ENGRD 202 Quiz 7 "Solutions"

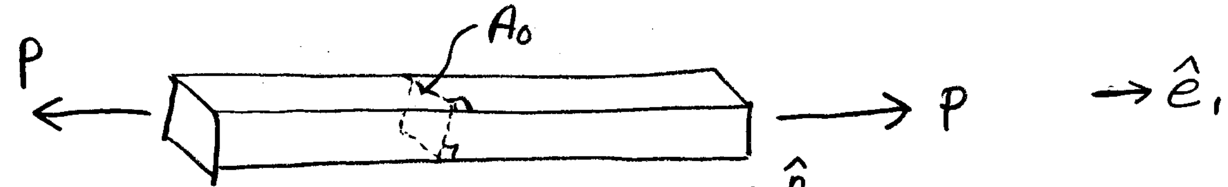
April 30, 2003

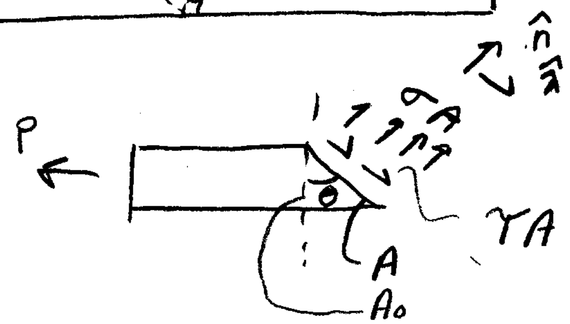
Your Name: RUINA

Section day & time:

TA name & section #:

13) (7 pts) A uniform bar is in tension. On what surface(s) does (do) the shear stress and normal stress have the same magnitude? Show your answer with a clear sketch (or sketches). And, as for all questions past and future, justify your answer well enough so a reader can distinguish a guess from an answer based on firm understanding.





$$\frac{A_o}{A} = \cos\theta \Rightarrow A = \frac{A_o}{\cos\theta}$$

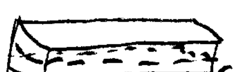
$\cos\theta = \hat{n}\cdot\hat{e}_1$

$$\sum F_{\hat{n}} = 0 \Rightarrow \sigma A - P\cos\theta = 0 \Rightarrow \sigma = \frac{P}{A_o}\cos^2\theta$$

$$\sum F_{\hat{\lambda}} = 0 \Rightarrow \tau A - P\sin\theta = 0 \Rightarrow \tau = \frac{P}{A_o}\cos\theta\sin\theta$$

$\tau = \sigma \Rightarrow \quad \cos^2\theta = \cos\theta\sin\theta \Rightarrow$ surfaces w/ $\hat{n} \perp$ axis

$\cos\theta = 0 \quad (\theta = \pi/2)$, top, bottom & side surfaces {also, by inspection $\tau = \sigma = 0$}

e.g. 

or $\cos\theta = \sin\theta \quad (\theta = \pi/4)$, any surface whose normal is at 45° to axis

e.g. 

diagonal cuts too.

14) **(10 pts)** A solid round shaft with length ℓ; the left end of the shaft is at $x = 0$ and the right end at $x = \ell$.

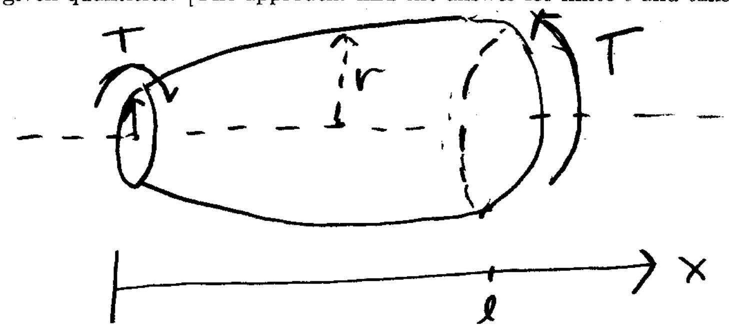
The shaft carries a torque T applied at the ends, and no other loads (Neglect gravity). It is made of material with elastic modulii E, G, and ν and density ρ. The radius of the shaft varies smoothely and gradually from one end to the other by the formula

$$r = a \cdot \sqrt{\frac{(x + x_0)}{x_0}} \quad (1)$$

where a and x_0 are given constants.

What is the rotation ϕ of one end of the shaft relative to the other in the limit that $\ell \to \infty$? Why, in words, isn't the answer ∞?

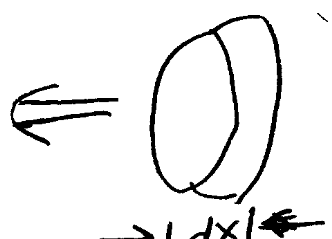
Of course, practically speaking there are no infinite length rods. But this just gets at the idea that not all very long rods that are narrow at one end have very big twist. Derive any formulas you use that you need but do not remember. Give your answer in terms of given quantities. [One approach: find the answer for finite ℓ and take the limit.]



Soln: put together eqs (1)-(4) & integrate

a short bar

$$\underbrace{d\phi}_{\text{a short bar}} = \frac{T}{JG}\,dx \quad (3) \quad \Leftarrow$$



$$J = \frac{\pi r^4}{2} \quad (2)$$
$$= \frac{\pi a^4}{2}\left(\frac{x}{x_0}+1\right)^2$$

$$\overbrace{\phi = \int_0^\infty d\phi}^{(4)} = \int_0^\infty \frac{T}{JG}\,dx = \int_0^\infty \frac{T/G}{\frac{\pi a^4}{2}\left(\frac{x}{x_0}+1\right)^2}\,dx$$

$$\phi = \frac{2T}{\pi G a^4}\int_0^\infty \frac{1}{(x/x_0+1)^2}\,dx \underset{u=\left(\frac{x}{x_0}+1\right)}{=} \frac{2T x_0}{\pi G a^4}\int_1^\infty \frac{1}{u^2}\,du$$

$du = dx/x_0$

$$\phi = \frac{2Tx_0}{\pi G a^4}\left[\frac{-1}{u}\right]_1^\infty$$

$$\boxed{\phi = \frac{2Tx_0}{\pi G a^4}}$$

$$[0] = [\phi] = \frac{[F\cdot\ell][\ell]}{[F/\ell^2][\ell^4]} \stackrel{?}{=} 0$$

units check

Fat stuff at right has so little twist per unit length that it doesn't build up as bar gets longer, Most of twist is near left end.

$$\boxed{\phi = \frac{2Tx_0}{\pi G a^4}}$$