

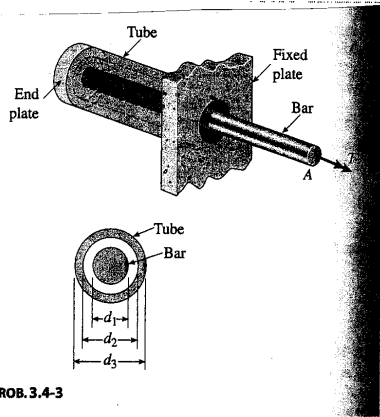
TAM202 Spring 03 HW13 Soln provided by Vijay.  
3.4-3, 3.4-10, 3.5-4, 4.3-1, 4.3-4, quiz 6 (due 04/22).  
See quiz 6 soln separately on website.

3.4-3 A circular tube of outer diameter  $d_3 = 2.75$  in. and inner diameter  $d_2 = 2.35$  in. is welded to a rigid end plate and at the left-hand end to a fixed plate and at the right-hand end to a rigid end plate (see figure). A solid circular bar of diameter  $d_1 = 1.60$  in. is inside of, and concentric with, the tube. The bar passes through a hole in the fixed plate and is welded to the end plate.

The bar is 40 in. long and the tube is half as long as the bar. A torque  $T = 10,000$  lb-in. acts at end A of the bar. Also, both the bar and tube are made of an aluminum alloy with shear modulus of elasticity  $G = 3.9 \times 10^6$  psi.

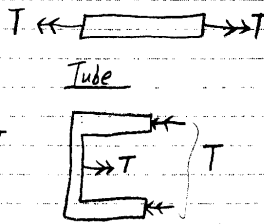
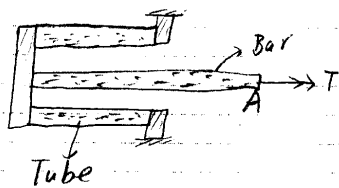
(a) Determine the maximum shear stresses in both the bar and tube.

(b) Determine the angle of twist (in degrees) at end A of the bar.



PROB. 3.4-3

Solution:



Tube  
 $d_3 = 2.75$  in,  $d_2 = 2.35$  in  
 $G = 3.9 \times 10^6$  psi  
 $L_{tube} = 20$  in  
 $(I_p)_{tube} = \frac{\pi}{32} (d_3^4 - d_2^4)$   
 $= 2.621$  in<sup>4</sup>

Bar  
 $d_1 = 1.60$  in,  
 $G = 3.9 \times 10^6$  psi  
 $L_{bar} = 40$  in  
 $(I_p)_{bar} = \frac{\pi}{32} d_1^4$   
 $= 0.6434$  in<sup>4</sup>  
 (Continued)

Torque  $T = 10,000$  lb-in

a) Maximum Shear Stresses

Bar:  $T_{bar} = \frac{16T}{\pi d_1^3}$   
 $= \frac{16 \cdot (10,000 \text{ lb-in})}{\pi (1.6 \text{ in})^3} = 12,430$  psi

Tube:  $T_{tube} = \frac{T (d_3/2)}{(I_p)_{tube}} = \frac{(10,000 \text{ lb-in}) (2.75 \text{ in})}{2.621 \text{ in}^4}$   
 $= 5,250$  psi

b)  $\phi_A = \phi_{bar} + \phi_{tube}$

Bar:  $\phi_{bar} = \frac{T L_{bar}}{G (I_p)_{bar}} = \frac{(10,000 \text{ lb-in}) (40 \text{ in})}{(3.9 \times 10^6 \text{ psi}) (0.6434 \text{ in}^4)}$   
 $= 0.1594$  rad

Tube:  $\phi_{tube} = \frac{T L_{tube}}{G (I_p)_{tube}} = \frac{(10,000 \text{ lb-in}) (20 \text{ in})}{(3.9 \times 10^6 \text{ psi}) (2.621 \text{ in}^4)}$   
 $= 0.0196$  rad

$\phi_A = \phi_{bar} + \phi_{tube} = 0.1790$  rad =  $10.3^\circ$

$\phi_A = 10.3^\circ = 0.179$  rad

3.4-10

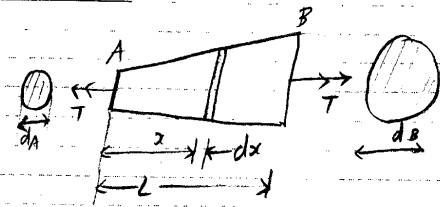


PROBS. 3.4-9 and 3.4-10

3.4-10 The bar shown in the figure is tapered linearly from end A to end B and has a solid circular cross section. The diameter at the smaller end of the bar is  $d_A = 25$  mm. The bar is made of steel with shear modulus of elasticity  $G = 82$  GPa.

If the torque  $T = 90$  N-m and the allowable rate of twist is  $0.5^\circ/\text{m}$ , what is the minimum allowable diameter  $d_B$  at the larger end of the bar? (Hint: Use the results of Example 3-5.)

(Continued)



To determine the general expression for Angle of twist:

Diameter  $d$  at a distance  $x$  from end A,

$d = d_A + \frac{d_B - d_A}{L} x \rightarrow (1)$

Polar moment of Inertia:

$I_p(x) = \frac{\pi d^4}{32} = \frac{\pi}{32} \left[ d_A + \frac{d_B - d_A}{L} x \right]^4$

$\phi = \int_0^L \frac{T dx}{G I_p(x)}$  [∵ Torque  $T$  is constant]

$\phi = \frac{32T}{\pi G} \int_0^L \frac{dx}{\left( d_A + \frac{d_B - d_A}{L} x \right)^4} \rightarrow (2)$

The above integral is of the form

$\int \frac{dx}{(a+bx)^4}$  where  $a = d_A, b = \frac{d_B - d_A}{L}$

Now,  $\int \frac{dx}{(a+bx)^4} = -\frac{1}{3b(a+bx)^3} \rightarrow (3)$

Using (3), (2) gets simplified to:

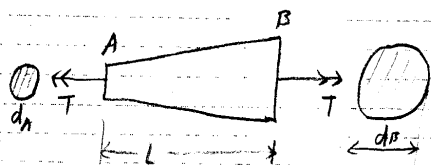
$\phi = \frac{32TL}{3\pi G (d_B - d_A)} \left( \frac{1}{d_A^3} - \frac{1}{d_B^3} \right) \rightarrow (4)$

A convenient form to (4) is

$\phi = \frac{TL}{G(I_p)_A} \left( \frac{\beta^2 + \beta + 1}{3\beta^3} \right) \rightarrow (5)$

where  $\beta = \frac{d_B}{d_A}, (I_p)_A = \frac{\pi d_A^4}{32}$  (Continued)

Tapered Bar



$d_A = 25\text{mm}$   $G = 82\text{GPa}$   $T = 90\text{N}\cdot\text{m}$

$\theta_{\text{allow}} = 0.5^\circ/\text{m}$

Using (5), we get

$$\theta = \frac{T}{G(J_P)_A} \left( \frac{\beta^2 + \beta + 1}{3\beta^3} \right)$$

where,  $\theta = \frac{\phi}{L}$ ,  $\beta = \frac{d_B}{d_A}$ ,  $(J_P)_A = \frac{\pi}{32} d_A^4$

$$\therefore 0.5^\circ/\text{m} \times \left( \frac{\pi}{180} \frac{\text{rad}}{\text{deg}} \right) = \frac{90\text{N}\cdot\text{m}}{(82\text{GPa}) \left( \frac{\pi}{32} \right) (25\text{mm})^4} \left( \frac{\beta^2 + \beta + 1}{3\beta^3} \right)$$

$$\Rightarrow 0.3049 = \frac{\beta^2 + \beta + 1}{3\beta^3}$$

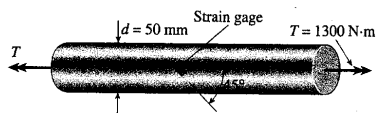
Solving,  $\beta = 1.9445$

$$\therefore d_B = \beta d_A \Rightarrow d_B = 48.6\text{mm}$$

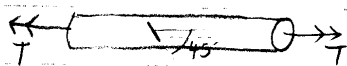
3.5-4

3.5-4 A solid circular bar of diameter  $d = 50\text{mm}$  (see figure) is twisted in a testing machine until the applied torque reaches the value  $T = 1300\text{N}\cdot\text{m}$ . At this value of torque, a strain gage oriented at  $45^\circ$  to the axis of the bar gives a reading  $\epsilon = 331 \times 10^{-6}$ .

Determine the shear modulus  $G$  of the material.



PROB. 3.5-4



Strain gage at  $45^\circ$ :  $\epsilon_{\text{max}} = 331 \times 10^{-6}$   
(Continued)

$d = 50\text{mm}$ ,  $T = 1300\text{N}\cdot\text{m}$

From Eq. 3-33 (here),

$$\gamma_{\text{max}} = 2 \epsilon_{\text{max}} = 662 \times 10^{-6}$$

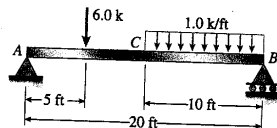
$$\tau_{\text{max}} = G \gamma_{\text{max}} = \frac{16 T}{\pi d^3}$$

$$\Rightarrow G = \frac{16 T}{\pi d^3 \gamma_{\text{max}}} = \frac{16 (1300\text{N}\cdot\text{m})}{\pi (0.05\text{m})^3 / (662 \times 10^{-6})}$$

$$G = 80.0\text{GPa}$$

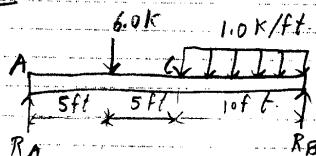
4.3-1

4.3-1 Determine the shear force  $V$  and bending moment  $M$  at the midpoint  $C$  of the simple beam  $AB$  shown in the figure.



PROB. 4.3-1

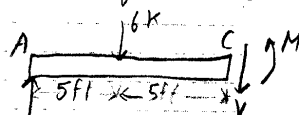
Solution:



$$\sum M_B = 0: (6\text{k} \times 15\text{ft}) + (10\text{k} \times 5\text{ft}) - R_A \times 20\text{ft} = 0$$

$$\Rightarrow R_A = 7.0\text{k}$$

FBD of segment AC



$R_A = 7\text{k}$

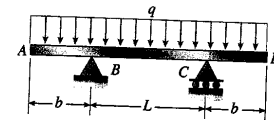
$$\sum F_y = 0: V = 7\text{k} - 6\text{k} = 1.0\text{k}$$

$$\sum M_C = 0: (6\text{k} \times 5\text{ft}) + M - (7\text{k} \times 10\text{ft}) = 0$$

$$\Rightarrow M = 40\text{k}\cdot\text{ft}$$

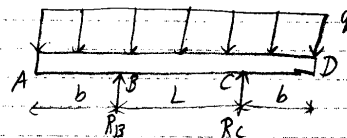
4.3-4

4.3-4 The beam  $ABCD$  shown in the figure has overhangs at each end and carries a uniform load of intensity  $q$ . For what ratio  $b/L$  will the bending moment at the midpoint of the beam be zero?



PROB. 4.3-4

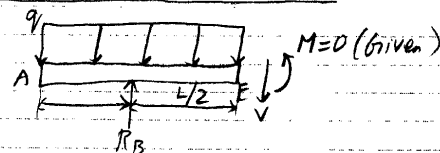
Solution:



From symmetry and equilibrium in vertical direction,

$$R_B = R_C = \frac{q(L + 2b)}{2}$$

FBD of left half of beam:



$$\sum M_E = 0 (+\uparrow):$$

$$-R_B \cdot \frac{L}{2} + q \cdot \frac{1}{2} \left( \frac{L}{2} + b \right)^2 = 0$$

$$\Rightarrow \frac{q}{2} \left( \frac{L}{2} + b \right)^2 = \frac{qL}{2} \left( \frac{L}{2} + b \right)$$

$$\Rightarrow \frac{L}{2} + b = L \Rightarrow b = \frac{L}{2}$$

$$\text{or } \boxed{\frac{b}{L} = \frac{1}{2}}$$