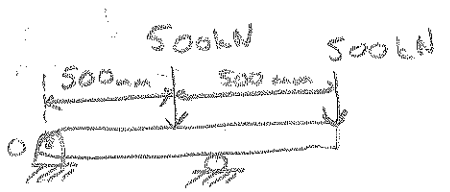
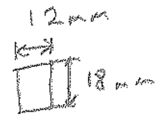
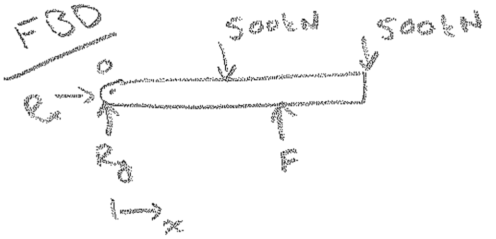
12.26

a) Determine the distance 'a' for which the maximum abs. value of the bending moment in the beam is as small as possible.







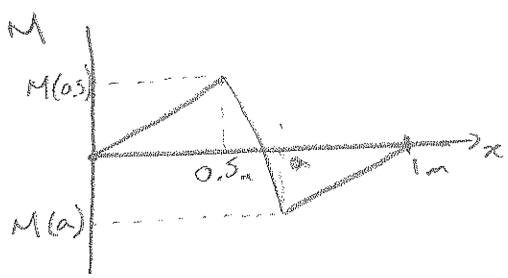
ΣF_x: $R_x = 0$

$\Sigma M_{/O}$: $aF = 500\,kN(0.5m) + 500\,kN(1m)$

$$F = \frac{750\,kN\cdot m}{a}$$

ΣF_y: $R_y = 1000\,kN - F$

$$R_y = 1000\,kN - \frac{750\,kN\cdot m}{a}$$

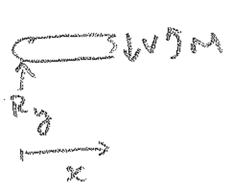


The maximum M is minimized when the magnitude of the 'peaks' are equal.

$\Rightarrow |M(0.5m)| = |M(a)|$

$\Rightarrow M(0.5m) = -M(a)$ (*)

① if $0 < x < 0.5$



$M = R_y x$

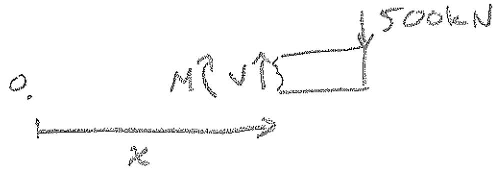
$= \left(1000\,kN - \frac{750\,kN\cdot m}{a}\right) x$

M is continuous $\Rightarrow M(0.5m) = \left(1000\,kN - \frac{750\,kN\cdot m}{a}\right)(0.5m)$

$= 500\,kN\,m - \frac{375\,kN\cdot m^2}{a}$

12.26 cont'd

② If $a < x < 1m$



$$M = -500kN(1m - x)$$

$$= 500kN(x) - 500kN \cdot m$$

$$M(a) = 500kN(a - 1m)$$

$$M(0.5m) = -M(a)$$

$$500\ kN \cdot m - \frac{375\ kN \cdot m^2}{a} = 500\ kN \cdot (1m - a)$$

$$\cancel{125\ kN}\left[4m - \frac{3m^2}{a}\right] = \cancel{125\ kN}\left[4(1m - a)\right]$$

$$4m - \frac{3m^2}{a} = 4m - 4a$$

$$4a^2 = 3m^2$$

$$\boxed{a = \frac{\sqrt{3}}{2} m}$$

b) $\sigma_{max} = \frac{-M_{max} c}{I} = -\frac{M_{max} c}{\frac{1}{12} b h^3}$

$$M_{max} = M\left(a = \frac{\sqrt{3}}{2} m\right) = -250(2 - \sqrt{3})\ kN \cdot m$$

$$\sigma_{max} = \frac{250(2 - \sqrt{3})(.009)}{\frac{1}{12}(0.012)(0.018)^3}\ kN/m^2$$

$$\boxed{\sigma_{max} = 103.4\ GPa}$$