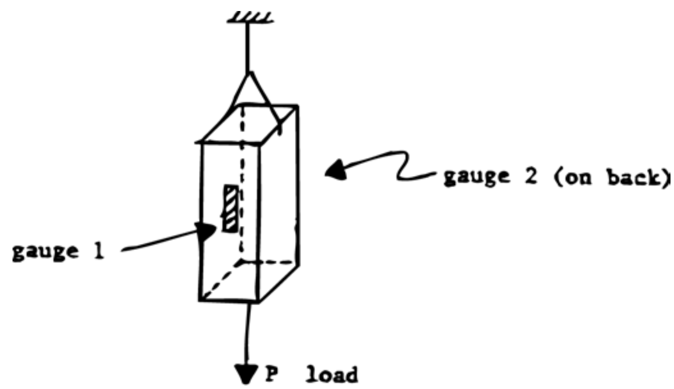


Figure I.13

Procedure - Hanging Weight Experiment

1) Set the strain indicator for *full bridge* operation.
2) With the weight on the floor, measure and record the zero-load strain, using channel 9 on the switch box.
3) Load the rod in tension with the 100 lb. weight, by lifting the lever handle and latching it at the top. Record the corresponding strain.
4) Unload to zero load and record again.
5) Repeat steps 3 and 4 until you have a good sense of the repeatability of the measurements (and thus an error estimate).
6) Using the equations derived for pre-lab question No. 6, calculate the measured load using the strain gage data. Remember that you will need to divide the strain reading by two at some point, because two gauges are being used.

IV. The Small Truss Experiment

1) As you perform the experiment fill in the two tables on the next page. Record your load in the first column, your raw strain indicator readings in columns 3 through 5, and an appropriate calculation of strain in the 'measured strain' column. When using gauges 1 and 2 you are actually measuring the sum of strain on opposite sides of the bar, so you must divide by two to get the actual strain. From the measured strain values, calculate the bar forces and enter them in the column labeled 'measured forces.' A list of the gauge or gauges that you used for calculating the force in a bar, plus any averaging formula, should go in the column labeled 'gauges used.' Enter the bar force found using the load cell reading and truss theory in column 5, and the percent difference between the two methods of determining bar force in column 6. The formulas you derived for your prelab questions may be helpful in the force calculations.

2) Check that the truss is not under any load. You should be able to see a small air gap between the truss and the test fixture on the crosshead. If in doubt, put the 'tension/compression' switch in the 'tension' (up) position and raise the crosshead slightly. Zero the load readout (Figure I.11-item 3), and take zero-load readings for all strain gauges.

SMALL TRUSS STRAIN DATA

Load Level	Gauge #	Initial Zero	ε reading at load of P	Final zero	Measured ε
	1				
	2				
	3				
	4				
P=______	5				
	1				
	2				
	3				
	4				
P=______	5				
	1				
	2				
	3				
	4				
P=______	5				

SMALL TRUSS FORCES

Load Level	**Bar of truss**	**Gauges(s) used**	**Measured force**	**Calculated force**	**Percent difference**
	AB				
	AC				
P=_________	**BC**				
	AB				
	AC				
P=_________	**BC**				
	AB				
	AC				
P=_________	**BC**				

For gauges 1 and 2 use the full bridge setting on the strain indicator. For gauges 3, 4, 5 use the 1/4, 1/2 bridge setting. Do not adjust the zero controls again until the test run is complete, and avoid brushing up against them accidentally. Ideally, we would like the zero-load reading for the strain gauges to be 0000. μ *in/in*. However, due to variations and drift in the gauges, it is inconvenient and sometimes impossible to adjust the readings to zero. Therefore, record whatever value the strain indicator shows. The engineering strain will be calculated as:

$$\varepsilon_{engineering} = \varepsilon_{meter} - \varepsilon_{zero-load}$$

3) Ask your TA to supervise as you lower the testing machine crosshead into contact with the truss for the first time. Note that the truss is very stiff, so the crosshead must be lowered slowly and carefully, with constant attention to the digital load readout and the amount of clearance remaining between the truss and the test fixture on the crosshead. <u>Warning: high loads can develop quickly</u>. The 'low/high' switch should be set to low.
4) Using the mechanical testing machine, gradually load the truss to 500 lbs. compression. Important!: The 'low/high' switch should be set to low, and the speed control knob should be operated at a setting of approximately 15. The load reading is often more stable if you load the truss slightly above your target load, and then back it down. Record the strains for all gauges or pairs of gauges (changing from full to quarter/half bridge as appropriate).
5) Repeat step 4 for 1000 lbs. and 1500 lbs. of load.
6) Unload the truss to zero load (you should see a small gap above the truss), and recheck the zero-load readings on the digital load readout and the strain gauges. If they are off by more than a few pounds or microstrain, you may have made a mistake, or there may be a problem with the equipment. Repeat the procedure; if the zero-load readings still aren't repeatable, check with your TA.
7) Repeat all steps 2-6 until you feel comfortable with the procedure and the data. You may want to average your data.
8) If you are shy or have a domineering lab partner go back and do all parts of the lab that you don't understand.

V. The Large Truss Experiment

A large truss is set up in the lab. It is built using the same bar material as for the hanging weight experiment and the small truss. You can stand on this truss and measure the consequent strains. All bars use "full bridges" with 2 active gauges. Strain indicator readings must be divided by 2 to measure strain in the bar.

1) Inspect the truss; notice the positions of the strain gauges.

2) Get zero-load readings.

3) Stand symmetrically on the truss (load = 2P), P for each half of truss)

4) Get strain readings.

5) Get off the truss and take zero-load readings again.

6) Repeat steps 3-5 until content.

7) Weigh yourself on the scale in the basement of Bard.

QUESTIONS

1) Complete the small truss data tables if you have not already done so. List any equations used, and show a sample calculation for each one.

2) Discuss your results from the small truss.
   a) Do the measured strains vary linearly with the applied load? Justify your answer with a graph of measured strain vs. load for at least one gauge. Should you include (0,0) as a data point?
   b) How do your measured data differ from your theoretical values? What simplifications in your theoretical analysis might have caused these differences? What errors might there be in your measurements? (Be bold; use your imagination.)
   c) Which of these simplifications and errors do you think are the most significant and why?

3) Discuss the measurements of strain.
   a) Why are there strain gauges on opposite sides of each bar? (Hint: what is the strain reading when the bar is bent, with no axial load?)
   b) In light of part a), why is the strain reported by gauge 5 different from the average strain reported by the two gauges at location 2? What does this tell you?

4) If you knew that the bars making up the truss will fail at 13,000 lbs. in tension and 10,000 lbs. in compression, what is the maximum load that could be applied to the truss? Show calculations to justify your answer.

5) Compare theory and experiment for the hanging weight lab.

6) Compare theory and experiment for the large truss using gauges on any one bar of your choosing. Remember that each side of the truss is loaded with half your weight.

LABORATORY II
THE TENSION TEST

Revised: July 2002

PRELAB QUESTIONS

(Due at the beginning of the lab period)

Read through the laboratory instructions and then answer these questions.

1. What is the first sample you will load under tension?

2. How can a load vs. displacement graph be converted into one of stress vs. strain?

3. Will the point of rupture be represented on your stress-strain curves of the testing of the popsicle sticks? Of the metal specimens? Why or why not?

4. At what point in the test will the extensometer be removed from the specimen?

5. How much of a specimen should be held in the grips, to avoid slipping or sticking?

6. How will you remove a specimen that has become stuck in the grips?

7. Research some of the properties of steel and aluminum alloys. What are typical ranges of Young's (elastic) modulus and yield stresses for each material? Compare the stiffness of the strongest steel you can find, to that of the weakest. Look in Gere's Appendix H.

INTRODUCTION

The objective of this laboratory is for you to learn more about the ideas of strain (a measure of deformation) and stress (a measure of the load carried by a material) and the relation between them in some engineering materials. You will also gain further experience with the testing machines and instrumentation.

MOTIVATION FOR THE TENSION TEST

The approach used in the truss experiment, testing a complete structure, can be expensive. One reason to study engineering is to learn how to predict when a structure will break and how much it will deform before it breaks, without actually breaking it (or even building it!) A basic idea that TAM 202 aims to communicate is this:

> A structure is built of separate substructures. These substructures are mechanically connected. The fact that they are connected says two things:
>
> 1) There is mechanical interaction between the substructures and thus the laws of mechanics apply.
>
> 2) There is some relation, which can be determined by geometry, between the amount of deformation in the different substructures.
>
> Thus, if one knows enough mechanics and geometry, the strength of a structure can be calculated by knowing the relation between load and deformation for the different substructures from which

it is built. If the above paragraph seems mysterious try to apply it to the truss experiment which you have already performed. (The truss is the structure; the bars are the substructures.)

A large part of your study in TAM 202 is learning about some particularly simple ‘substructures’: bars, shafts, beams and columns. These relatively simple structures can be tested for strength and stiffness in the laboratory. (There is currently no TAM 202 lab on shafts, unfortunately.)

MECHANICS OF LINEAR ELASTIC BARS

Tension specimens are something like bars in structures. When the load is well below the greatest load that the bar can bear, the behavior (in the case of most metals) is linear elastic. The basic mechanics of linear elastic bars was summarized in the truss lab but may also be read in Gere’s text *Mechanics of Materials* (5^{th} ed., Ch. 2.1-2.3).

INELASTIC DEFORMATION

In the truss lab the deformation was described as linear-elastic, (*i.e.* the deformation was linear and reversible). In this lab you will load several samples past their elastic limits into nonlinear and irreversible deformation. This is the way metals deform soon before a structure fails. Thus understanding inelastic deformation is important for determining structural integrity. The basic ideas of metal deformation in the tensile test are discussed in the Gere *Mechanics of Materials* (5^{th} ed.) text in sections 1.3, 1.4, and 2.2.

Inelastic behavior is often crudely characterized by two simply defined numbers. These numbers are the maximum stress and the yield stress.

The “maximum” stress occurs during the process of failure and is often referred to by the names of “ultimate,” “failure”, or “peak” stress. Maximum stress should not be confused with the stress at which sudden rupture occurs, which (perhaps surprisingly) is not a material property. Maximum stress is important to the practicing engineer because at this stress some substructure collapses.

Yield stress is roughly defined as the point on the stress-strain curve where the stress-strain relationship is noticeably no longer linear. For some metals, including mild or low carbon steel, this point is identifiable by a noticeable “dip” that occurs in the curve. The 6061- T6 aluminum alloy and cold-rolled AISI 1018 steel this experiment uses do not have this characteristic “dip”; instead, their curves make a more gentle transition from the elastic to the plastic regions. In the latter case, determination of yield stress is traditionally and arbitrarily done by the 0.2% offset method. This method involves starting a line at the point $\varepsilon = 0.002$, $\sigma = 0$. A straight line is then drawn parallel to the linear portion of the stress-strain curve, as in Figure II.1. The point where this line intersects the stress-strain curve is then defined as the yield stress. See section 1.3 of *Mechanics of Materials* by Gere (5^{th} ed.) for a more complete description of this method.

Most structures are designed so that the stresses never exceed yield stress. This prevents the structure from having any large permanent deformation.

STRAIN HARDENING AND ANNEALING

Strain hardening or *work hardening* is a property of metals in which plastic deformation results in higher strength and hardness. In Figure II.1, which shows a typical stress-strain curve for aluminum or cold-rolled steel, notice that the stress continues to increase as loading continues beyond the yield point. If load on the specimen is removed after some plastic deformation has occurred, in a later tension test the specimen will behave linearly and elastically up to the point where it left off (see the unloading/reloading curve in Figure II.1). Thus, the partially deformed specimen has a new, higher yield stress than it did before. By carefully controlling the amount of plastic deformation, the new yield stress can be moved up the stress-strain curve until it approaches the maximum stress. This is an often-used technique in metal processing to increase the strength of an otherwise soft metal. However, work-hardening uses up some of the ductility and toughness of the original metal, and too much can cause brittleness and cracks.

The steel specimen for this lab was cold-rolled during its manufacture; i.e., it was squeezed down to the desired size between rollers, at approximately room temperature. This process resulted in a large plastic deformation, and thus the yield point moved up the curve considerably from the yield point of the original steel.

In the process of *annealing*, the metal is heated to a temperature high enough to cause recrystallization, thus erasing most of the strain-hardening effects. When very large deformations are needed during a manufacturing operation (for example, in making a thin sheet or foil from a thick plate of steel), the metal may be taken through multiple stages of cold rolling and annealing, first rolling the steel until it starts to become brittle, and then annealing it to make it ready for more rolling. In hot-rolling, the temperature is high enough to anneal the metal throughout the process, and work-hardening is minimal.

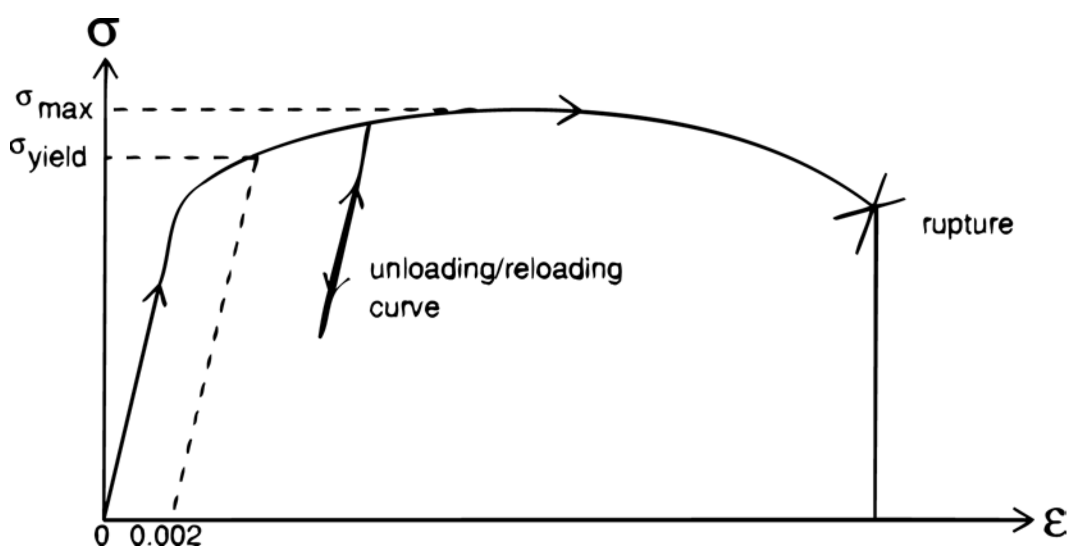


FIGURE II.1

LABORATORY PROCEDURE

I. Basics

1) Read this set of instructions all the way through.
2) Write down the name and phone number of your lab partner.
3) Eye protection is required whenever any of the testing machines are in use, not just while your test is running.
4) Ask your teaching assistant any general questions you have.

II. Testing Machine Review

1) Pick a machine to work with.
2) Refresh your memory of the testing machine and its controls.
3) Before pushing any buttons or switches notice the following items:
   a) The extensometer. This is a small golden-colored device that allows the measurement of strains in the specimens without having to glue on strain gauges. It has two knife-edges spaced one inch apart, which are pressed against the specimen. Any elongation of the section of metal between the knife edges causes them to move apart, thus bending the strip of aluminum they are attached to. The aluminum strip has strain gauges attached to convert the degree of bend into an electrical signal useable by the plotter.
   To mount the extensometer on a specimen, first squeeze the spring-loaded cylindrical barrel; then put the specimen between the knife blades and the backing plate, align the specimen with the backing plate, and carefully release the spring. You will probably find it easier to mount the extensometer before installing the specimen in the testing machine grips.
   b) On machines 1 through 4, note the LVDT (Linear Variable Differential Transformer) on a bracket encircling the right drive screw, for measuring large displacements of the crosshead. The LVDT has a rod pressed against the top of the crosshead. Movement of the rod changes the amount of magnetic coupling between two coils inside the LVDT's cylindrical case, resulting in a change in the voltage going to the plotter. For proper operation, the LVDT bracket should be about 3 inches above the crosshead.
   On machines 5 and 6, the LVDT is replaced by the crosshead motion indicator, which displays crosshead position and velocity on a digital display in the lower left corner of the console. It will also work with the X-axis of the plotter if you set the ‘strain/crosshead’ switch to ‘crosshead.’ The crosshead motion indicator is easy to use, but you must always remember to press the “reset” button before starting your test; when used with the plotter, it has a useable range of only ± 1 inch.
   c) The test fixtures (grips) in this lab use wedge-shaped jaws, which clamp the specimen in tightly as tension is applied to it. Look at the grips and move the handles, to get a feel for how to open and close them. When first starting a test you may need to hold the handle of the lower grip until the specimen catches. If you need to remove an unbroken specimen, remember that the grips will not release if the specimen is under any tension whatsoever. The grips may stick; to free a stuck specimen, <u>do not</u> hammer on the handles. Use the brass block provided, and tap on the ends of the jaws, pushing them back into the grips.
   d) The specimens. These are called ‘dog bone’ specimens, because of their shape. The center portion is reduced in width, so that strain and rupture will occur near

the center. Otherwise, the specimen would tend to break near the grips because of the large forces and sharp edges involved in clamping the ends. You'll be testing two specimens, one of cold-rolled 1018 steel, and the other of 6061-T6 aluminum alloy. You'll also be practicing with wood specimens (tongue depressors), which are not dog-bone-shaped.

*III. The Tension Tes*t

1) Practice doing tension tests with popsicle sticks (actually, tongue depressors). Don't use the extensometer. Use the LVDT to measure crosshead displacement (machines 1 through 4 -- swap wires if necessary, unscrewing the outer ring of the connector, not the connector body), or the crosshead motion indicator (machines 5 and 6 --set strain/crosshead switch to 'crosshead'). Use the 100% displacement (X) range on the plotter, and the 10% load (Y) range. If you are using machine 5 or 6, remember to press "reset" before the test.

2) Disconnect the LVDT or crosshead motion indicator (reverse of above), and hook up the extensometer. Set up the plotter so that it responds to the extensometer, using the same X range as you will use for your tests of the metal specimens (see table below).

3) Calibration: determine both the X and Y axes scales on the plotter. The Y-axis on the plotter records the load (from which you can calculate stress) being imposed on the specimen. The calibration factor in pounds/inch is calculated as:

   (20,000 lbs)/(10 inches)*(Scale in Percent)/100%.

   For machine 4 only the calibration factor is

   (30,000 lbs.)/(10 inches)*(Scale in Percent)/100%.

   The appropriate 'Scale in Percent' setting is listed in the table on page 6.

The extensometer is calibrated using the stage-mounted micrometer. First, squeeze the spring-loaded barrel of the extensometer, separating the knife blades, and attach it carefully to the rods of the micrometer. Avoid letting the blades slide as you release the tension. With paper in the plotter, lower the pen, then rotate the micrometer's thumbscrew 0.010 inches (ten divisions) counterclockwise, so as to separate the knife blades. After turning the thumbscrew, the pen moves Q inches on the plotter. Thus 1 inch of displacement on the paper corresponds to 0.010/Q inches of actual displacement. The X-axis is now plotting ΔL using the scale factor just calculated. This ΔL can be changed to strain by dividing by the length between the knife edges of the extensometer. (Note: the extensometer is both delicate and expensive (about $1000); use it carefully.)

4) Plot load vs. extension for steel and aluminum specimens. In each case:
   a) Make sure to record the cross sectional dimensions of the specimens before the test, (measure with calipers and ruler).
   b) Set the x-axis of the plotter to "standby" and attach the extensometer. Make sure it is centered and squarely positioned.
   c) Gently measure the separation distance of the two points of attachment of the extensometer (it should be close to one inch). If the knife blades slide you will need to remount the extensometer. Record the distance (this is your gauge length). It will be very similar for both specimens, so you need not measure it more than once.
   d) With the specimen not installed in the grips, adjust the digital load readout to zero. It's also helpful to mark a zero load line on your plot, using the X axis zero control.

e) Install the specimen in the grips, being careful not to disturb the extensometer. The grip jaws should grasp the specimen by about one inch on each end. You may need to have a partner hold the lower jaw handle until a load is applied.

f) Use the following scales for the X and Y-axes on the plotter:

	X	Y
Aluminum	10%	20%
Steel	10%	50%

g) Check that the plotter X and Y axes are "on", not set to "standby". The reset/sweep switch should be set to "reset".

h) Position the plotter pen about an inch from the lower left corner of your graph paper and lower the pen.

i) Check that the machine is set to "tension" and "low speed", and start the test (20-30 is usually about right on the speed dial). Do <u>not</u> allow the specimen to break while the extensometer is attached.

j) Shortly after the specimen begins to yield (the graph starts to level out), stop the test, put the machine in compression, and unload the specimen to about 100 pounds or so. Then reload it until it begins to yield once again.

k) Stop the test, put the X-axis of the plotter on standby, and <u>remove the extensometer</u>.

l) Continue loading the specimen until it breaks. Record the maximum load achieved (you can set the digital load readout to "lock" to hold the highest reading).

m) Stop the test after the specimen breaks. If the pieces are stuck in the grips, free them by tapping lightly on the jaw faces with the brass block provided. Do not pound on the handles or anywhere else on the grips, or use any other item in the lab as a hammer. If you need to remove an unbroken specimen from the grips, do not attempt to do so while it is under tension. Bring the crosshead down until there is clearly some slack when you push the top grip from side to side.

5) Record your maximum load and the slope of your load vs. displacement curve (in pounds/inch) on the data chart on the cabinet (ask your TA if you don't see it). If your initial slope does not seem very straight, use the unloading/reloading curve. Do this for both aluminum and steel, and also record the date and the number of the testing machine you used.

Copy down the ten most recent sets of data from the chart, including your own. You may copy down more than ten if you wish. You will need this data for your lab report.

QUESTIONS

[Remember to follow the format given in the Introduction]

1) Re-label both axes of all plots for the steel and aluminum specimens in appropriate physical units of stress and strain. Show your calculations determining the scales. Sketch specimens and give dimensions.

2) Calculate the mean and standard deviation for each of the four sets of data that you copied down (i.e., maximum strength for aluminum, maximum strength for steel, slope for aluminum, slope for steel.)

3) Answer questions a) and b) below for both the group data and your data. Question c) applies only to your data, and d) applies only to the group data. Assume that the specimen sizes and gauge lengths in other students' tests are equal to those in your tests. Do the calculations for both aluminum and steel.

a) What is the Young's modulus?

b) What is the maximum stress?

c) Using your data only, what is the yield stress? (Do not use the unloading/ reloading curve for this.)

d) The 6061 T6 aluminum alloy is listed (Juvinall and Marshek, *Fundamentals of Machine Component Design*) as having a maximum stress of about 45 ksi, a yield stress of 40 ksi, and a Young's modulus of 10 Mpsi. For annealed 1018 steel, the listed maximum stress is 50 ksi, the yield stress is 32 ksi, and the Young's modulus is 30 Mpsi. How large is the discrepancy between the textbook values and the measured values? How much variation was there between tests of the same material? Do you think the variations in test results are mostly due to random errors in measurement, or due to variability in the specimens? If not stated otherwise on the testing machine, you may assume that the error in your load cell, plotter, and extensometer readings is equal to ± 3 percent or the error in reading your graph, whichever is greater. Show sample calculations.

4) Besides the Young's modulus and stress values discussed above, what differences did you observe in the behavior and appearance of the two metals during the tension test? Sketches may be helpful in answering this question.

5) If a specimen twice as thick had been tested, what measured or computed quantities would be different in the tension test? Which would be the same? Why?

6) If the overall length of the specimens had been twice as long, what measured and computed quantities would be different?

LABORATORY III
DEFORMATION AND FAILURE OF BEAMS

Revised: July 2002

PRE-LAB QUESTIONS
(Due at the beginning of the lab period)

Read through the laboratory instructions and then answer these questions.

1) What is the maximum load you will place on the I-beam?

2) What data will be recorded in the I-beam part of the experiment?

3) Draw shear and bending-moment diagrams for the I-beam, in terms of the applied load P. See Meriam and Kraige *Statics* (5^{th} ed., Sec. 5.7) or Gere's *Mechanics of Materials* (5^{th} ed., Sec. 4.5) for details. These diagrams will allow you to determine the moment and shear force at various positions on the beam, so that you can calculate the theoretical bending and shear stresses.

4) What are the sizes of the rectangular beams that you will be testing?

5) What is the purpose of testing an aluminum rectangular beam?

INTRODUCTION

The beam is one of the most commonly used structural elements. It is used in situations where a long narrow structure must resist transverse loads; applications include floor and roof beams (joists), diving boards, leaf springs and wrench handles. Several geometrical and physical variables are used in the analysis of beams. This lab is intended to acquaint you with these variables in a physical way. The equations that you will use and 'discover' here are based on theory that will be presented in your text and lectures. As has been pointed out before, your goal as a structural engineer (should you choose to become one) will be to use your knowledge of basic simple structures (like beams and bars) to design safe buildings, bridges, airplanes, etc.

This experiment has two parts:

1) You will measure strain and deflection in an aluminum 'I-beam'. The measurements can be compared with theoretical predictions.

2) You will load five rectangular beams to failure and measure their stiffness and strength.

The two parts of the lab can be done in any order.

RECTANGULAR BEAMS

Large rectangular beams are not used as often as I-beams in metals because they are wasteful of material. However, they are quite common in wood. Many non-optimal structures that are not large also use rectangular beams. In this part of the lab you will load five rectangular metal beams to failure while plotting load vs. displacement. The geometry is as pictured below.

Each lab group is given one set of beams. There is a standard beam (1/4 in. × 1/8 in. × 4 in.) made of annealed AISI 1018 low carbon steel and then one beam each that is twice as thick, twice as wide, or twice as long. In each of these three cases there is twice as much material as in the standard beam. There is also a beam made of 6061-T6 aluminum alloy that is identical in size to one of the steel beams but has a different Young's modulus. The question is: How do these parameters of a beam affect its stiffness and strength? You will learn theoretical answers in lecture. Here you are supposed to discover the relations (or at least the trends) experimentally.

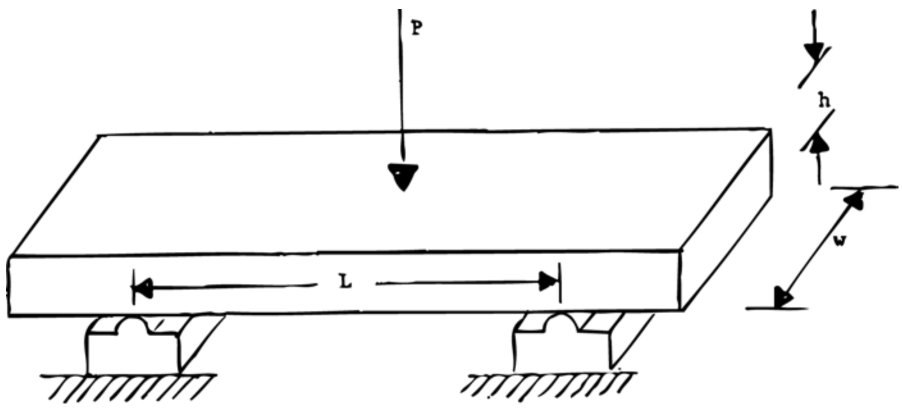


Figure III.1

The stiffness of the beam is the ratio of load to displacement while the beam deforms elastically. The strength of the beam is the total load it can carry.

PROCEDURE

1) You are required to wear eye protection in the lab, whenever a testing machine is in use.

2) Check the dimensions of each beam. Do minor length variations matter?

3) Calibrate the LVDT (X-axis) by inserting a 1/4 inch bar between the LVDT plunger and the crosshead of the testing machine, measuring the pen deflection, and then taking the specimen out again. Calibration should be performed using the 100 % range for the X-axis, with the crosshead very close to the height it will be at during a test. (If you are using one of the newer, beige testing machines, use this procedure: stack two 1/4 inch specimens in the test fixture, bring down the crosshead until the load display barely registers a load, and use the Y -axis zero knob to mark the pen location. Then raise the crosshead and remove the top specimen, bring the crosshead down again as before, and make a second mark.) When testing the beams, set the displacement scale (X-axis) and the load scale (Y-axis) in accordance with the chart below or your TA's instructions. The rated capacity of the load cells used in this lab is 2000 lb. (3000 lb. for machine 4) instead of the 20,000 and 30,000 lb. load cells used in the other experiments. The nominal calibration factors are given below. If the results of your calibration of the X-axis differ from these, you should use yours instead, modified as necessary for the lower ranges. Mount the graph paper squarely on plotter.

Specimen #	L x W x H (in.)	Material	X-Scale	Y-scale
1	4 x 1/4 x 1/8	steel	50%	10%
2	4 x 1/4 x 1/4	steel	20%	20%
3	8 x 1/4 x 1/8	steel	100%	10%
4	4 x 1/2 x 1/8	steel	20%	10%
5	4 x 1/4 x 1/4	aluminum	50%	20%

X-Scale:	100%	1 in. on plot	→ 0.1	in motion in crosshead
(LVDT)	50%	"	→ 0.05	"
	20%	"	→ 0.02	"
	10%	"	→ 0.01	"

			Mach. 1,2,3,5,6	Mach. 4
Y-Scale	100%	1 in. on plot	→ 200 lb.	300 lb.
(Load)	50%	"	→ 100 lb.	150 lb.
	20%	"	→ 40 lb.	60 lb.
	10%	"	→ 20 lb.	30 lb.

4) Make sure the digital load readout is set to zero, then set up one of the beams in the test fixture. The distance between the blocks should be 4 inches for the standard length specimens and 8 inches for the twice-as-long specimen. Lay rectangular specimens flat.

5) Lower the crosshead until the load display just barely registers a load, then raise the crosshead slightly so that it reads zero. (If you are using one of the newer beige testing machines, press the white reset button on the crosshead motion indicator at this point.) Next, zero the plotter so that the pen is near the upper right corner of the graph paper. Lower the pen and mark the zero coordinates.

6) Set the load/lock switch to 'lock,' and the tension/compression switch to 'down/compression.' Slowly run the experiment, using the low speed setting. Record all pertinent data on the plot: X and Y scales, beam size, material, etc.

7) Record the maximum load from the digital readout. Important! For this lab the display reads in tenths of a pound (i.e., divide reading by ten to get the actual load). The newer machines show the decimal point, so you don't have to worry about dividing.

8) Repeat the above procedure with the other beams.

QUESTIONS

1) Fill in a copy of the rectangular beam table (also see Figure III.2). List the formulae used and show an example calculation for each.
2) Using data from the table, how does stiffness depend upon width, height, modulus, and length? Use words or formulae as you see fit.
3) How does strength depend on the above parameters?

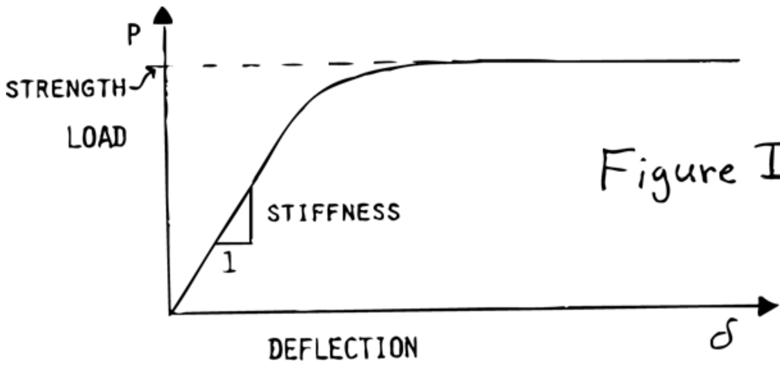


Figure III.2

4) What do you think were the most important sources of error in this experiment?

5) What would you predict for the stiffness and strength of bar 1 if it was loaded edgewise instead of flat? Compare the new (edgewise) stiffness and strength to the old (flat). Use your data from 2) and 3) to estimate these results. What would probably happen if you loaded bar 4 edgewise?

6) Can you use what you have seen here to justify the use of I-beams?

TABLE FOR RECTANGULAR BEAM LAB

Specimen Number	1	2	3	4	5
Nominal Size (in.)	4 x 1/4 x 1/8	4 x 1/4 x 1/4	8 x 1/4 x 1/8	4 x 1/2 x 1/8	4 x 1/4 x 1/4
Measured Width					
Measured Height					
Young's Modulus	$30 \times 10^6\ psi$	$30 \times 10^6\ psi$	$30 \times 10^6\ psi$	$30 \times 10^6\ psi$	$10 \times 10^6\ psi$
Length Ratio	$\frac{L\#1}{L\#1} = 1$	$\frac{L\#2}{L\#1} = 1$	$\frac{L\#3}{L\#1} = 2$	$\frac{L\#4}{L\#1} = 1$	$\frac{L\#5}{L\#2} = 1$
Width Ratio	$\frac{W\#1}{W\#1} = 1$	$\frac{W\#2}{W\#1} =$	$\frac{W\#3}{W\#1} =$	$\frac{W\#4}{W\#1} =$	$\frac{W\#5}{W\#2} =$
Height Ratio	$\frac{H\#1}{H\#1} = 1$	$\frac{H\#2}{H\#1} =$	$\frac{H\#3}{H\#1} =$	$\frac{H\#4}{H\#1} =$	$\frac{H\#5}{H\#2} =$
Modulus Ratio	$\frac{E\#1}{E\#1} = 1$	$\frac{E\#2}{E\#1} = 1$	$\frac{E\#3}{E\#1} = 1$	$\frac{E\#4}{E\#1} = 1$	$\frac{E\#5}{E\#2} = \frac{1}{3}$
Strength (lb.)					
Strength Ratio	$\frac{F\#1}{F\#1} = 1$	$\frac{F\#2}{F\#1} =$	$\frac{F\#3}{F\#1} =$	$\frac{F\#4}{F\#1} =$	$\frac{F\#5}{F\#2} =$
Stiffness (lb./inch)					
Stiffness Ratio	$\frac{S\#1}{S\#1} = 1$	$\frac{S\#2}{S\#1} =$	$\frac{S\#3}{S\#1} =$	$\frac{S\#4}{S\#1} =$	$\frac{S\#5}{S\#2} =$

THE I-BEAM

In this experiment you will measure the strain in and deflection of an aluminum 'I-beam' loaded as in Figure III.3. This beam gets its name from the fact that its cross section looks like the capital letter 'I'. Metal I-beams are used frequently in structures because they provide large strength and stiffness for a given amount of material. This is because the primary stresses in beams arise from bending moments in the beam. These moments are carried by internal tension and compression stresses in the beam. The I-beam spatially separates the material carrying these tension and compression stresses and thus can carry a large bending moment for a given amount of material.

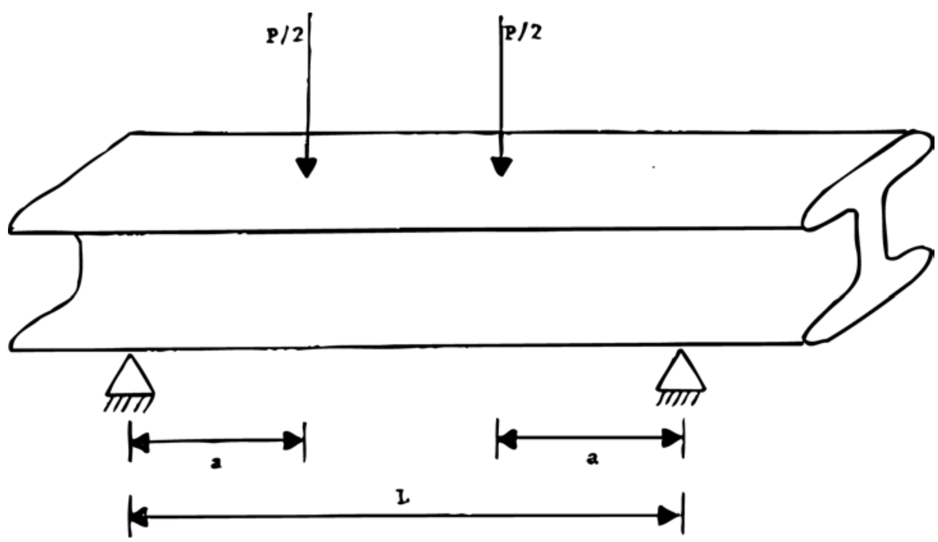


Fig. III. 3

From the measurements of strain, the stress in the beam can be calculated using the elastic moduli of aluminum. The beam shear force *V(x)* (the vertical force on a free body diagram of the beam cut at *x*) and bending moment *M(x)* (the couple on a free body diagram of the beam cut at *x*) for the beam above are left for you to determine. Given *M* and *V* it is possible, using 'technical beam theory', to quite accurately determine the stress in different parts of the beam. The theoretical predictions (from beam theory) for stress are based on numbers and equations given here and in Figures III.4 and III.5 as well as the measured value of the load.

The deflection in the middle of the beam (using beam theory and assuming a linear elastic beam) is given by

$$\text{midpoint deflection} = Pa(3L^2 - 4a^2)/(48EI)$$

where *P, a*, and *L* are defined in Figure III.3, *E* is Young's modulus and I is the area moment of inertia.

APPARATUS

- Hand cranked testing machine
- I-beam with strain gauges
- Dial gauge indicator, for measuring midpoint deflection of the beam
- Two SB-10L switch and balance boxes
- P-3500 digital strain indicator box

PROCEDURE

1) Read this handout.

2) Look over the equipment. Note the location and orientation of the gauges. The P3500 digital strain indicator box should be set to "1/4-1/2 bridge".

3) Get zero-load readings for the strain gauges and the dial gauge. Record the strain readings in the I-beam table. For gauges 1 through 10, set the rotary selector switch on the right-hand SB-10L box to "open," and set the switch on the left-hand box to the appropriate gauge number. For gauges 11 through 15, set the rotary switch on the left-hand SB-10L box to "open," and use positions 1 through 5 on the right-hand box.

4) Load the beam to 1000 pounds, watching the dial gauge carefully. If necessary, count full revolutions of the dial. A full dial revolution is equal to 0.01".
Make sure the load never exceeds 4000 pounds.
Read the dial gauge, adding in any full revolutions of the dial, then take readings for all of the strain gauges.

5) Unload the beam and repeat the zero-load readings. If either the displacement or the strain reading does not return to its original zero-load reading then something has gone wrong and you should repeat the procedure above (a few thousandths of an inch or a few microstrain are nothing to worry about, though).

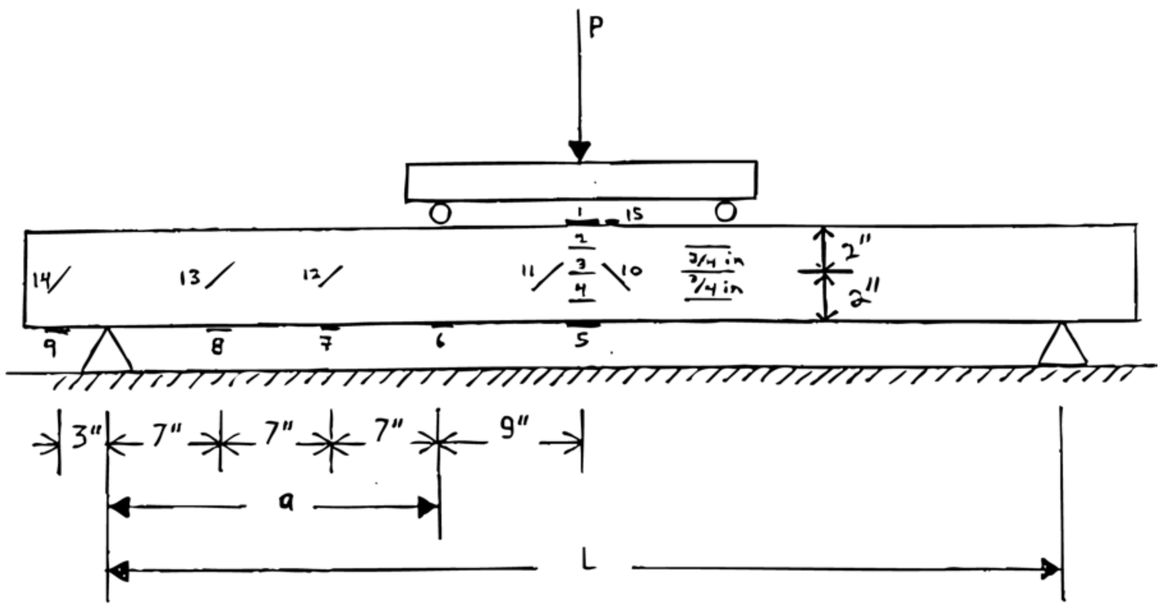
I-BEAM STRAIN GAUGE LOCATIONS

ALUMINUM:

$E = 10 \times 10^6$ LB/IN^2 $= 6.9 \times 10^6$ N/CM^2

$\nu = 0.33$

$G = E/[2(1+\nu)]$



GAUGES: 1 - 9 FOR TENSION
10 - 14 FOR SHEAR
15 FOR TRANSVERSE TENSION

Fig. III. 4

I-BEAM FORMULAE AND DIMENSIONS

FORMULAE

$\sigma = -My/I$

- I: AREA MOMENT OF INERTIA OF BEAM CROSS SECTION
- y: POSITION (DISTANCE ABOVE 'NEUTRAL AXIS') OF THE POINT OF INTEREST
- M: BENDING MOMENT IN THE BEAM
- σ: TENSION STRESS ON THE SURFACE WHOSE NORMAL IS PARALLEL TO THE BEAM AXIS

$\tau = VA'\bar{y}'/It$

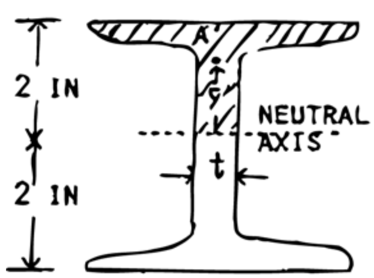
- $A'\bar{y}'$: 'Q' (SEE FIGURE)
- V: SHEAR FORCE IN THE BEAM
- τ: SHEAR STRESS AT THE CENTER OF THE BEAM

$\sigma = E\epsilon$ (TENSION STRESS) = (YOUNG'S MODULUS)(TENSION STRAIN)

$\tau = G\gamma$ (SHEAR STRESS) = (SHEAR MODULUS)(SHEAR STRAIN)

$\gamma = -2\epsilon_{45^\circ}$ (ϵ_{45° IS WHAT THE "SHEAR" GAUGES MEASURE)

DIMENSIONS



(CROSS SECTION) "4 x 9.5"

$I = \int_{\text{WHOLE SECTION}} y^2\, dA = 6.79\ \text{IN}^4$

$\bar{y}'$ = CENTROID OF THE AREA ABOVE THE CENTERLINE = 1.44 IN

A' = AREA ABOVE THE CENTERLINE = $A/2$ = 1.4 IN^2

t = 0.326 IN

Fig. III. 5

TABLE FOR I-BEAM LAB

Gauge No.	Initial Reading	Loaded Reading	Final Reading	Strain	Meas. Stress	Theor. Stress
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						
13						
14						
15						
Dial				N.A.	N.A.	N.A.

QUESTIONS ON THE I-BEAM

[Remember to follow the format given in the Introduction]

1) There are a number of strain gauges placed at various points along the beam. Using the attached formulae and your diagrams from above, calculate the predicted tension stress σ at the various gauges. Calculate the predicted shear stress τ at the shear gauge locations. (Hint: τ and σ are zero at some locations).

2) Calculate stresses from your measured strain values by multiplying by the appropriate modulus, and record them in the I-beam table.

3) Calculate your prediction of the beam deflection at a load of 1000 pounds, using the formula given above.

4) Compare all readings or calculations that you think point out agreement or disagreement between theory and experiment. Try to make clear conclusions about the relation of the theory to the experiment. Your answer should include three graphs:

   a) tension stresses vs. position along beam length,

   b) tension stresses vs. position across beam depth, and

   c) shear stress vs. position along beam length. Graph the theoretical curves and plot the data points on these graphs. Use any other clearly labeled graphs and charts to illustrate your points. Only show the data from the appropriate gauges on the appropriate graphs.

   d) What do you think were the major sources of error in this experiment?

5) Gauge 15 is mounted perpendicularly to the axis of the beam. Should it read zero? Why or why not?

LABORATORY IV
BUCKLING OF COLUMNS

Revised: July 2002

PRELAB QUESTIONS

[Due at the beginning of the lab period.]

Read through the laboratory instructions and then answer these questions:

1) What end conditions will be used in the buckling tests?
2) Of what will you make sketches?
3) Besides the sketches, what other data will be recorded?
4) Is a long column stronger than a short column?
5) Plot the predicted buckling load P_{cr} versus the column length, L, assuming elastic buckling and pinned-pinned end conditions. See sections 11.1-11.3 of *Mechanics of Materials* (Gere, 5th ed.). Make a copy of this graph to use in the lab for plotting data as it is collected.

INTRODUCTION

The strength of a structure depends upon its shape and material properties. Most structural failures occur when there is material failure in some part of the structure due to an applied load. However, some structures collapse even though none of the material in the structure has failed. This second kind of failure is generally called 'buckling' and is associated with relatively large elastic deformations. The most common example of buckling is the failure of long slender columns loaded axially in compression. In the lectures and text you will learn a simple theory of buckling which you can use to predict the buckling load of a column.

BUCKLING OF COLUMNS

The word 'column' is used to describe a rod that is loaded in compression as in Figure IV.1. At first one might think that resistance to large changes of length is important for making a column strong. For example, the yield stress in tension is what makes ropes, wires and rods resistant to tensile (pulling) loads. However, a bicycle chain is very resistant to change of its arc length but could not withstand the loading shown in Figure IV.1. Under compressive load the chain undergoes lateral deflection rather than shortening. The strength of columns comes from their resistance to bending. In classical Euler-Bernoulli buckling theory, columns loaded in compression are modeled as elastic beams with bending resistance EI. You might think about the following fairly subtle question: How does an axial load applied to a straight column cause a bending moment?

The load at which a column buckles depends, among other things, upon how the ends are constrained. This is why columns are often classified by how they are held at their ends. If an end of the rod is loaded axially but is free to move laterally and free to rotate, the end is said to be 'free.' If an end is not free to move laterally but is free to rotate, it is said to be 'pinned.' If an end is neither free to move laterally nor to rotate, it is said to be 'clamped.' There is no word for an end that is free to move laterally but not free to rotate, since there is no common structure that approximates this constraint.

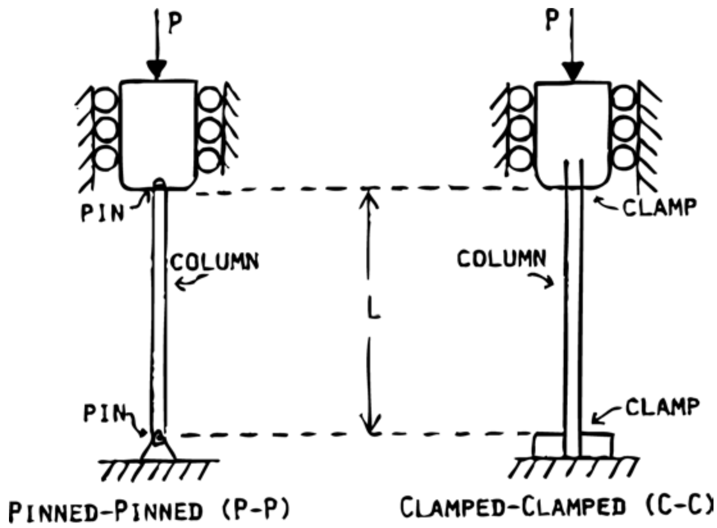


Figure IV.1

In this lab you will mostly be loading columns that are pinned at both ends (‘pinned- pinned’ columns). The ends are not in fact constrained by pinned joints but are pushed at their round ends by flat plates. We assume that this type of connection is well idealized by a ‘pin.’ Columns that are ‘pinned-pinned’ are sometimes called ‘free-free’ even though the description is not exactly appropriate. You will also load one ‘clamped-clamped’ rod to see the effect of a different end condition. How much weaker or stronger is a clamped-clamped column than a pinned-pinned column of the same length?

The classical elastic buckling theory that you will learn in class is only accurate for columns that are much longer than they are thick. For short columns collapse is due, at least in part, to compressional yielding, and the buckling failure load is not so strongly dependent upon length (see Gere’s *Mechanics of Materials*, 5th ed., Sec. 11.7).

APPARATUS:

Universal Testing Machine and X-Y recorder: You should know this fairly well by now.

SPECIMENS:

All of the specimens are nominal 1/4 inch diameter steel rods. For the pinned-pinned experiments a typical set of rods will have lengths of 2, 3, 5, 7, 10 and 15 inches. For the clamped-clamped experiment there is one 12 inch rod. This will count as a 10 inch clamped-clamped rod since each clamp uses about one inch of a rod end.

PROCEDURE:

1) You are required to wear eye protection in the lab, whenever a testing machine is in use.

2) Make careful measurements of the original dimensions of your specimens.

3) Set the load channel of the *X-Y* recorder at the level required for each sample from the table below.

4) Zero the digital load readout and set the 'lock' switch.

5) Put the longest rod (15") between the flat loading plates, putting the ends in the slight depressions in the centers of the plates. Lower the crosshead slowly so that it lightly touches the sample, and center the specimen as best you can.

6) Lower the pen on the plotter.

7) Very slowly load the sample. At least one person should carefully watch the sample while someone else stands next to the plotter. One person should mark the precise point on the force-deflection curve where lateral deflection of the sample first occurs. A folded edge of a piece of paper (or another rod) held next to the rod may aid in this observation. The approximate shape of the slightly deflected rod should be sketched.

8) Unload the rod to about 1/2 the peak load and notice that the bar straightens out.

9) Again load the rod. As the rod is further deflected you should note how the amplitude and shape of the lateral deflection evolve. Sketch the shape of the specimen for at least two deflection levels. You will need your sketches to answer question number 4.

10) Stop the experiment before the rod becomes severely bent. The ends of the rod should not exceed an angle of 20 degrees or so from the vertical. Record the peak load displayed on the digital load readout. Note on the *X-Y* plot the rod length, end conditions, and any other pertinent information. Plot P_{cr} on your plot from pre-lab question number 5.

11) Repeat the above procedure for the remaining specimens, in approximately the order shown. However, you may skip the partial unloading step. Be sure to sketch the shapes of the clamped-clamped and the 2-inch specimens as they are loaded (additional sketches are optional).

BUCKLING LAB

Exp. #	Length (in)	Boundary Condition	*X*-Scale	*Y*-Scale
	15	Pinned-Pinned (P-P)	50%	10%
	12 (10)	(10 in) Clamped-Clamped (C-C)	50%	20%
	10	P-P	50%	20%
	7	P-P	50%	20%
	5	P-P	50%	20%
	3	P-P	50%	50%
	2	P-P	50%	50%
	Soda Can	Not Applicable	100%	10%

X-Scale:	100%	1 in. on plot	→ 0.1	in motion in crosshead
(LVDT)	50%	"	→ 0.05	"
	20%	"	→ 0.02	"
	10%	"	→ 0.01	"

			Mach. 1,2,3,5,6	Mach. 4
Y-Scale	100%	1 in. on plot	→ 2000 lb.	3000 lb.
(Load)	50%	"	→ 1000 lb.	1500 lb.
	20%	"	→ 400 lb.	600 lb.
	10%	"	→ 200 lb.	300 lb.

12) For all of the above you should be thinking about two things and their relation:
    a) Load when the rod visibly buckles.
    b) Peak load (on load cell and on plot).

13) *Optional:* Test a 4-inch specimen with clamped ends, and compare its buckling load to that of the 2-inch pinned specimen. Or, test an 8-inch specimen with only one clamp, and compare its buckling load to that of a 7-inch pinned specimen.

COMPRESSION FAILURE OF CANS

1) Find a machine set up for compression.
2) Adjust the crosshead so a soda can will barely fit between the compression flats.
3) Using the LVDT, plot load vs. deflection as you very slowly compress the can at a constant rate. Watch the can as the curve is being drawn. Don't exceed a load of 1000 lbs. and be sure to use the low speed range.
    - precisely mark pen position at first noticeable deflection of can
    - sketch shape of can
    - seriously deform, then sketch shape of can.
    - stand on a can (carefully!) and have your partner gently tap it with a long rod.

QUESTIONS

[Remember to follow the form given in the Introduction]

1) If you have not already done so, complete the plot giving experimental and (elastic) theoretical values of P_{cr} vs. L for the pinned{pinned columns. Label the plot, etc., so it is neat and informative. Also calculate and compare the (elastic) theoretical values of P_{cr} to your experimental values for clamped specimen(s).
2) Discuss the results.
    a) At what value of L do the theoretical and experimental curves deviate? What accounts for the difference? What other differences are there, and what do you think caused them?
3) How was the load at first noticeable lateral deflection related to the peak load?
4) How did the shape of the columns as they bent change with length and end conditions?
5) Comment on the load vs. displacement plots. Label any interesting features on them.
6) Comment on the soda can experiment.