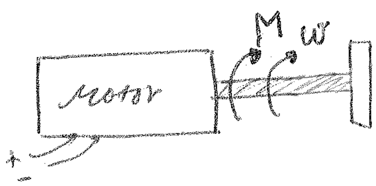
Help for HW-8

NOTE $\mathfrak{r} = r$ — Indian, ?

Q1. This is a unit conversion problem, people can do that I guess.

Q2 Basic information you need to understand:

→ Motor curve

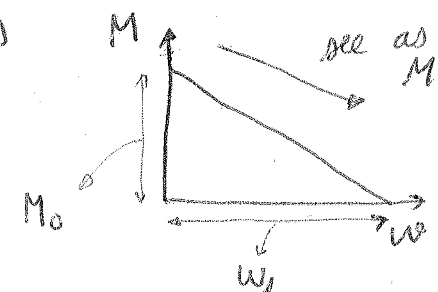


- motor gives a torque (moment) = M
- and rotates at angular velocity omega = ω

- M and ω of a motor are not independent
- if you expect high speed ω, 'M' will be small
  if you expect high torque M, 'ω' will be small
- Ideally this is expressed as

$$\boxed{M = M_0 - c\,\omega}$$

M_0, c → constants



see as ω increases M decreases!

→ Moment $M = M_0$ if $\omega = 0$ → stalling, Max Moment)

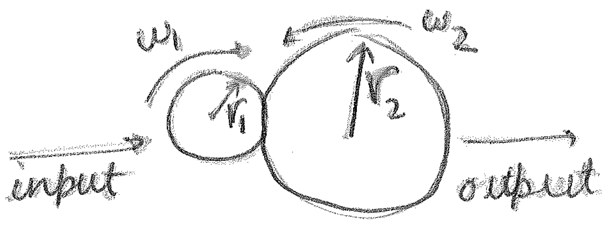
→ when $M = 0$, $\omega = \omega_f$ } $M_0 - c\omega_f = 0$ → no load rpm

$\therefore$ $\boxed{\omega_f = \dfrac{M_0}{c}}$

→ • ~~gea~~ muscles, like of the biker, in this problem behave similar to motor → they give more force (torque) at low velocity → ~~low~~ low force at high speed.

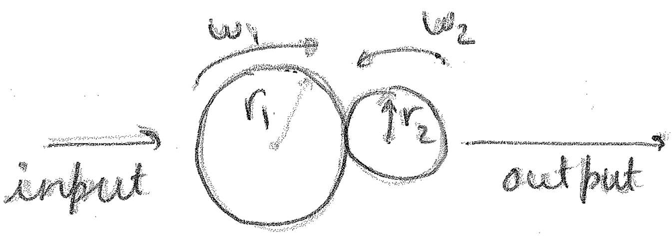
gears

→ they are used to step up or (more commonly) step down the speed.

•) 

output input

$$\omega_2 = \frac{r_1}{r_2}\,\omega_1 \quad \Bigg\} \quad \begin{array}{c}\text{step down}\\ r_2 > r_1\end{array}$$

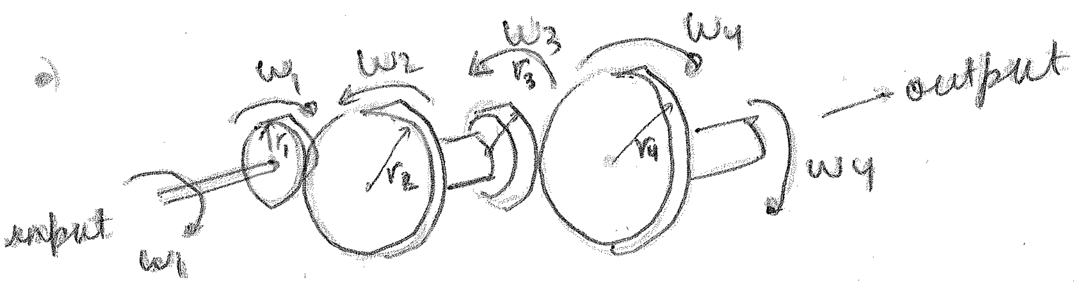
•) 

output input

$$\omega_2 = \omega_1 \cdot \frac{r_1}{r_2} \quad \Bigg\} \quad \begin{array}{c}\text{step up}\\ r_2 < r_1\end{array}$$

Hint: if confused between $\frac{r_1}{r_2}$ or $\frac{r_2}{r_1}$ just remember smaller gear is faster always.

• or simply $\omega_1 r_1 = \omega_2 r_2$

•) 

→ see that $\omega_2 = \omega_3$ → ∵ on same shaft

→ $\omega_1 r_1 = \omega_2 r_2$

$\omega_3 r_3 = \omega_4 r_4$

$$\Longrightarrow \omega_1 r_1 = r_2\left(\frac{\omega_4 r_4}{r_3}\right)$$

$$\Rightarrow \omega_4 = \frac{r_1 r_3}{r_2 r_4}\,\omega_1$$

ω_4 → output, ω_1 → input

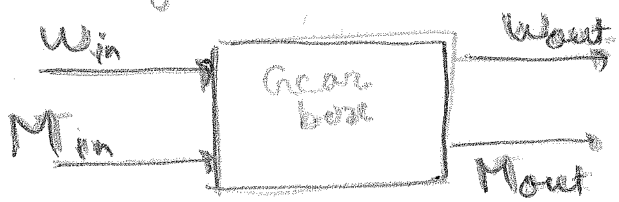
•) NOTE: in all of above, radius of gear can be replaced with number of teeth it has, because for meshing teeths have same thickness and hence more radius, means more circumference, means more number of teeth.

- if speed is stepped up (or stepped down) Torque is stepped down (or up) by same factor.

so that $\boxed{\text{power}_{in} = \text{power}_{out}}$

$$(M_{in}\, \omega_{in}) = (M_{out}\, \omega_{out})$$

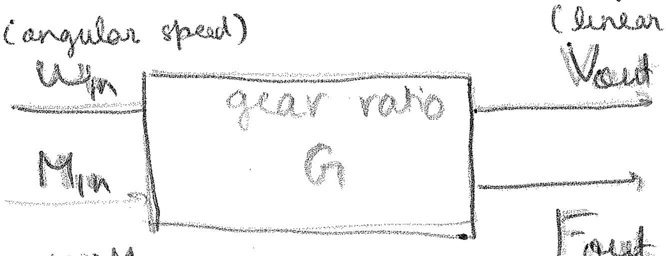
- therefore you should picture gear box as told in class

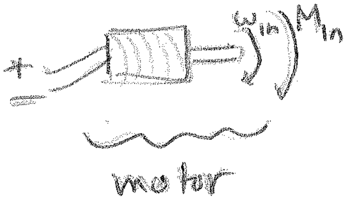


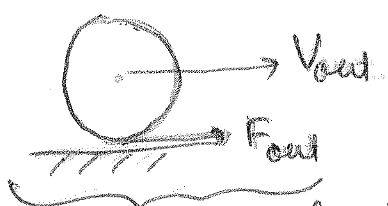
$$\omega_{out} = \frac{\omega_{in}}{(\text{same factor})}, \quad M_{out} = M_{in} \cdot (\text{same factor})$$

(if this factor > 1, speed is reduced, torque is increased
" " < 1, speed is increased, torque is decreased

- In class for easiness we define gear ratio 'G' as







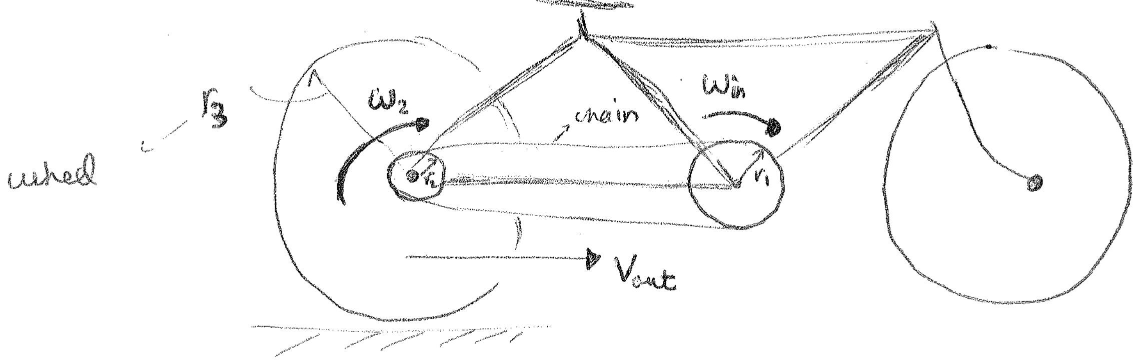
velocity and friction of wheel

$$\therefore \boxed{V_{out} = \frac{\omega_{in}}{G}, \quad F_{out} = G\, M_{in}}$$

$\boxed{\text{power}_{in} = M_{in}\,\omega_{in} = \text{power}_{out} = F_{out}\, V_{out}}$

- $\omega_{in}, M_{in} \longrightarrow$ motor speed & Torque $(M_{in} = M_0 - C\,\omega_{in})$
- $V_{out}, F_{out} \longrightarrow$ speed of wheel (hence bike) & force of friction which drives the bike.

Q2a find 'G' for a real simple bike.



$\omega_{in} \rightarrow$ angular speed of pedalling (rad/sec)

$V_{out} \rightarrow$ " " bike (m/s)

$r_1 \rightarrow$ radius of front 'gear'

$r_2 \rightarrow$ " " rear 'gear'

$r_3 \rightarrow$ " " wheel

$\omega_2 \rightarrow$ ang. speed of rear 'gear' (rad/sec).

∴

- $\omega_2 r_2 = \omega_{in} r_1$
- ω_2 is also angular speed of wheel (rear gear and wheel are one 'body')
- V_{out} = velocity of wheel = $\underbrace{\omega_2}_{\text{angular speed of wheel}} \times \underbrace{r_3}_{\text{radius of wheel}}$ } equation for a rolling wheel

$$\therefore V_{out} = r_3 \frac{r_1}{r_2} \omega_{in} = \frac{\omega_{in}}{\left(\frac{r_2}{r_3 r_1}\right)} = \frac{\omega_{in}}{G}$$

$$\therefore \boxed{G = \frac{r_2}{r_1 r_3}} = \text{also } \frac{\text{number of teeth on rear 'gear'}}{\text{radius of wheel} \times \text{number of teeth on front 'gear'}} = \boxed{\frac{N_2}{r_3 N_1}}$$

NOW PLUG IN YOUR NUMBERS

Q2 b) • fastest the rider turns crank $= \omega_f =$ no load rpm

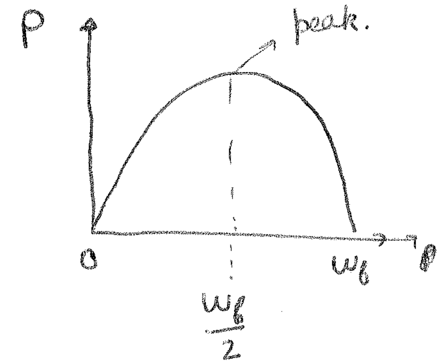
$= 180$ rev per min

• peak power of the rider $= .5$ Hp.

→ $\text{Power} = M\omega = (M_0 - c\,\omega)\,\omega$

$= M_0\omega - c\,\omega^2$

• power of rider is function of angular velocity

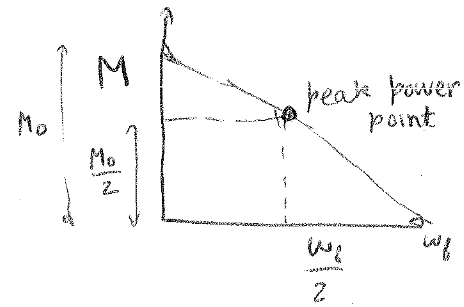


• $P = 0$ when $\omega = 0$

$P = 0$ when $\omega = \omega_f$

because $\boxed{\omega_f = \frac{M_0}{C}}$ → see page (1)

$$P = M_0\left(\frac{M_0}{C}\right) - C\left(\frac{M_0}{C}\right)^2 = 0$$



→ at peak, ~~slope~~ slope $= \frac{dP}{d\omega} = 0$

$$\therefore \frac{d}{d\omega}\left(M_0\omega - c\,\omega^2\right) = 0$$

$$M_0 - 2c\,\omega = 0 \quad \text{at peak.}$$

REMEMBER

$$\therefore \boxed{\omega_{\text{at peak}} = \frac{M_0}{2C} = \frac{1}{2}\,\omega_f}$$

$$\boxed{M_{\text{at peak}} = M_0 - C\,\omega_{\text{at peak}} = M_0 - C\,\frac{M_0}{2C} = \frac{1}{2}M_0}$$

$$\boxed{\text{Power at peak} = \omega_{\text{peak}} \times M_{\text{peak}} = \frac{1}{4}M_0\,\omega_f}$$

$$\therefore\ .5\text{ hp} \times \frac{746\text{ W}}{1\text{ hp}} = \frac{1}{4}M_0 \times 180\,\frac{\text{rev}}{\text{min}} \times \frac{2\pi\text{ rad}}{1\text{ rev}} \times \frac{1\text{ min}}{60\text{ sec}}$$

(6)

$$\therefore M_0 = \frac{746 \times .5 \times 4 \times 60}{180 \times 2\pi}$$

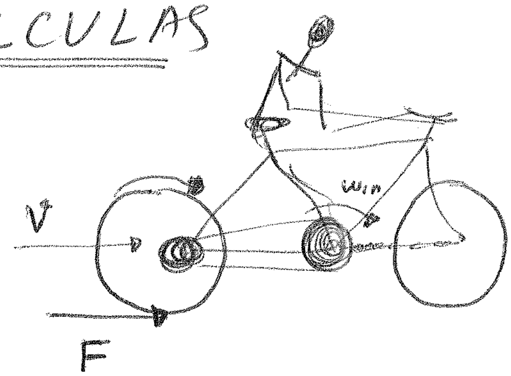
$$\boxed{M_0 = 79.153 \quad Nm}$$

$$\omega_f = \frac{M_0}{C} \Rightarrow C = \frac{M_0}{\omega_f} = \frac{79.153 \ Nm}{\left(\frac{180 \times 2\pi}{60}\right) \ rad/s}$$

$$\boxed{C = 4.199 \quad \frac{Nm\,s}{rad}}$$

2c i)

BY CALCULAS



approximate Free body diagram of bike + rider

i)

- assuming very little (negligible) force on front wheel, because its light compared to (rider + bike) mass and needs very little force to rotate

(you'll understand more in 203)

- only force (external to bike + rider) = F on rear wheel
- by Newton's law

$$F = m\,a = m\frac{dv}{dt}$$

$$\boxed{\begin{aligned} m &= 150 \ lbm \\ &= 68.038 \ kg \end{aligned}}$$

when you pedal, you rotate the rear wheel., if there was no friction it would slip its due to friction F; that opposes this slipping; that the bike moves forward & wheel rolls.

from before

$$F = GM \quad \text{(moment by biker)}$$

$$V = \frac{\omega}{G} \quad \text{(angular speed by bike at crank)}$$

page 3

$$\therefore \cancel{GM = m \frac{d}{dt}\left(\frac{\omega}{G}\right) = \frac{m}{G}}$$

and

$$M = M_0 - c\omega = M_0 - c\,(GV)$$

page 1

$$\therefore F = m\frac{dv}{dt} = G\,(M_0 - GcV)$$

$$\therefore \boxed{m\frac{dv}{dt} = GM_0 - G^2 c\,V}$$

but also $c = \dfrac{M_0}{\omega_f}$ from page 6, $\omega_f = \dfrac{4P_{peak}}{M_0}$ from page 5

$$\therefore c = \frac{M_0^2}{4P_{peak}}$$

$$\boxed{m\frac{dv}{dt} = GM_0 - \frac{G^2 M_0^2}{4P_{peak}}\,V}$$

very imp will be used in future HWs

Now from this diff equation, we can integrate to get $V(t)$, $x(t)$ and all good things of life.

NOTE you can also use power given out by motor = change of K.E of bike.

$$M\omega = \frac{d}{dt}\left(\frac{1}{2}mv^2\right)$$

this gives the same above differential equation

$$\frac{dv}{dt} = \underbrace{\frac{GM_0}{m}}_{A} - \underbrace{\frac{(GM_0)^2}{4P_{peak}\, m}}_{B} V = A - BV$$

$$\int_0 \frac{dv}{A - BV} = \int_0 dt$$

$$\frac{\ln(A - BV)}{-B} = t + C$$

$\boxed{\text{at } t = 0 \quad V = 0}$ $\therefore$ $C = \frac{\ln(A)}{-B}$

$$\therefore -\frac{\ln(A - BV)}{B} = t - \frac{\ln A}{B}$$

$$t = \frac{\ln A - \ln(A - BV)}{B} = \frac{\ln\left(\frac{A}{A - BV}\right)}{B} = \frac{\ln\left(\frac{1}{\left(1 - \frac{B}{A}V\right)}\right)}{B}$$

$$\boxed{t = -\frac{\ln\left(1 - B/A\, V\right)}{B}}$$

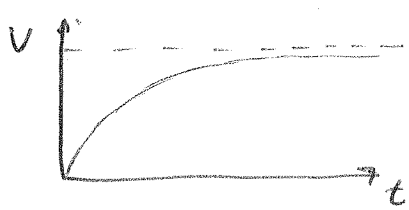
$$\therefore \left(1 - \frac{B}{A}V\right) = e^{-Bt}$$

$$\therefore V = \frac{A}{B}\left(1 - e^{-Bt}\right)$$

$$\boxed{V = \frac{4P_{peak}}{GM_0}\left(1 - e^{-\frac{(GM_0)^2}{4mP_{peak}} t}\right)}$$

now u can plot this using
$P_{peak} = .5 \times 746$ W
$M_0 = 79.153$ Nm
$G =$ whatever you had in 2a $\left(\frac{1}{m}\right)$
$m = 68.038$ kg

plot will look like



$\boxed{\frac{4P_{peak}}{GM_0}}$ $\rightarrow$ V as $t \rightarrow \infty$

Q2c ii

$$\rightarrow \quad v = \frac{dx}{dt} = \frac{4P_{peak}}{GM_0}\left(1 - e^{-\frac{(GM_0)^2}{4mP_{peak}}t}\right)$$

$$\rightarrow \quad \int dx = \frac{4P_{peak}}{GM_0}\left[\int \left(1 - e^{-\frac{(GM_0)^2}{4mP_{pe}}t}\right) dt\right]$$

$$x = \frac{4P_{peak}}{GM_0}\left[t + \frac{e^{-\frac{(GM_0)^2}{4mP_{peak}}t}}{\frac{(GM_0)^2}{4mP_{peak}}}\right] + C$$

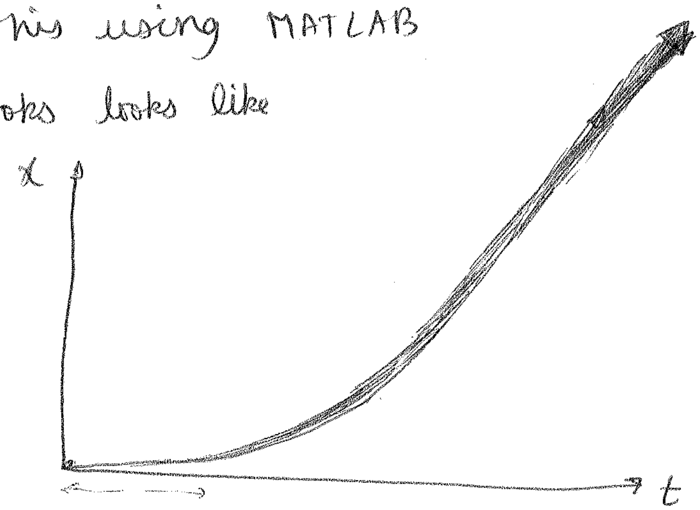
$x = 0$ at $t = 0$

$$\therefore \quad 0 = \frac{4P_{peak}}{GM_0}\left(\frac{4mP_{peak}}{(GM_0)^2}\right) + C$$

$$\therefore \quad \boxed{x = \frac{4P_{peak}}{GM_0}\left[t + \frac{\left(e^{-\frac{(GM_0)^2}{4mP_{peak}}t} - 1\right)}{\frac{(GM_0)^2}{4mP_{peak}}}\right]}$$

plot this using MATLAB

it looks ~~looks~~ like



if you are plotting ~~for~~ small time range it might look like a line

Q2c iii

inverting the velocity equation on page 8

$$\boxed{t = \frac{-\ln\left(1 - \frac{G M_0}{4 P_{peak}} v\right)}{\frac{(G M_0)^2}{4 P_{peak} m}}}$$

plug in $v = 20 \text{ mph} = \frac{20 \times \cancel{16}09.3 \text{ m}}{3600 \text{ sec}} = 8.94$

get t

$\boxed{\text{your answer depends on } G}$

Q 2c iv

use above t and plug it in the distance relation on page 9

Q2c USING MATLAB

idea:

- we need to find x and v for all times.
- we know their rate of change, in terms of each other. **

$$\dot{x} = v$$
$$\dot{v} = a = F/m = \frac{G M_0}{m} - \frac{G^2 M_0^2}{4 m P_{peak}} v$$

conceptualize / visualize

- ~~define~~ a vector z of two numbers, let first be x and second be v

$z(1) = x; \quad z(2) = v; \qquad z = [\,:\,]$

- define a function (lets call it myfun) which takes in two numbers in form of a vector z ; and return the rate of change of both numbers in a vector (lets say zdot)

```
function    zdot = myfun (t, z)

% a number t and a vector z is given to this function
% we need to find zdot just using that info.

% to make it more intuitive extract and rename the
% numbers from z.

    x = z(1);
    v = z(2);

% to find their rate of change we need G, m, Mo, Ppk.

    G = ---- ;          % put it here
    m = 68.038 ;
    Mo = 79.153 ;
    Ppk = .5 * 746 ;

% define rate of change in terms of some new variables

    xdot = v;           % v is defined above.
    vdot = G*Mo/m - (G*Mo)^2/(4*m*Ppk) * v ;

% repack these into a vector which tells rate of
% change of numbers in z, thats what we want

    zdot = [xdot ; vdot] ;

end.
```

• now write a main function which uses this info

• to solve ode we need 1) a function which defines it (myfun)

2) initial values of the two numbers (zzero)

3) time interval you want to ~~find~~ solution for. call it tspan.

```
function ode

tspan = [0 100] ;    % put a big number like 100 hopefully
zzero = [0 ; 0] ;    % v will reach 20 mph during that
                     % otherwise increase it.

[t zarray] = ode45 ( @myfun , tspan, zzero) ;

    % this solves the ode ie finds the
    % values of both number is z for
    % various times between 0 and 100
    % and stores them in [t zarray]
    % which is
```

$$\begin{array}{cl} \begin{bmatrix} 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 100 & \cdot & \cdot \end{bmatrix} & \begin{array}{l} \rightarrow t=0 \quad Z = [0 ; 0]; \\ \\ \rightarrow t=100 \quad Z = [\text{someth} ; \text{someth}]; \end{array} \\ t \quad \text{zarray} & \end{array}$$

```
% extract the x data and v data from zarray, these are vectors
% containing x and v values at times in 't'
  x = zarray (: , 1) ;      % all rows and 1st column of zarray
  v = zarray (: , 2) ;      %   "    "   2nd   "    "    "

plot (t, v) ;    % for part 2ci
plot (t, x) ;    %  "   "   2cii

% for parts 2ciii and 2civ you can manually zoom
% in these plots and get time where v = 8.9408m/s and
% then find x for this time by zooming in on x plot
% otherwise we can interpolate as follows.
% 'v' contains velocity only for only some selected times specified in 't'
% chosen by ode 45 bet 0 and 100 , automatically.
```

```
% it probably wont have a number. 8.94. So we
% can find a number just below 8.94 and just above it
% and interpolate to get t for 8.9408

i = 0;
n = length (t);    % the size of t, v, x vectors.

while ( v(i) < 8.9408)
        i = i+1;
end.

% at the end of it  i = number such that v(i) is just
% greater than 8.9408.

display ('t. for v= 20 mph is ')
T= t(i-1) + ((t(i) - t(i-1))/(v(i) - v(i-1)))*(8.9408 - v(i-1));

display ('distance at that time')
X = x(i-1) + ((x(i) - x(i-1))/(v(i) - v(i-1)))*(8.9408 - v(i-1))

end
```

---

without all the comments code is real small

```
function ode.
[t zarray]= ode45 (@ myfun, [0 100], [0;0]);
x = zarray (:,1) ; v = zarray (:,2)
plot (t,v); plot (t,x);
T= copy from above ;   X = copy from above ;
end
function zdot = myfun (t,z)
zdot = [ z(2) ; (G* 79.153/68.038 - (G*79.153)^2/(4*68.038*.5*746)* z(2))];
                                                     % put value of G
end.
```

∘) define a function which takes in two numbers
ie the vector z and time t and returns the rate of change of
these, at that particular 't'.
calle it · my rhs.

∴ function zdot = myrhs (t, z)

% a time number t and a vector 'z' is given to this.
% lets extract x & v from z, just to make it inductive

x = z(1) ;
v = z(2) ;

% find their rate of change

xdot = v ;
vdot =