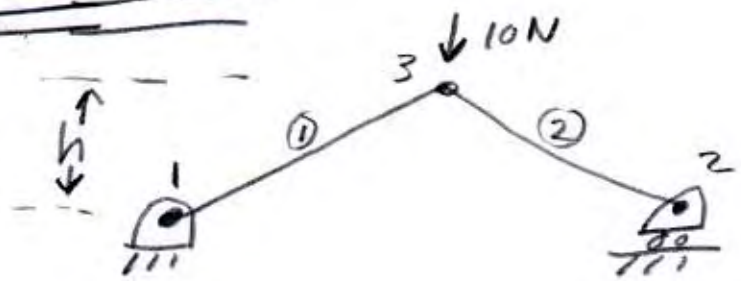
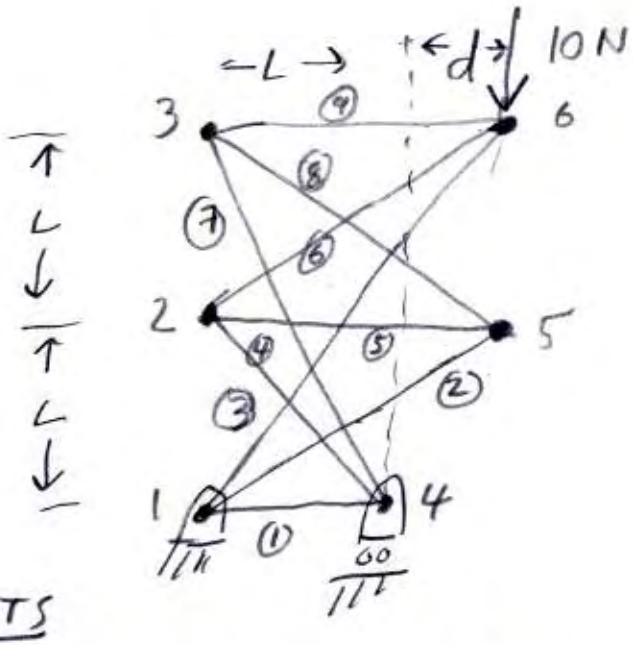
5, 5, 10 a & d

a)



What happens when $h \to 0$?

b)



What happens when $d \to 0$?

CONTENTS

Truss program: pages 2-3] Most of the work ≈ 4 hrs (good for lots of HW probs though)

Problem a: pages 4-5] 20 minutes

Problem b: pages 6-8] 20 minutes

```
% Truss Solver   ANDY RUINA
% -Andy Ruina (modified Oct 25, 2010)
fprintf('\n**********************************************\n')
disp(['Calculation started at:'  datestr(now)])

%Read in data from a users truss data file
datafile = 'data5_5_10a';  %Enter data file as a textstring
%datafile = 'data5_5_10d';  %Enter data file as a textstring
[J B F R] = eval(datafile);


njoints = length(J(:,1)); %number of joints
nbars   = length(B(:,1)); %number of bars
nloads  = length(F(:,1)); %number of loads
nreacts = length(R(:,1)); %number of reactions
if (nbars+nreacts) ~= 2*njoints  % ERROR CHECK
    disp('Number of equations not equal number of unkowns!')
    return %Stops the function from running
end

%For each bar find the x and y coordinates of its base
%and tip. Each of the arrays as the same number of elements
%as does B.
basex = J(B(:,2),2);    basey = J(B(:,2),3);
tipx  = J(B(:,3),2);    tipy  = J(B(:,3),3);


% Find the direction cosines of the bars
x  =  tipx - basex;      y  =  tipy - basey;
D  =  sqrt(x.^2 + y.^2);
cx =  x./D;              cy = y./D; % Direction cosines

%Make arrays of zeros
A  = zeros(2*njoints);   L  = zeros(2*njoints,1);

%Fill in the A matrix. This represents the coefficients
%in all of the joint equilibrium equations.
for i=1:nbars   %once through for each bar
   A(2*B(i,2)-1  , i ) =  cx(i);
   A(2*B(i,2)    , i ) =  cy(i);
   A(2*B(i,3)-1  , i ) = -cx(i);
   A(2*B(i,3)    , i ) = -cy(i);
end

for j = 1:nreacts %once through for each reaction
    A(2*R(j,2)-1, nbars+j ) = R(j,3);
    A(2*R(j,2)  , nbars+j ) = R(j,4);
end

%Fill in the column vector of loads
for k = 1:nloads % once through for each load
    L(2*F(k,1)-1) = -F(k,2);
    L(2*F(k,1)  ) = -F(k,3);
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
T=A\L; %THIS IS _THE_ CALCULATION
```

TRUSS SOLVER

Slightly modified from class version.

Two data files used for this problem.

Counting bars/joints/ reactions

Check for possibility of static determinacy

$n_{bars} + n_{reacts} = 2\, n_{joints}$

If not ⇒ error

Various things about the bars

Fill in the first n_{bars} columns of A matrix

Fill in the last n_{reacts} cols of A matrix

Fill in applied loads

Solves big set of linear eqs. corresponding to joint equilib.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%OUTPUT%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
drawtruss(J, B, F, R) %The m-file drawtruss draws the truss
%fprintf('\n***********************************************\n')
disp(['Data file name is:    '  datafile])
disp('Force units determined by applied loads.')
for i = 1:nbars  %Print bar tensions
    disp([ 'Tension in bar ' num2str(i)...
           '  is:  '         num2str(T(i))  ])
end

for i = 1:nreacts %Print reaction forces
    disp(['Reaction '        num2str(i) ...
          ' at joint '       num2str(J(R(i,2))) ...
          ' in ['            num2str(R(i,3:4)) ...
          '] direction is: ' num2str(T(i+nbars))])
end

disp(['Calculation complete at:'  datestr(now)])
disp('***********************************************')
```

calls drawing program (not used here, but useful for checking.)

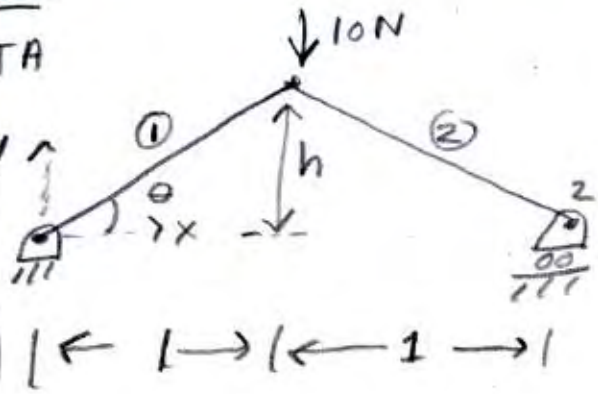
helps keep track of output

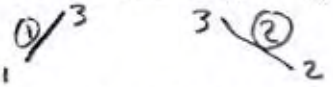
5.5.10a

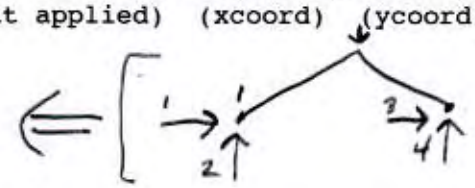
DATA

```
function [J B F R] = data5_5_10a()
%TRUSS DATA: Data for a simple truss
%  4 arrays  define a truss and loading

% Joint positions, one row for each joint
%  #  xcoord  ycoord
h= 1;  % Height of middle bar VARIED
J=[ 1 -1 0
    2  1 0
    3  0 h];

% Bar connectivity, one row for each bar
%   (bar #)  (joint # at one end)   (joint # at other end)
B=[ 1 1 3
    2 2 3];

% Reactions, one row for each reaction direction
% (xcoord, ycoord) is  a unit vector in dir. or force
% (React #) (joint applied)  (xcoord)  (ycoord)
%
R=[ 1 1 1 0
    2 1 0 1
    3 2 1 0
    4 2 0 1];

% Applied Forces, one row for each force
% (joint) (x comp of force)  (y comp of force)
F=[3 0 -10];
end
%%%%%%%%%%%%%
```

10N↓







h=1 $\theta = 45°$

```
**********************************************
Calculation started at:26-Oct-2010 16:40:09
Data file name is:   data5_5_10a
Force units determined by applied loads.
Tension in bar 1  is:  -7.0711
Tension in bar 2  is:  -7.0711
Reaction 1 at joint 1 in [1  0] direction is: 5
Reaction 2 at joint 1 in [0  1] direction is: 5
Reaction 3 at joint 2 in [1  0] direction is: -5
Reaction 4 at joint 2 in [0  1] direction is: 5
Calculation complete at:26-Oct-2010 16:40:09
**********************************************
```

Normal truss.
Similar to lab.
Bars in compression with reasonable values.
Note vertical reactions add to 10
[horiz. reactions add to zero]

h=.01 $\theta = 0.01$ rad $\approx 0.6°$

```
**********************************************
Calculation started at:26-Oct-2010 16:40:46
Data file name is:   data5_5_10a
Force units determined by applied loads.
Tension in bar 1  is:  -500.025
Tension in bar 2  is:  -500.025
Reaction 1 at joint 1 in [1  0] direction is: 500
Reaction 2 at joint 1 in [0  1] direction is: 5
Reaction 3 at joint 2 in [1  0] direction is: -500
Reaction 4 at joint 2 in [0  1] direction is: 5
```

Flat truss
Like a toggle,
Forces are amplified!
add to 10N↑

5.5.10 5/8


```
Calculation complete at:26-Oct-2010 16:40:46
**********************************************
h= .000001
**********************************************
Calculation started at:26-Oct-2010 16:41:51
Data file name is:   data5_5_10a
Force units determined by applied loads.
Tension in bar 1  is:  -5000000
Tension in bar 2  is:  -5000000
Reaction 1 at joint 1 in [1  0] direction is: 5000000
Reaction 2 at joint 1 in [0  1] direction is: 5
Reaction 3 at joint 2 in [1  0] direction is: -5000000
Reaction 4 at joint 2 in [0  1] direction is: 5
Calculation complete at:26-Oct-2010 16:41:51
**********************************************
```

Super flat truss

Forces are getting huge.
Seems like as $h \to 0$ forces $\to \infty$.

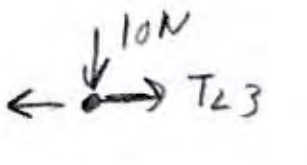
adds to 10N↑

h=0 TROUBLE

```
**********************************************
Calculation started at:26-Oct-2010 16:42:28
Warning: Matrix is singular to working precision.
> In trusssolver at 57
Data file name is:   data5_5_10a
Force units determined by applied loads.
Tension in bar 1  is:  NaN
Tension in bar 2  is:  NaN
Reaction 1 at joint 1 in [1  0] direction is: NaN
Reaction 2 at joint 1 in [0  1] direction is: NaN
Reaction 3 at joint 2 in [1  0] direction is: -Inf
Reaction 4 at joint 2 in [0  1] direction is: Inf
Calculation complete at:26-Oct-2010 16:42:28
**********************************************
```

TOTALLY FLAT TRUSS

joint 3 looks like this:



No way to solve for equilib.
STRUCTURE CANNOT carry load.

Eqs. have no solution. Computer gives up, saying "NaN" & "inf".

Computer gives up

"Not a Number"

So: Ability to carry load depends on geometry, not just topology!
(distances) (what is connected to what)

5.5.10 d

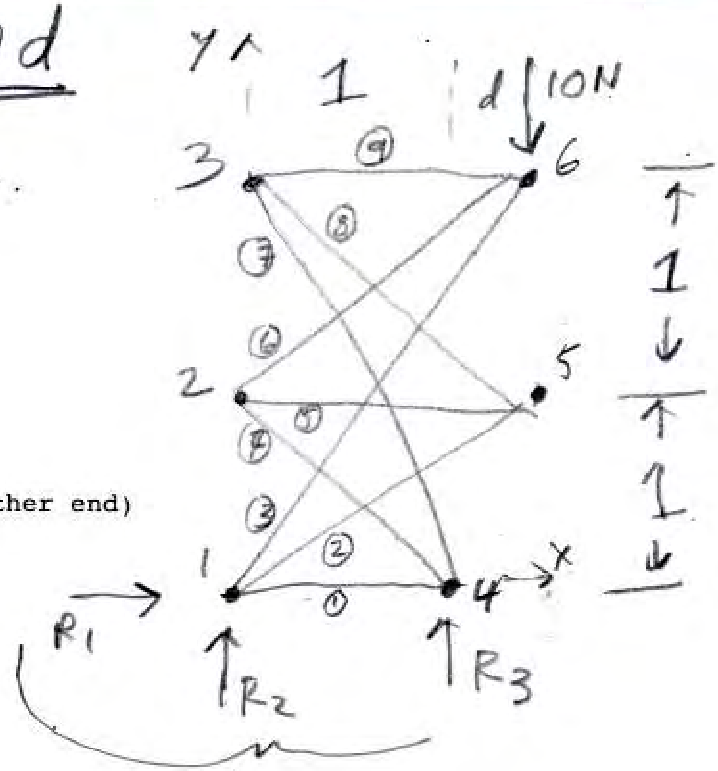
```
function [J B F R] = data5_5_10d()
%TRUSS DATA: Data for a simple truss
%  4 arrays  define a truss and loading

% Joint positions, one row for each joint
%  #  xcoord  ycoord
d=1;  % Horizontal offset of joints 5 & 6.
J=[ 1 0 0
    2 0 1
    3 0 2
    4 1 0
    5 1+d 1
    6 1+d 2];
```

Set to 1, 0.1, 0.0001, 0 in calculations

JOINT LOCATIONS

```
% Bar connectivity, one row for each bar
%    (bar #)  (joint # at one end)   (joint # at other end)
B=[ 1 1 4
    2 1 5
    3 1 6
    4 2 4
    5 2 5
    6 2 6
    7 3 4
    8 3 5
    9 3 6];
```

Bars




```
% Reactions, one row for each reaction direction
% (xcoord, ycoord) is  a unit vector in dir. or force
% (React #) (joint applied)  (xcoord)  (ycoord)
%
R=[ 1 1 1 0
    2 1 0 1
    3 4 0 1];

% Applied Forces, one row for each force
% (joint) (x comp of force)  (y comp of force)
F=[6 0 -10];

end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

The results for a variety of values of d are on the following pages.

5.5,10 7/8

d, cont'd

d=1

```
**********************************************
Calculation started at:27-Oct-2010 22:49:25
Data file name is:   data5_5_10d
Force units determined by applied loads.
Tension in bar 1  is:  50
Tension in bar 2  is:  -134.1641
Tension in bar 3  is:  98.9949
Tension in bar 4  is:  -113.1371
Tension in bar 5  is:  240
Tension in bar 6  is:  -178.8854
Tension in bar 7  is:  67.082
Tension in bar 8  is:  -134.1641
Tension in bar 9  is:  90
Reaction 1 at joint 1 in [1  0] direction is: 0
Reaction 2 at joint 1 in [0  1] direction is: -10
Reaction 3 at joint 4 in [0  1] direction is: 20
Calculation complete at:27-Oct-2010 22:49:25
**********************************************
```

adds to 10 ✓

d=.1

```
**********************************************
Calculation started at:27-Oct-2010 22:50:17
Data file name is:   data5_5_10d
Force units determined by applied loads.
Tension in bar 1  is:  126.5
Tension in bar 2  is:  -343.4062
Tension in bar 3  is:  264.7749
Tension in bar 4  is:  -342.2397
Tension in bar 5  is:  508.2
Tension in bar 6  is:  -359.7589
Tension in bar 7  is:  258.2659
Tension in bar 8  is:  -343.4062
Tension in bar 9  is:  138.6
Reaction 1 at joint 1 in [1  0] direction is: -7.1942e-14
Reaction 2 at joint 1 in [0  1] direction is: -1
Reaction 3 at joint 4 in [0  1] direction is: 11
Calculation complete at:27-Oct-2010 22:50:17
**********************************************
```

adds to 10 ✓

d cont'd

d=0.00001

```
**********************************************
Calculation started at:27-Oct-2010 22:50:53
Data file name is:   data5_5_10d
Force units determined by applied loads.
Tension in bar 1  is:  1000025.0002
Tension in bar 2  is:  -2828483.6938
Tension in bar 3  is:  2236105.9911
Tension in bar 4  is:  -2828483.6937
Tension in bar 5  is:  4000100.001
Tension in bar 6  is:  -2828497.8362
Tension in bar 7  is:  2236101.5187
Tension in bar 8  is:  -2828483.6938
Tension in bar 9  is:  1000035.0004
Reaction 1 at joint 1 in [1  0] direction is: -1.0896e-10
Reaction 2 at joint 1 in [0  1] direction is: -0.0001
Reaction 3 at joint 4 in [0  1] direction is: 10.0001
Calculation complete at:27-Oct-2010 22:50:53
**********************************************
```

huge forces, like: Looks like trouble as $d \to 0$!

adds to 10 ✓

d = 0

```
**********************************************
Calculation started at:27-Oct-2010 22:52:14
Warning: Matrix is singular to working precision.
> In trussSolver at 57
Data file name is:   data5_5_10d
Force units determined by applied loads.
Tension in bar 1  is:  NaN
Tension in bar 2  is:  NaN
Tension in bar 3  is:  NaN
Tension in bar 4  is:  NaN
Tension in bar 5  is:  NaN
Tension in bar 6  is:  NaN
Tension in bar 7  is:  NaN
Tension in bar 8  is:  NaN
Tension in bar 9  is:  Inf
Reaction 1 at joint 1 in [1  0] direction is: -Inf
Reaction 2 at joint 1 in [0  1] direction is: 10
Reaction 3 at joint 4 in [0  1] direction is: 10
Calculation complete at:27-Oct-2010 22:52:14
**********************************************
```

TROUBLE

Not a determinate structure!

This special configuration is analogous to

from part (a)

adds to 20! (computer is confused!)

General result from wierd abstract theory of structures: This structure [each of joints 1,2,3 connected to all of 4,5,6] is rigid unless all 6 points are on a conic section. Parallel lines are a limiting case of a hyperbola hence $d=0 \Rightarrow$ not rigid. (Bob Connelly, Cornell Math dept.).