2

HW 11: due on Tue Dec 1, in class.

Final HW :)

1) Draw a truss with at least 3 bars, more if you dare. Load it with forces as you like. Draw this neatly. Calculate the tensions in all of the bars.

2) Assume a motor following the standard motor law always operating at its peak (i.e., like an electric motor at fixed
voltage, not peak power). The transmission is lossless gear box. The motor has peak power P_p. Assume the wheels never skid. Other key numbers:

m = total mass of car (1000 kg)
P_p = 100 hp
rho = density of air (1 kg/m^3)
A = cross sectional area (2 m^2)
v = present speed of car (variable)
c_d = drag coefficient (1.0)
F_f = constan friction force (1000 N)
F_0 = force at wheel if the car is still (depends on gear ratio).
v_f = speed of car when motor supplies no torque.

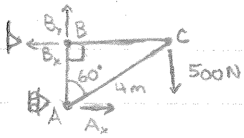
The car is slowed by air drag:

Air drag = rho * A * c_d * v ^2/2.

and by a constant friction force F_f.

a) Assuming the gearing is such that the stall force at the wheels is F_0 = 10,000 N (this would be the
gearing that would make the car almost skid if the friction coefficient was 1). Graph the position vs time for 20 seconds.
b) Find the fixed gear (characterized by F_0) that minimizes the time to reach 60 mph. What is that time?
c) Assume that you had a continuously variable transmission so that the P_p was always delivered to the wheels. What then would be the time to reach 60 mph? (This problem is easier than (b)).

10/10

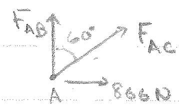
1. 

$\Sigma M_B = (4\cos 60°)A_x - 500(4\sin 60°) = 0$

$A_x = 500\tan 60° = 866\text{ N}$

$\Sigma F_x = A_x - B_x = 0 \quad A_x = B_x = 866\text{ N}$

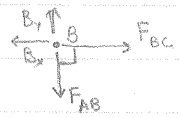
$\Sigma F_y = B_y - 500 = 0 \quad B_y = 500\text{ N}$



$\Sigma F_x = F_{AC}\sin 60° + 866 = 0 \quad F_{AC} = \frac{-866}{\sin 60°} = -1000\text{ N}$

$\Sigma F_y = F_{AB} + F_{AC}\cos 60° = 0 \quad F_{AB} = -F_{AC}\cos 60°$

$F_{AB} = 1000\cos 60° = 500\text{N}$



$\Sigma F_x = F_{BC} - B_x = 0 \quad F_{BC} = B_x = 866\text{N}$ ✓

$\Sigma F_y = B_y - F_{AB} = 0 \quad F_{AB} = 500\text{N}$

$F_{AB} = 500\text{N} \quad F_{AC} = -1000\text{N} \quad F_{BC} = 866\text{N}$

2. a. $F_{net} = F - F_{drag} - F_f \quad F = GM \quad M = M_o - CW \quad GM_o = F_o$

$F = F_o - CGW \quad W = GV \quad C = \frac{M_o}{W_f} \quad P_p = \frac{M_o W_f}{4} \quad W_f = \frac{4P_p}{M_o}$

$C = \frac{M_o^2}{4P_p} \quad F = F_o - \frac{M_o^2}{4P_p}G^2V = F_o - F_o^2\frac{V}{4P_p}$

$F_{net} = F_o\left(1 - F_o\frac{V}{4P_p}\right) - \rho A c_d \frac{V^2}{2} - F_f \quad F = ma$ ✓

$a = \left[F_o\left(1 - F_o\frac{V}{4P_p}\right) - \rho A c_d\frac{V^2}{2} - F_f\right]/m$

Matlab function and graph attached. ✓

b. Matlab function and graph attached. ✓

c. $P = \text{constant} = P_p \quad P = FV \quad F = \frac{P}{V}$

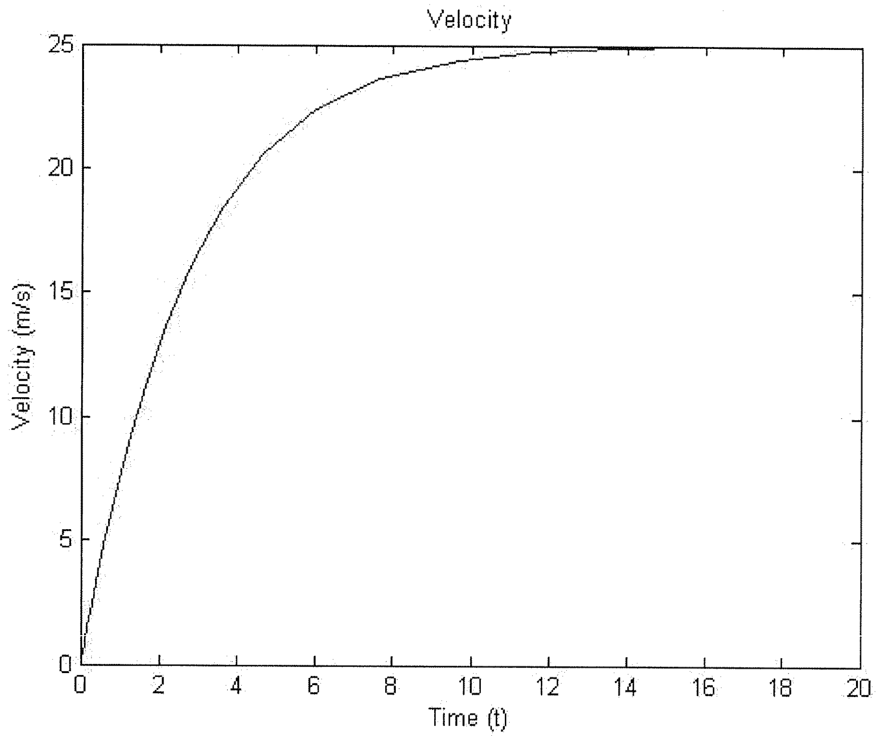
$F_{net} = \frac{P}{V} - F_{drag} - F_f \quad F = ma$

$a = \left(\frac{P}{V} - \rho A c_d\frac{V^2}{2} - F_f\right)/m$ ✓

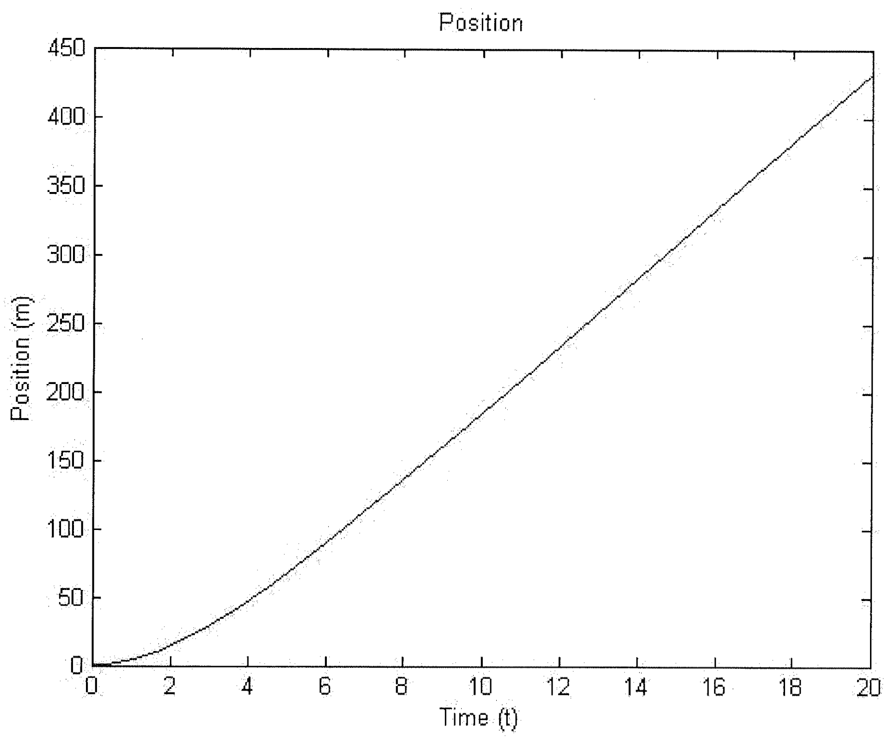
Matlab function and graph attached.

2.
a.

```
function myfun
p.P_p = 100*746; %Watts
p.rho = 1; %kg/m^3)
p.m = 1000; %kg
p.A = 2; %m^2)
p.c_d = 1;
p.F_f = 1000; %N
p.F_0 = 10000; %N

v_0 = 0;
x_0 = 0;
z_0 = [v_0 x_0];
tspan = [0 20];

[t z] = ode23(@myfn,tspan,z_0,[],p);

v = z(:,1);
x = z(:,2);

figure(1)
plot(t,v)
xlabel('Time (t)')
ylabel('Velocity (m/s)')
figure(2)
plot(t,x)
xlabel('Time (t)')
ylabel('Position (m)')
end

function dz = myfn(t,z,p)
v = z(1);
x = z(2);

prop = p.F_0*(1 - (p.F_0/(4*p.P_p))*v);
drag = p.c_d*p.A*p.rho*v^2/2;

dv = (prop - drag - p.F_f)/p.m;
dx = v;

dz = [dv dx]';
end
```



Velocity
25
20
15
10
5
0
Velocity (m/s)
0
2
4
6
8
10
12
14
16
18
20
Time (t)

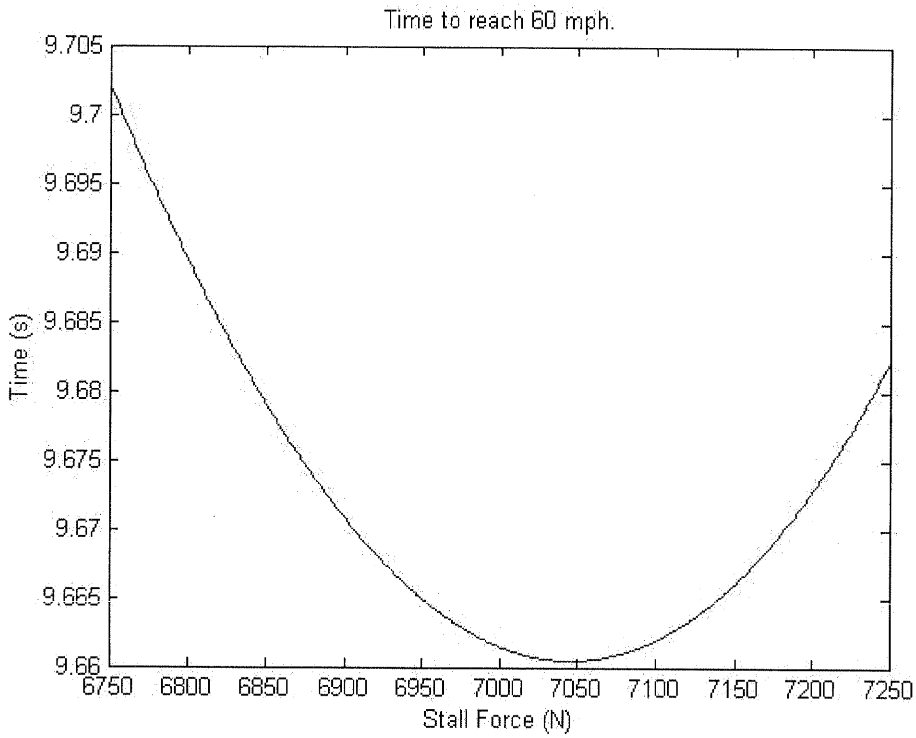


Position
450
400
350
300
250
200
150
100
50
0
Position (m)
0
2
4
6
8
10
12
14
16
18
20
Time (t)

b.

```
function myfun
vel = 26.8224; %m/s
n = 501;
F_0 = linspace(6750,7250,n);
time = zeros(1,n);

for i = 1:n
    time(i) = fsolve(@(t)myerror(t,F_0(i),vel),vel);
end

plot(F_0,time)
title('Time to reach 60 mph.')
xlabel('Stall Force (N)')
ylabel('Time (s)')

[x y] = min(time);
F_op = F_0(y);

disp(['The optimal stall force is about ' num2str(F_op) ' N.'])
end

function error = myerror(t,F_0,vel)
error = myvel(t,F_0) - vel;
end

function vel = myvel(t,F_0)
p.F_0 = F_0;
p.P_p = 100*746; %Watts
p.rho = 1; %kg/m^3)
p.m = 1000; %kg
p.A = 2; %m^2)
p.c_d = 1;
p.F_f = 1000; %N

tspan = [0 t];
v_0 = 0;

[t v] = ode23(@myfn,tspan,v_0,[],p);

vel = v(end);
end

function dv = myfn(t,v,p)

prop = p.F_0*(1 - (p.F_0/(4*p.P_p))*v);
drag = p.c_d*p.A*p.rho*v^2/2;

dv = (prop - drag - p.F_f)/p.m;
end
```

The optimal stall force is about 7045 N.



c.

```
function myfun
p.P_p = 100*746; %Watts
p.rho = 1; %kg/m^3)
p.m = 1000; %kg
p.A = 2; %m^2)
p.c_d = 1;
p.F_f = 1000; %N

v_0 = .1;
x_0 = 0;
z_0 = [v_0 x_0];
tspan = [0 20];

[t z] = ode23(@myfn,tspan,z_0,[],p);

v = z(:,1);
x = z(:,2);
m = linspace(0,20,10001);

figure(1)
plot(t,v,m,26.8224)
title('Velocity')
xlabel('Time (t)')
ylabel('Velocity (m/s)')
figure(2)
plot(t,x)
title('Position')
xlabel('Time (t)')
ylabel('Position (m)')
end

function dz = myfn(t,z,p)
v = z(1);
x = z(2);

prop = p.P_p/v;
drag = p.c_d*p.A*p.rho*v^2/2;

dv = (prop - drag - p.F_f)/p.m;
dx = v;

dz = [dv dx]';
end
```

The time it takes to reach 60 mph is about 7.9 s.