Your TA, Section # and Section time:

"SOLUTIONS"

Your name:

RUINA

Cornell TAM 2020

No calculators, books or notes allowed.

3 Problems, 90 minutes (+ up to 90 minutes overtime)

Prelim 2

Nov 4, 2010

Directions.
To ease your TA's grading and to maximize your score, please:

- Draw **Free body diagrams** whenever force, moment, linear momentum, or angular momentum balance are used.
- Use correct **vector notation**.
- A+ Be (I) neat, (II) clear and (III) well organized.
- ☐ TIDILY REDUCE and $\boxed{\text{box in}}$ your answers (Don't leave simplifiable algebraic expressions).
- >> Make appropriate `Matlab code` clear and correct.
  You can use shortcut notation like "$T_7 = 18$" instead of, say, "`T(7) = 18`".
  Small syntax errors will have small penalties.
- Clearly **define** any needed dimensions ($\ell, h, d, \ldots$), coordinates ($x, y, r, \theta \ldots$), variables ($v, m, t, \ldots$), base vectors ($\hat{\imath}, \hat{\jmath}, \hat{e}_r, \hat{e}_\theta, \hat{\lambda}, \hat{n} \ldots$) and signs ($\pm$) with sketches, equations or words.
- →| **Justify** your results so a grader can distinguish an informed answer from a guess.
- If a problem seems *poorly defined*, clearly state any reasonable assumptions (that do not oversimplify the problem).
- ≈ Work for **partial credit** (from 60–100%, depending on the problem)
  - Put your answer is in terms of well defined variables even if you have not substituted in the numerical values.
  - Reduce the problem to a clearly defined set of equations to solve.
  - Provide Matlab code which would generate the desired answer (and explain the nature of the output).
- **Extra sheets.** Put your name on each extra sheet, fold it in, and refer to it at the relevant problem.
  Note the last page is **blank** for your use. Ask for more extra paper if you need it.

Problem 4: ____ /25

Problem 5: ____ /25

Problem 6: ____ /25

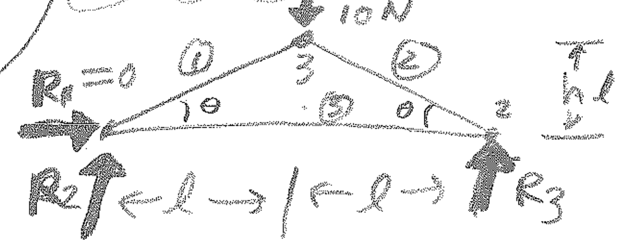
1) The Matlab script file below is exactly that which was explained in 3 lectures and that which you used, if you did not write your own, for homework. The comments and fancy output formatting have been removed. The output is a list of numbers. *For the given data file, provide that list of numbers (that is, you calculate them by hand and write down the numerical values).*
Where appropriate please use the small angle approximations for $\theta \ll 1$ that $\sin\theta \approx \tan\theta \approx \theta$ and $\cos\theta \approx 1$.

```
% Truss Solver,  Stripped down for prelim  -Andy Ruina (modified Nov 3, 2010)
[J B F R] = prelimdata;

njoints = length(J(:,1));   nbars   = length(B(:,1));
nloads  = length(F(:,1));   nreacts = length(R(:,1));

basex = J(B(:,2),2);    basey = J(B(:,2),3);
tipx  = J(B(:,3),2);    tipy  = J(B(:,3),3);

x  =  tipx - basex;      y  =  tipy - basey;
D  =  sqrt(x.^2 + y.^2);
cx =  x./D;              cy = y./D; % Direction cosines

A  = zeros(2*njoints);   L  = zeros(2*njoints,1);

for i=1:nbars   %once through for each bar
   A(2*B(i,2)-1  , i ) =  cx(i);
   A(2*B(i,2)    , i ) =  cy(i);
   A(2*B(i,3)-1  , i ) = -cx(i);
   A(2*B(i,3)    , i ) = -cy(i);
end

for j = 1:nreacts %once through for each reaction
    A(2*R(j,2)-1, nbars+j ) = R(j,3);
    A(2*R(j,2)  , nbars+j ) = R(j,4);
end

%Fill in the column vector of loads
for k = 1:nloads % once through for each load
    L(2*F(k,1)-1) = -F(k,2);
    L(2*F(k,1)  ) = -F(k,3);
end

T=A\L
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

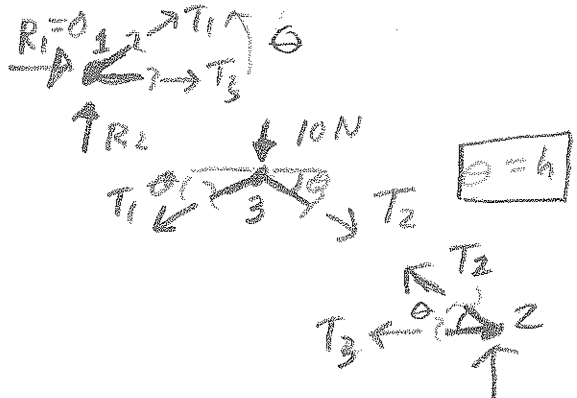
Not needed for exam but for last line.)

FIND T's & R's.

As per the front cover: Here is "Matlab code that would generate the desired output"!! [Just kidding ☺]



FBDs of joints




```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [J B F R] = prelimdata()
J=[ 1 -1 0
    2  1 0
    3  0 0.01];
B=[ 1 1 3
    2 2 3
    3 1 2];
R=[ 1 1 1 0
    2 1 0 1
    3 2 0 1];
F=[3 0 -10];
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

joint location: joint 1 at x=-1, y=0; 2 " 1 " 0; 3 " 0 " h=.01 ($\boxed{h}$)

Bar connection: bar 1 connects joint 1 to 3 etc

React. comps: reacts in x & y dir. at joint 1; " in y " " " 3

Loads: load of "-10 N" (say) in y dir. at joint 3.

$$T = \begin{bmatrix} -500 \\ -500 \\ 500 \\ 0 \\ 5 \\ 5 \end{bmatrix} = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ R_1 \\ R_2 \\ R_3 \end{bmatrix}$$

ANSWER

Structure FBD ⇒ $R_1 = 0$, $R_2 = 5N$, $R_3 = 5N$

Joint 2: $\Sigma F_y = 0 \Rightarrow$

$R_3 + T_2 \sin\theta = 0$

$\Rightarrow T_2 = \frac{-R_3}{\sin\theta} = \frac{-R_3}{h} = \frac{-5N}{.01}$

$= -500N$

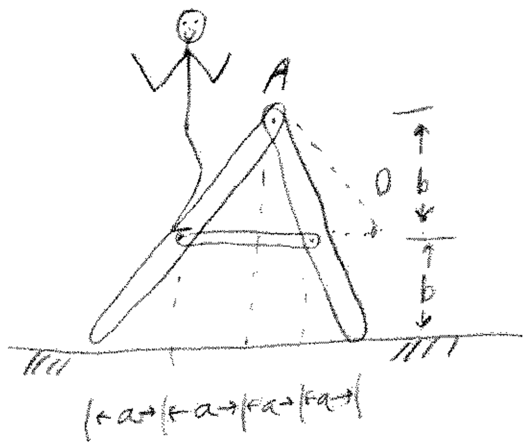
$\Sigma F_x = 0 \Rightarrow -T_2 \underbrace{\cos\theta}_{\cong 1} - T_3 = 0$

$T_3 = -T_2 = 500N$

JOINT 1: $\Sigma F_y = 0$

$\Rightarrow T_1 = -500N$ (like T_2)

2) A person of with weight W stands on a symmetric ladder that is made of 3 rigid parts with negligible weight. It sits on a ground with negligible friction. In terms of some or all of a, b, W, and any coordinates or base vectors you define, find the force of the left piece on the right piece at A.



FBD ①

$\sum M_{/E} = 0 \Rightarrow N_2 = W/4$

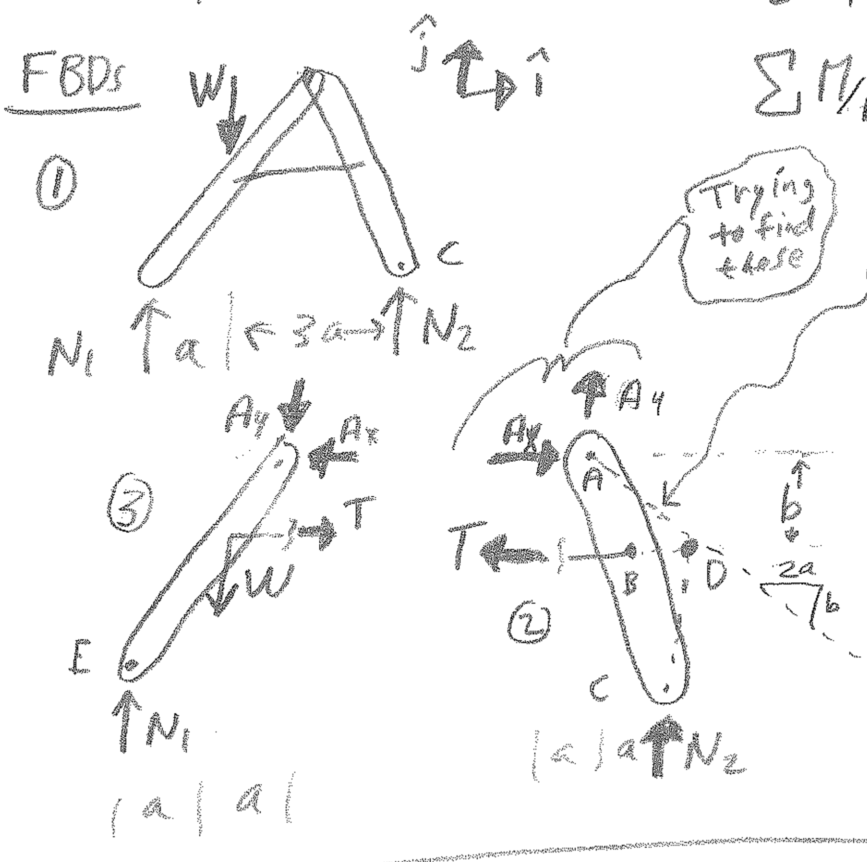
$\sum M_{/C} = 0 \Rightarrow N_1 = 3W/4$

Σ FBD ②

$\sum F_y = 0 \Rightarrow \boxed{A_y = -N_2 = -W/4}$

$\sum M_{/D} = 0 \Rightarrow \dfrac{-A_y}{A_x} = \dfrac{b}{2a}$

$\vec{A}$'s line of action goes through D



$\Rightarrow A_x = \dfrac{-2a}{b} A_y$

$A_x = \dfrac{-2a}{b}\left(\dfrac{-W}{4}\right)$

$\boxed{A_x = \dfrac{aW}{2b}}$

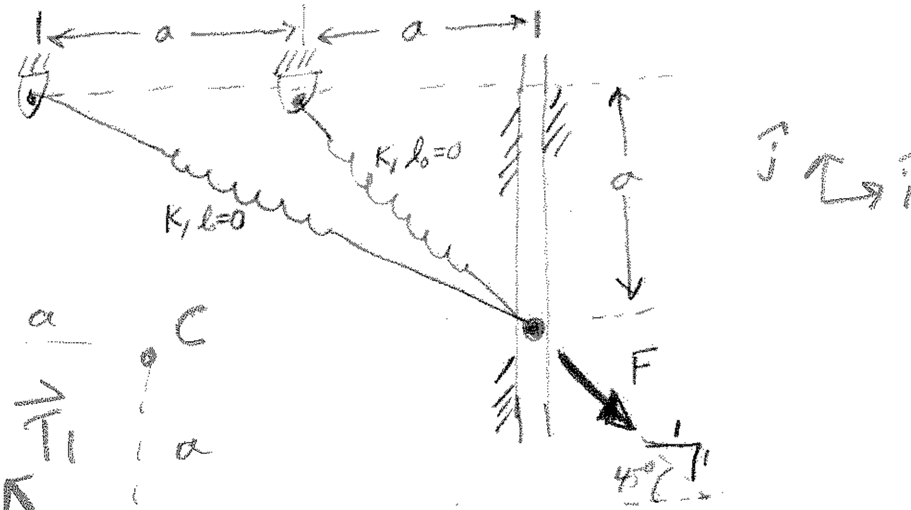
$$\boxed{\vec{A} = \frac{aW}{2b}\,\hat{i} - \frac{W}{4}\,\hat{j}}$$

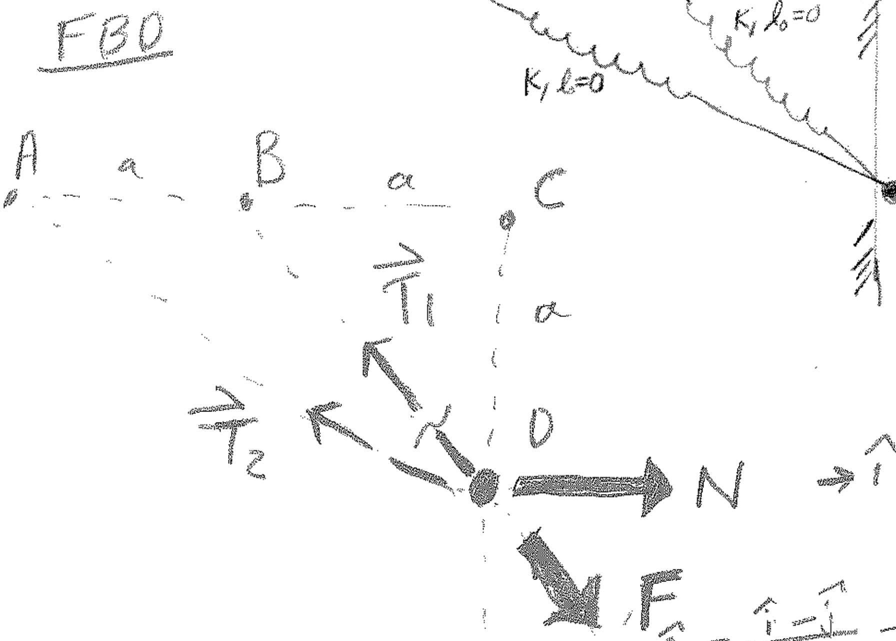
force of left piece onto right

Note! A_y ind. of a or b,

$A_x \to \infty$ as $b \to 0$ (like a toggle).

3) A bead slides in a massless rigid frictionless slot. It is held in place by the force F. Find F in terms of k and a.





$$\hat{a} = \frac{\hat{i}-\hat{j}}{\sqrt{2}}$$

$$\sum \vec{F} = \vec{0} \Rightarrow \vec{T}_1 + \vec{T}_2 + N\hat{i} + F\hat{a} = \vec{0} \qquad (1)$$

$$\vec{T}_1 = K(\underbrace{l}_{|\vec{r}_{DB}|} - \cancelto{0}{l_0})\frac{\vec{r}_{DB}}{|\vec{r}_{DB}|} = K\vec{r}_{DB} = K(a\hat{i} - a\hat{j})$$

$$\vec{T}_2 = K(2a\hat{i} - a\hat{j})$$

$$(1) \Rightarrow \left\{K(a\hat{i}-a\hat{j}) + K(2a\hat{i}-a\hat{j}) + N\hat{i} + F\frac{\hat{i}+\hat{j}}{\sqrt{2}} = 0\right\}$$

$$\{\ \}\cdot\hat{j} \Rightarrow -2Ka + 0 + \frac{F}{\sqrt{2}} = 0$$

$$\boxed{F = 2\sqrt{2}\,Ka}$$