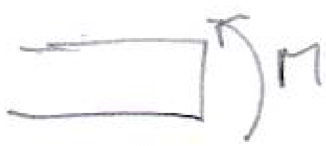
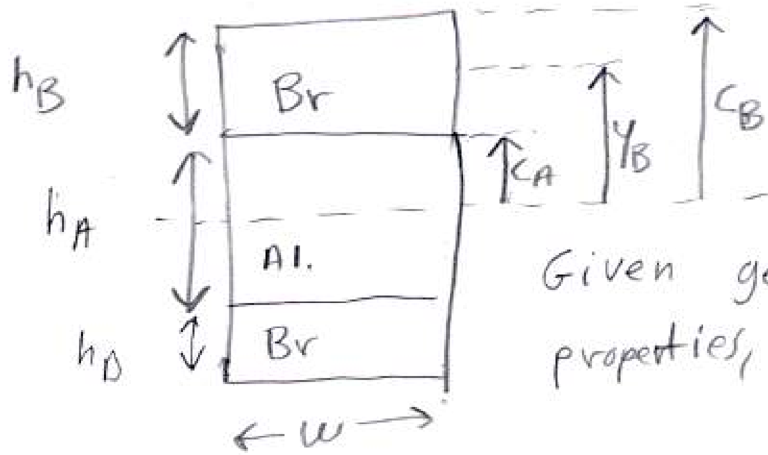
B&J 11.25

Here is a quick (minimal) solution. Detailed soln on later pages.





Given geometry, properties, find M_{max}?

$$M = \frac{K}{\rho_r} = \frac{K\sigma_{max}}{Ec} \qquad K = \text{"}EI\text{"} = \sum E_i(I_i + d_i^2 A_i)$$

$$\left|\frac{c}{\rho}\right| = \epsilon_m \Rightarrow \rho = \frac{c}{\epsilon_{max}}$$

$$\sigma_{max} = E\epsilon_{max} \Rightarrow \epsilon_{max} = \frac{\sigma_{max}}{E}$$

Do calc. twice, take smaller value

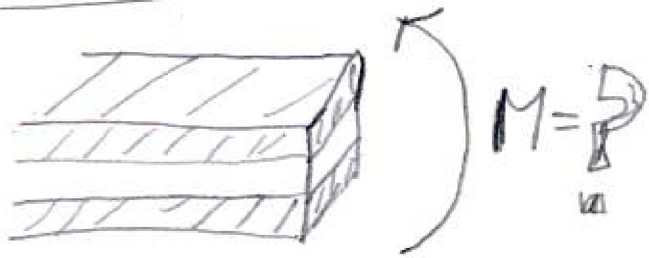
```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Minimal version of BJ 11.25, Use units of N, m
EB   = 105E9;   EA   =  70E9;
SigB = 160E6;   SigA = 100E6;
cA   = 0.015;   cB   = 0.021;
yB   = 0.018;
w    = 0.030; hB   = 0.006; AB   = w*hB;
hA   = 0.030;
k = 2*EB*( w* hB^3/12 + yB^2*AB) ... % k = "EI" net.
    + EA*( w* hA^3/12 +  0        );
MmaxA = k*SigA/(EA*cA),  MmaxB = k*SigB/(EB*cB)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

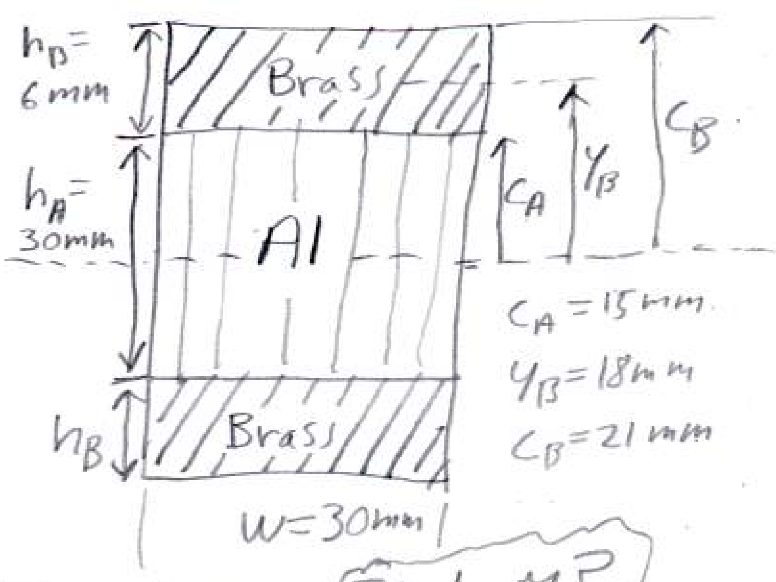
```
%Output
MmaxA =
   1.6272e+03

MmaxB =
   1.2398e+03
```

smaller of two ⟹ $M_{max} = 1.24\ kN \cdot m$

B&J 11.25 Detailed Solution





Neutral Axis in middle

$E_B = 105$ GPa, $\sigma_{YB} = 160$ MPa

$E_A = 70$ GPa, $\sigma_{YA} = 100$ MPa

All geometry & properties given. Find M?

Note, this is 2 problems in 1. First have to find what M will cause Al to fail. Then find what M will cause Brass to fail. Then take smaller of the two.

Failure of Al

at $\sigma_{YA} = E_A \epsilon_{YA} \Rightarrow \epsilon_{YA} = \frac{\sigma_{YA}}{E_A}$ (1)

But for all pts. in cross section: $\epsilon = -y/\rho$

biggest ϵ at biggest y $\Rightarrow$ $\rho_{failA} = \frac{\epsilon_{maxAL}}{c_A}$ (2)

But for composite beam $M = \frac{K}{\rho}$ ← "EI"

$\Rightarrow M_{maxA} = \frac{K}{\rho_{failA}}$ (3)

(1), (2), (3) $\Rightarrow$ $\boxed{M_{maxA} = \frac{K \sigma_{YA}}{E_A c_A}}$ Gives max Moment if failure due to yield of Al. (4)

11.25

Detailed sol'n Continued, 2

Failure of Brass

Exactly the same logic ⇒

$$\boxed{M_{maxB} = \frac{K\,\sigma_{yB}}{E_B\,C_B}}$$ Max Possible moment before Brass yields. (5)

Soln: $$\boxed{\min\left(M_{maxA},\, M_{maxB}\right) = M_{max}}$$ (6)

Failure is when first of two critical loads is reached.

Need Section stiffness K = "EI"

$$K = \text{"}EI\text{"} = \sum_{\substack{\text{All regions}\\ \text{[One Al, two Br.]}}} E_i\left(I_i + \bar{y}_i^2 A_i\right) \quad (7)$$

(I_i = "$bh^3/12$")

All calculations done in Matlab.

First calc. K from (7)

Then M_{maxA} & M_{maxB} from (4) & (5)

Then M_{max} from (6)

See following page for arithmetic.

CODE

```
%BJ11point25
% Use all consistent units of m, N
% -A. Ruina   Nov 16, 2010 (corrected .017->.018)
EB   = 105E9; % modulus of Brass      = 105 GPa
EA   =  70E9; % modulus of Al         =  70 GPa
SigB = 160E6; % yield stress of Brass = 160 MPa
SigA = 100E6; % yield stress of Al    = 100 MPa

cA   = 0.015; % max dist of Al from Neut Axis = 15 mm
cB   = 0.021; % max dist of Br from Neut Axis = 21 mm
yB   = 0.018; % dist of brass center from NA  = 18 mm

w    = 0.030; % width of rect beam     = 30 mm
hB   = 0.006; % height of brass region = 6 mm
hA   = 0.030; % height of Al region    = 30 mm
AB   = w*hB;  % area of each brass part

% Find stiffness of cross section = k = "EI" net.

k = 2*EB*( w* hB^3/12 + yB^2*AB) ...
    + EA*( w* hA^3/12 +  0        );

MmaxA = k*SigA/(EA*cA);  % Max moment is Al yields first
MmaxB = k*SigB/(EB*cB);  % Max moment if Br yields first


Mmax = min(MmaxA,MmaxB);


%Check, take Mmax as given, find stresses.
rho = k/Mmax; %"Bending moment's given by EI over rho"

SigAmax = (cA/rho)*EA;    % should be < or =  SigA
SigBmax = (cB/rho)*EB;    % should be < or =  SigB


%Uncomment lines above to see what you want.
%Or, to waste time, make a fancy output like this.
disp(['*********************************'])
disp(['Output at:     '  datestr(now)])
disp(['Net bending stiffness of cross section=    '...
      num2str(k)  '    N m^2'] )
disp(['Max moment load to keep Al happy =            '...
      num2str(MmaxA)  '     N m '] )
disp(['Max moment load to keep Br happy =            '...
      num2str(MmaxB)  ' N m '] )
disp(['THE ANSWER, min of above two =                '...
      num2str(Mmax)   ' n m '] )
disp(['CHECKS:   '] )
disp(['   rho, (just for fun) =             '...
      num2str(rho)   ' m^-1 '] )
disp(['   Stresses are less or equal what is allowed?:    '] )
disp(['     Stress in Al at given Mmax=      '...
      num2str(SigAmax)  ' m^-1 '] )
disp(['     Stress in Br at given Mmax=      '...
      num2str(SigBmax)  '      m^-1 '] )
```

OUT-PUT

```
*********************************
Output at:     16-Nov-2010 19:06:42
Net bending stiffness of cross section=    17085.6    N m^2
Max moment load to keep Al happy =         1627.2     N m
Max moment load to keep Br happy =         1239.7714 N m
THE ANSWER, min of above two =             1239.7714 n m
CHECKS:
   rho, (just for fun) =             13.7813 m^-1
   Stresses are less or equal what is allowed?:
     Stress in Al at given Mmax=     76190476.1905 m^-1
     Stress in Br at given Mmax=     160000000     m^-1
```

= M_{max} = 1.24 kN·m ← ANSWER