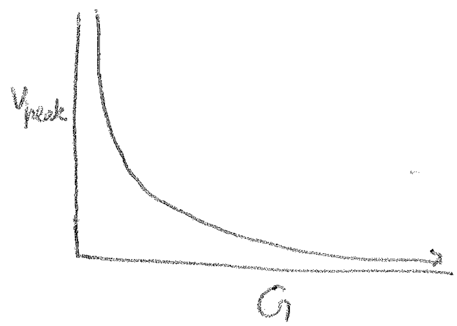
Help for HW-9

Q. 1a). all equations are same as HW-8

from page (8) of HW-8 see the figure at the end.

$$V_{peak} = \frac{4 P_{peak}}{G M_0} = \frac{4 \times .5 \times 746 / 79.153}{G}$$

plot is

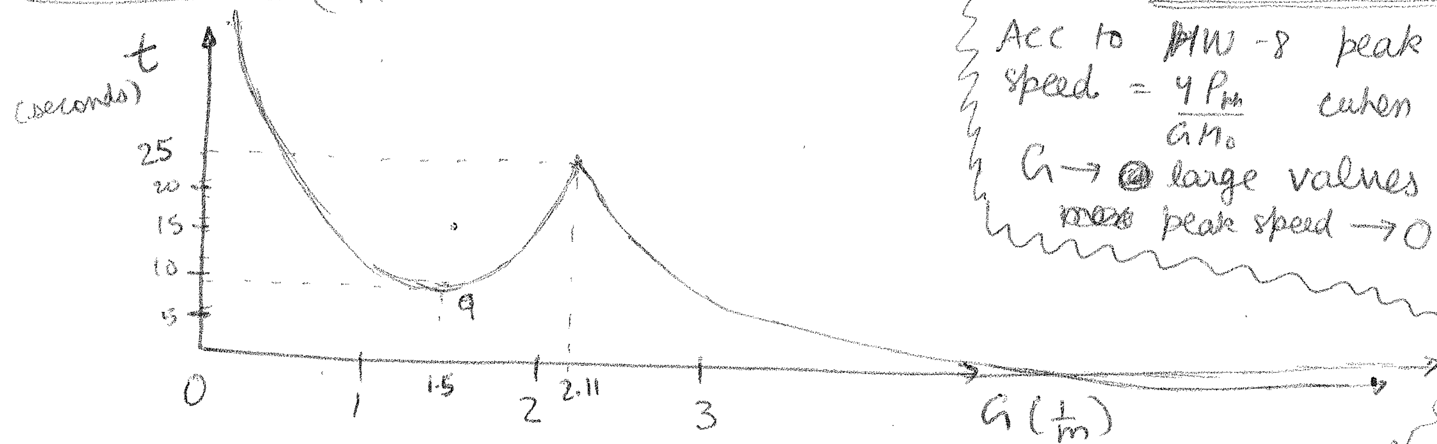


Q 1b) from page (10) of HW-8

$$t = \frac{-\ln\left(1 - \frac{G M_0}{4 P_{peak}} v\right)}{\frac{G^2 M_0^2}{4 m P_{peak}}}$$

Here put
- $V = 8.94$ (20 mph)
- $P_{peak} = .5 \times 746$ W
- $m = 68.038$ kg
- $M_0 = 79.153$

MATLAB plot looks like (approximately)



FOR CURIOUS PEOPLE

Acc to HW-8 peak speed $= \frac{4 P_{pm}}{G M_0}$ when $G \to$ large values peak speed $\to 0$

So it will be impossible to go to 20 mph After a particular G

1b is slightly misleading. NOTICE ~~at~~ t goes negative!! MATLAB also gives warning that its ignoring imaginary parts because ~~log~~ ~~is imaginary~~ log (negative numbers is imaginary

c) from page (9) of HW-8

$$x = \frac{4P_{peak}}{GM_0}\left[t + \frac{\left(e^{-\frac{(GM_0)^2 t}{4mP_{peak}}} - 1\right)}{\frac{(GM_0)^2}{4mP_{peak}}}\right]$$

from Hoo. • t = 10 find x.

• use any G you like

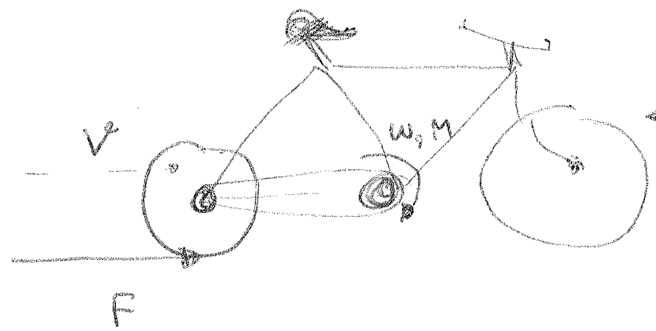
<u>using CODE</u>

~~>> x0 = 0 ; V0 = 0 ; t = 0 / h = .001 ; t = 0 ;~~
~~>> while (t < 10)~~

```
% initialize
  x=0 ; V=0 ; t=0 ; h= .001 ;   % h is time step.
  G= whatever; Mo = 79.153; m = 68.038; Ppk = .5*746 ;
while ( t <= 10 )
  a = G*Mo/m - (G*Mo)^2/(4*Ppk*m)*V ;   % acceleration
                                        % from page (7)
                                        % HW-8
  x = x + h*V ;
  V = V + a*h ;
  t = t + h ;
end

% this loop keeps on updating x and V every time step
% of length h = .001   untill t = 10 s

 display('x at t=10 is')
 display( x )

% answer depends on G, if it doesn't match the
% calculus answer from above , make the time
% step h smaller , this increases accuracy but takes
% more computer time ( why? )
```

Q2 a). now we have drag force.

lets revisit page ⑥ of HW-8 to get new equation for motion.



$F_{drag} = \frac{1}{2} C_d \rho A V^2$

g air density; frontal area of bike + biker; speed

by Newton's law

- $F - F_{drag} = ma$.
- as before $F = GM$

$\omega = GV$

$M = M_0 - C\omega = M_0 - \frac{M_0^2 \omega}{4 P_{peak}}$ (motor law) ($C = \frac{M_0^2}{4 P_{peak}}$ go to page ⑦ HW-⑧)

$$\boxed{G M_0 - \frac{(G M_0)^2}{4 P_{peak}} V - \frac{1}{2} C_d \rho A V^2 = m \frac{dV}{dt}}$$

NOTE: Power by motor = $M\omega$ = change in K.E + lost in drag

$M\omega = \frac{d}{dt}\left(\frac{1}{2} m v^2\right) + F_{drag} V$ → this gives same equation

to find peak speed

- its difficult for this course level to integrate the above differential equation (probably).
- we don't need to solve
- at peak speed $\frac{dV}{dt} = 0$

$$\therefore G M_0 - \frac{(G M_0)^2}{4 P_{peak}} V - \frac{1}{2} C_d \rho A V^2 = 0 \Big|_{at\ V = V_{peak}}$$

solve this quadratic formula

$$V_{peak} = \frac{\dfrac{(G M_0)^2}{4 P_{peak}} \pm \sqrt{\dfrac{(G M_0)^4}{16 P_{peak}^2} + 2 C_d \rho A G M_0}}{- C_d \rho A}$$

<u>NOTE</u> term of under $\sqrt{\ }$ is bigger than $\frac{G M_0^2}{4 P_{peak}}$ because $\sqrt{x^2 + \text{something}} > x$

- and denominator is negative.
- we want $V_{peak} > 0$ $\therefore$ numerator is negative too.
- $\therefore$ of the two roots above pick the one with $-$ve sign so that $V_{peak} > 0$

Many people chose negative root in their HW, its wrong.

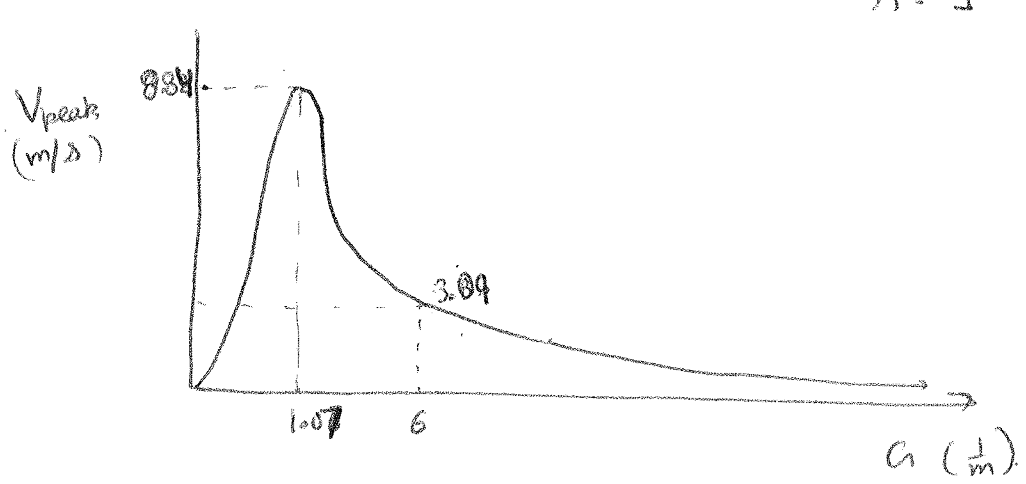
<u>Hence</u>

$$\boxed{V_{peak} = \frac{\sqrt{\left[\dfrac{(G M_0)^2}{4 P_{peak}}\right]^2 + 2 C_d \rho A G M_0} - \dfrac{(G M_0)^2}{4 P_{peak}}}{C_d \rho A}}$$

<u>its plot wrt G from MATLAB:</u>

$M_0 = 79.153$, $P_{peak} = .5 \times 746$

$A = 1$ $C_d = .9$ $\rho = 1.2$



Q 2b

we will just numerically integrate the expression

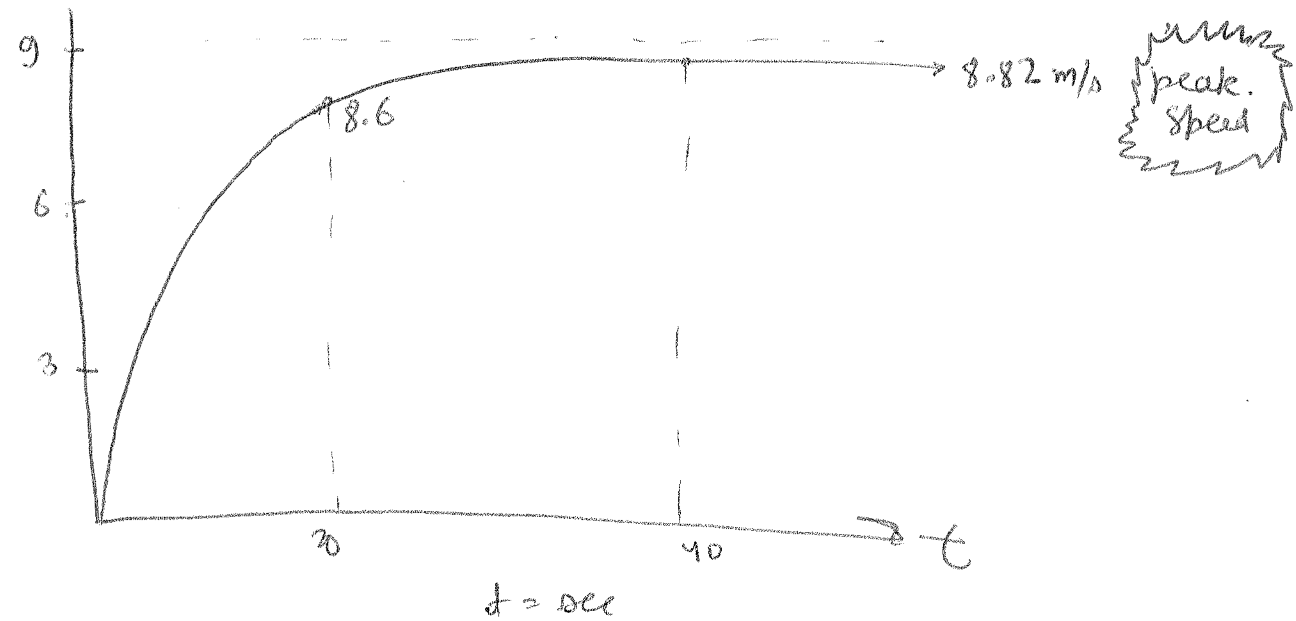
$$m\frac{dv}{dt} = GM_0 - \frac{(GM_0)^2}{4P_{peak}} V - \frac{1}{2} C_d \rho A V^2$$

using the routine in 1(b)

```
% initialise
   x=0 ; V=0 ; t=0
   h = .01          % time step
   G=1 ; Ppk= .5*746 ; m= 68.038 ; Mo = 79.153 ; Cd = .9 ; R=1.2 ; A=1
while  ( t <= 60 )
   a = G*Mo/m - (G*Mo)^2 /(4*Ppk m)*V - 1/2*Cd*R*A*V^2 ;
   x = x + h*V;
   V = V + a*h ;
   t = t + h ;
end
```

```
% initialise
  h = .01 ;          % time step
  t = 0:h:60 ;
  x = zeros(1, length(t)); V = zeros (1; length(t));
% parameters
  G=1; Ppk = .5*746 ; m = 68.038 ; Mo = 79.153 ; Cd = .9 ; R=1.2 ; A=1;
for i = 1:(length(t)-1)        % so that final length of v is same as t  i=1 to length(t)-1
  a = G*Mo/m - (G*Mo)^2/(4*Ppk*m)*V(i) - 1/2*Cd*R*A*V(i)^2 /m ;
x(i+1) = x(i) + h*V(i) ;
v(i+1) = V(i) + a*h ;
end
   plot ( t, x ) ;
```

result is



NOTICE that in 2(a)

→ the $(V_{peak})_{max}$ is at $G = 1.07$ and $V_{peak} = 8.84$

→ Here in 2(b). $G = 1$ and $V_{peak} = 8.82$

close!