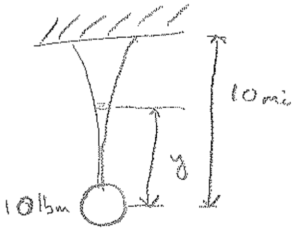
7.1.17

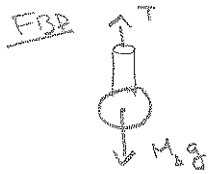


$\sigma_{max} = 60{,}000 \ lbf/in^2$

$\rho = 500 \ lbm/ft^3$

a) Neglecting the weight of the wire, what is the weight of the wire needed to hold the weight?

FBD



$\Sigma F_y: \ T = M_b g$

$= 10 \ lbm \cdot 32.17 \ ft/s^2$

$T = 321.7 \ lbf$

$A = \frac{T}{\sigma_{max}} = \frac{321.7 \ lbf}{60{,}000 \ lbf/in^2}$

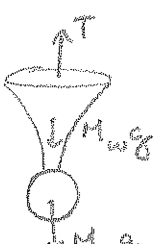
$= 0.0054 \ in^2$

$W = \rho A L \ (lbm)$

$= \frac{500 \ lbm}{ft^3} \cdot 0.0054 \ in^2 \cdot \left(\frac{ft}{12 \ in}\right)^2 \cdot 10 \ mi \cdot \frac{5280 \ ft}{mi}$

$\boxed{W = 983 \ lbm}$

b) Now with the weight of the wire



$T = (M_w + M_b) g$

$A(y) = \frac{T}{\sigma_{max}}$

differential element of wire

$dM_w = \rho A(y) dy$

$\Rightarrow M_w = \int \rho A(y) dy$ (*)

$A(y) = (M_w + M_b) \frac{g}{\sigma_{max}}$

(*) $A(y) = \frac{\rho g}{\sigma_{max}} \int A(y) dy + \frac{M_b g}{\sigma_{max}}$

7.1.17 Cont'd

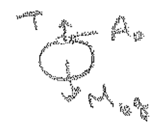
$$\frac{dA(y)}{dy} = \frac{\rho g}{\sigma_{max}} A(y)$$

$$\frac{dA(y)}{A(y)} = \frac{\rho g}{\sigma_{max}} dy$$

$$\ln A(y) = \frac{\rho g}{\sigma_{max}} y + C_0$$

$$A(y) = A_0 \exp\left(\frac{\rho g}{\sigma_{max}} y\right)$$

at $y = 0$, the wire has no mass



$\Rightarrow A_0 = \frac{T}{\sigma_{max}} = \frac{M_b g}{\sigma_{max}}$

$$A(0) = \frac{(M_w^{\,0} + M_b)g}{\sigma_{max}} = \frac{M_b g}{\sigma_{max}} = 0.0054 \text{ in}^2 = A_0 = 3.72\times10^{-5}$$

$$A(y) = (3.72\times10^{-5})\exp\left(\frac{\rho g}{\sigma_{max}} y\right)$$

$$M_w = \int_0^{10\text{ mi}} \frac{M_b g}{\sigma_{max}} \exp\left(\frac{\rho g}{\sigma_{max}} y\right) dy \cdot \rho$$

$$= \left[\rho \cdot \frac{\sigma_{max}}{\rho g} \cdot \frac{M_b g}{\sigma_{max}} \exp\left(\frac{\rho g}{\sigma_{max}} y\right)\Bigg|_0^{10\text{ mi}}\right]$$

$$= M_b\left[\exp\left(\frac{\rho g}{\sigma_{max}}(10\text{ mi})\right) - 1\right]$$

$$= 10\text{ lbm}\left[\exp\left(\frac{500\text{ lbm/ft}^3 \cdot 32.17\text{ ft/s}^2}{60{,}000\text{ lbf/in}^2 \cdot 144\text{ in}^2/\text{ft}^2} \cdot 52800\text{ ft}\right) - 1\right]$$

$$\boxed{M_w = 4.90\times10^{43}\text{ lbm}}$$