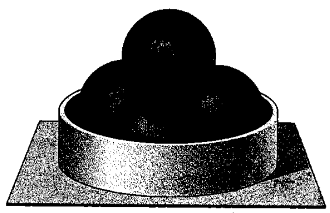
TAM 202 Spring03 HW 9 Soln provided by Peeyush and Tian
3/108, 6/131, 5/189, 6/59 (due 03/25/03)

3/108 Three identical steel balls, each of mass m, are placed in the cylindrical ring which rests on a horizontal surface and whose height is slightly greater than the radius of the balls. The diameter of the ring is such that the balls are virtually touching one another. A fourth identical ball is then placed on top of the three balls. Determine the force P exerted by the ring on each of the three lower balls.

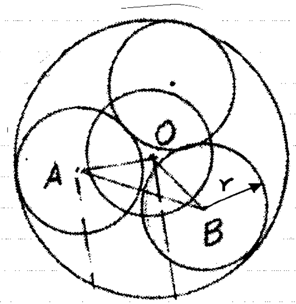


Problem 3/108

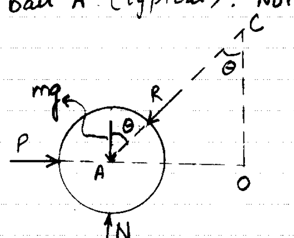
Solution:

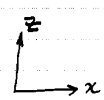
Top View →

Angles OAB and ABO are 30°.



FBD for the ball A (typical). Note, assume no contact between ball A, B, D

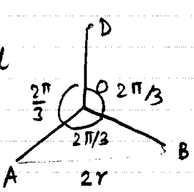




Geometry

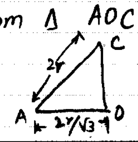
C is the centre of the upper ball

Length of AB = $\bar{AB} = 2r = \bar{AC}$



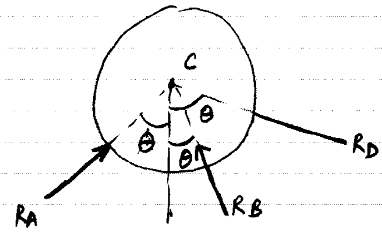
From Δ AOB, $\bar{AO} = \dfrac{r}{\cos 30°} = \dfrac{2r}{\sqrt{3}}$.

From Δ AOC $\quad \bar{OC} = \sqrt{AC^2 - OA^2} = r\sqrt{2^2 - (2/\sqrt{3})^2}$



or $\quad \bar{OC} = 2r\sqrt{\dfrac{2}{3}}$, (continued)

FBD for the upper (top) ball.



Equilibrium implies. $\Sigma F_z = 0$.

so $\Sigma F_z = 3R\cos\theta - mg = 0$

or $3R\left(\dfrac{\bar{OC}}{\bar{AC}}\right) - mg = 0 \qquad \left[\cos\theta = \dfrac{\bar{OC}}{\bar{AC}} \text{ from } \Delta\ OAC\right]$

or $R = \dfrac{mg \cdot 2r}{2r\sqrt{\frac{2}{3}}} \cdot \dfrac{1}{3} = \dfrac{mg}{\sqrt{6}}$

Equilibrium for ball (see FBD), ball A

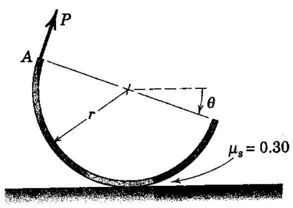
$\Sigma F_x = 0 \Rightarrow P - R\sin\theta = 0$.

or $P = R\sin\theta = \dfrac{mg}{\sqrt{6}} \cdot \dfrac{2r/\sqrt{3}}{2r} = \dfrac{mg}{3\sqrt{2}}$

So $\boxed{P = \dfrac{mg}{3\sqrt{2}}}$ lb.

***6/131** The semicylindrical shell of mass m and radius r is rolled through an angle θ by the force P which remains tangent to its periphery at A as shown. If P is slowly increased, plot the tilt angle θ as a function of P up to the point of slipping. Determine the tilt angle θ_{max} and the corresponding value P_{max} for which slipping occurs. The coefficient of static friction is 0.30.

Ans. $\theta_{max} = 59.9°$
$P_{max} = 0.295mg$

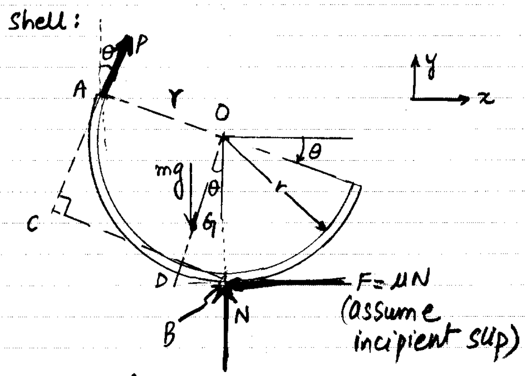


$(\mu_s = \mu_k = \mu)$

Problem 6/131

Solution:

FBD for the shell:



$\overline{OG} = d = \frac{2r}{\pi}$; (See Table D/14, Pg 285 Meriam & Kraige)
$\mu = 0.3$

Equilibrium:

$\Sigma F_x = 0 \Rightarrow P\sin\theta - F = 0$ (1)

$\Sigma F_y = 0 \Rightarrow N - mg + P\cos\theta = 0$ (2)

$\Sigma M_B = 0 \Rightarrow mg\,d\sin\theta - P(\overline{BC})$ (3)

$\overline{BC} = \overline{BD} + \overline{BC} = r + r\sin\theta.$

At the start of slipping $F = \mu N$. (4)

(continued)

Relation between P & θ can be derived from (3)

$$P = \frac{2\sin\theta * (mg)}{\pi(1+\sin\theta)} \quad (5)$$

Substitute $F = \mu N$ in (1), we get

$$P\sin\theta = \mu N \Rightarrow N = \frac{P\sin\theta}{\mu}. \quad (6)$$

Substitute $N = \frac{P}{\mu}\sin\theta$ in (2) to find relation between P and mg.

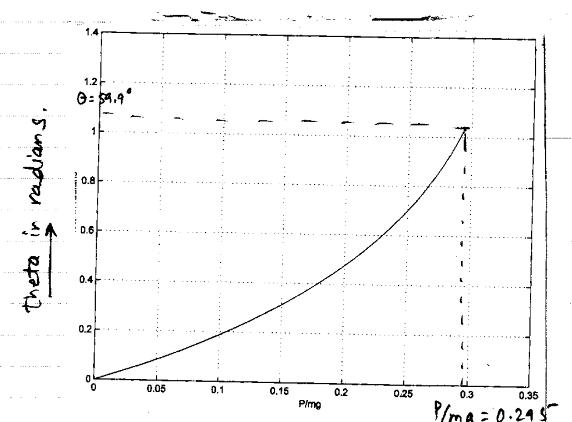
$$\frac{P\sin\theta}{\mu} - mg + P\cos\theta = 0 \Rightarrow \boxed{P = \frac{\mu mg}{(\mu\cos\theta + \sin\theta)}} \quad (7)$$

Comparing (5) & (7)

$$\frac{\mu}{(\mu\cos\theta + \sin\theta)} = \frac{2\sin\theta}{\pi(1+\sin\theta)} \quad (8)$$

Solve (8) numerically to obtain $\theta_{max} = 59.9°$

& corresponding $P_{max} = 0.295\,mg$ using (5).



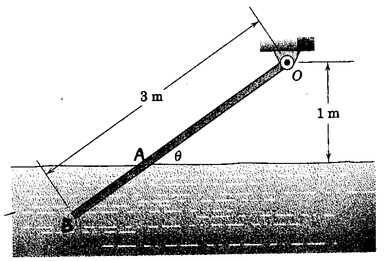
Matlab Program.

```
theta=0:0.1:(59.9*pi/180);
P=2.*sin(theta)./(pi.*(1+sin(theta)));
plot(P,theta)
xlabel('P/mg')
ylabel('theta(in radians')
grid on
```

5.189

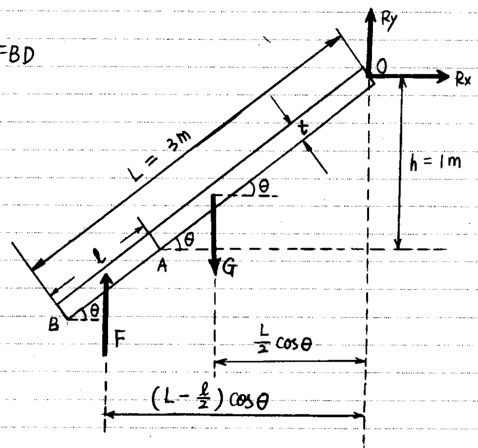
5/189 The 3-m plank shown in section has a density of 800 kg/m^3 and is hinged about a horizontal axis through its upper edge O. Calculate the angle θ assumed by the plank with the horizontal for the level of fresh water shown.

Ans. $\theta = 48.2°$



Problem 5/189

① FBD



Assume:

width of the plank (into the paper) $= b$

thickness of the plank $= t$

then

$V_P \equiv$ Volume of the plank $= Lbt$

$V_s \equiv$ Volume of the part of the plank submerged into the water

$= \ell bt$

$\Rightarrow G = \rho_P g V_P = \rho_P g Lbt$

where ρ_P is the density of the plank.

(Continued)

5.189 (Cont'd)

Buoyancy $F = \rho_W g V_s = \rho_W g \ell bt$

where ρ_W is the density of the water

② Equilibrium eqn

$$\Sigma M_{/O} = 0 = G\left(\frac{L}{2}\cos\theta\right) - F\left(L - \frac{\ell}{2}\right)\cos\theta$$

$$= \rho_P g Lbt\left(\frac{L}{2}\cos\theta\right) - \rho_W g \ell bt\left(L - \frac{\ell}{2}\right)\cos\theta$$

$$\Rightarrow \rho_P L^2 - \rho_W \ell (2L - \ell) = 0$$

$$\Rightarrow \ell^2 - 2L\ell + \frac{\rho_P}{\rho_W} L^2 = 0$$

substitute numbers in:

$$\ell^2 - 2(3m)\ell + \frac{800\ kg/m^3}{1000\ kg/m^3}(3m)^2 = 0$$

$$\Rightarrow \ell = \frac{2(3m) \pm \sqrt{(6m)^2 - 4(4/5)(3m)^2}}{2}$$

$= 1.66\ m$ or $4.34\ m$ (rejected)

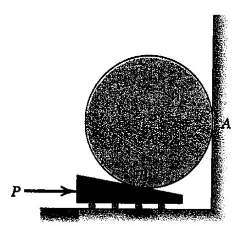
$$\Rightarrow \ell = L - \frac{h}{\sin\theta} = 1.66m$$

$$\Rightarrow 3m - \frac{1m}{\sin\theta} = 1.66\ m$$

$$\Rightarrow \boxed{\theta = 48.2°}$$

6/59 Calculate the horizontal force P on the light 10° wedge necessary to initiate movement of the 40-kg cylinder. The coefficient of static friction for both pairs of contacting surfaces is 0.25. Also determine the friction force F_B at point B. (*Caution:* Check carefully your assumption of where slipping occurs.)

Ans. $P = 98.6$ N, $F_B = 24.6$ N



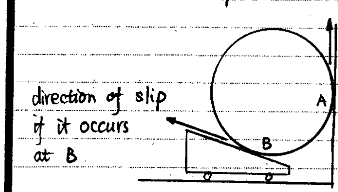
Problem 6/59

(Continued)

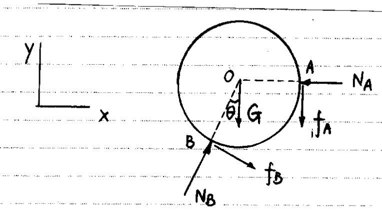
6.59 (Cont'd)

① FBD of the cylinder.

First, we should determine the directions of friction forces at A & B. Since we are pushing the wedge in, if the cylinder slips, it will slip upward at A OR slip outward at B (we don't know whether it will slip at A or B yet), i.e.



Since friction force is in the opposition direction to the slip, we get the FBD of the cylinder:



$\theta = 10°$

Assume radius of the cylinder to be R.

② Equilibrium Eqns:

$\Sigma M_{/O} = 0 = f_B R - f_A R \Rightarrow f_A = f_B$ (1)

$\Sigma F_x = 0 = N_B \sin\theta + f_B \cos\theta - N_A$ (2)

$\Sigma F_y = 0 = N_B \cos\theta - f_B \sin\theta - f_A - G$ (3)

Now we have 3 eqns but 4 unknowns (N_A, N_B, f_A, f_B), so we need one more eqn. This is the equation relating friction force and normal force when the cylinder starts to slip, i.e. this eqn is either $f_A = \mu N_A$ or $f_B = \mu N_B$, depending on at which point (A or B) the cylinder slips. Since we don't know at this time, let's try:

(Continued)

If it slips at B

$f_B = \mu N_B$ sub into (2) $\Rightarrow N_B \sin\theta + \mu N_B \cos\theta - N_A = 0$

$\Rightarrow N_A = N_B(\sin\theta + \mu\cos\theta) = N_B(\sin 10^\circ + 0.25\cos 10^\circ) = 0.42 N_B$

From (1) $\Rightarrow f_A = f_B = \mu N_B$

$\Rightarrow \frac{f_A}{N_A} = \frac{\mu N_B}{0.42 N_B} = \frac{\mu}{0.42} > \mu$

$\Rightarrow$ When B slips, the friction at A already exceeds μN_A $\Rightarrow$ Bad assumption!

So the slip should first occur at point A. Let's check:

If it slips at A

$f_A = \mu N_A$. from (1) $\Rightarrow f_B = \mu N_A$

sub into (2) $\Rightarrow N_B \sin\theta + \mu N_A \cos\theta - N_A = 0$

$\Rightarrow N_B = \frac{N_A(1-\mu\cos\theta)}{\sin\theta} = \frac{N_A(1-0.25\cos 10^\circ)}{\sin 10^\circ} = 4.34 N_A$

$\Rightarrow \frac{f_B}{N_B} = \frac{\mu N_A}{4.34 N_A} = \frac{\mu}{4.34} < \mu$

$\Rightarrow$ when A slips, B doesn't slip

$\Rightarrow$ No paradox!

So the slip first occurs at A.

Another equation: $f_A = \mu N_A$ (4)

Solve (1) ~ (4):

Sub (4) into (1) $\Rightarrow f_B = \mu N_A$ (5)

sub (5) into (2) $\Rightarrow N_B = \frac{N_A(1-\mu\cos\theta)}{\sin\theta}$ (6)

Sub (5), (6) into (3)

$\Rightarrow \cos\theta \frac{N_A(1-\mu\cos\theta)}{\sin\theta} - \mu N_A \sin\theta - \mu N_A - G = 0$

$\Rightarrow N_A = \frac{G\sin\theta}{\cos\theta - \mu(1+\sin\theta)}$

$= \frac{(40\,kg)(9.81\,N/kg)\sin 10^\circ}{\cos 10^\circ - 0.25(1+\sin 10^\circ)} \Rightarrow \boxed{N_A = 98.6\,N}$

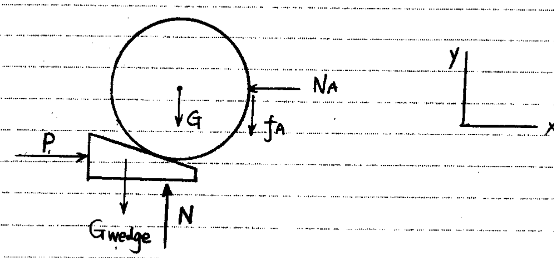
(Continued)

Sub $N_A = 98.6\,N$ into (5)

$\Rightarrow \boxed{f_B = 0.25\,(98.6\,N) = 24.6\,N}$

③ P

FBD of the "cylinder + wedge" system



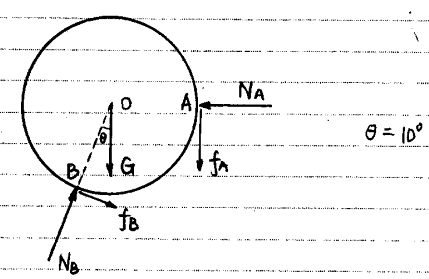
$\Sigma F_x = 0 = P - N_A$

$\Rightarrow \boxed{P = N_A = 98.6\,N}$

Comments

One can also use a graphic method to determine whether the cylinder will slip at A or B.

First, the FBD of the cylinder is

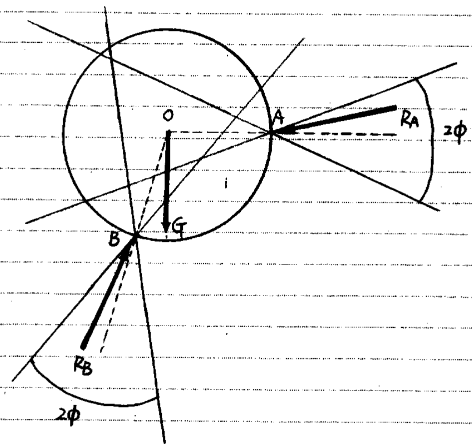


Second, for the equilibrium of the cylinder, the resultant of f_A and N_A (denoted as R_A) can only lie in a wedge at A with angle 2ϕ, where

$\phi = \tan^{-1}\mu = \tan^{-1}(0.25) \approx 14^\circ$.

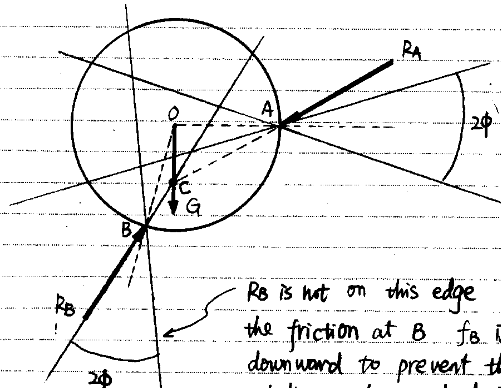
For the same reason, the resultant of f_B and N_B (denoted as R_B) can also only lie in a wedge at B with same angle 2ϕ (See figure below)

(Continued)



Third, the cylinder is a 3-force member (G, R_A and R_B), so these three forces have to pass through a same point. Now let's see whether A or B will slip first.

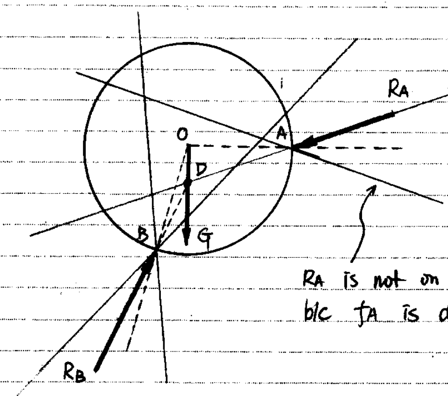
If B slips first, then R_B is on the edge of the wedge, i.e.



R_B is not on this edge b/c the friction at B f_B is downward to prevent the relative motion, which is upward.

So R_B and G intersect at point C. For equilibrium, R_A must pass through C too, as shown. But then R_A doesn't lie in the wedge any more, which is a paradox. (Continued)

If A slips first, then R_A is on the edge of the wedge, i.e



R_A is not on this edge b/c f_A is downward

R_A and G intersect at D, so R_B must pass through D to maintain equilibrium, as shown above. We can see that R_B is still in the wedge, so there is no paradox, and we get the conclusion that A must slip first.