

TAM 202 Spring 2003 HW12 soln provided by Pankaj
2.6-2, 2.6-7, 2.6-14, 3.2-4, 3.3-4, 3.3-17 (due 04/15/13)

Q.1

2.6-2 A circular steel rod of diameter d is subjected to a tensile force $P = 90 \text{ kN}$ (see figure). The allowable stresses in tension and shear are 110 MPa and 50 MPa , respectively. What is the minimum permissible diameter d_{\min} of the rod?



PROB. 2.6-2



$A =$ Cross-sectional area.

Max Normal stress $\sigma_x = \frac{P}{A}$

Max. Shear stress $\tau_{\max} = \frac{\sigma_x}{2} = \frac{P}{2A}$
($\theta = \pm 45^\circ$)

$\sigma_{\text{allow}} = 110 \text{ MPa}$ $\tau_{\text{allow}} = 50 \text{ MPa}$

Because τ_{allow} is less than one half σ_{allow}

($\frac{\sigma_{\max}}{\tau_{\max}} = 2$) the shear stress governs

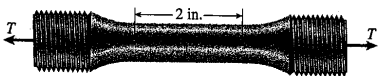
so $\tau_{\max} = \frac{P}{2A} \Rightarrow 50 \text{ MPa} = \frac{90 \text{ kN}}{2 \left(\frac{\pi d^2}{4} \right)}$

$\Rightarrow d_{\min} = 33.9 \text{ mm}$

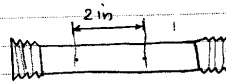
Q.2

2.6-7 During a tension test of a mild-steel specimen (see figure), the extensometer shows an elongation of 0.00140 in. with a gage length of 2 in. Assume that the steel is stressed below the proportional limit and that the modulus of elasticity $E = 30 \times 10^6 \text{ psi}$.

(a) What is the maximum normal stress σ_{\max} in the specimen? (b) What is the maximum shear stress τ_{\max} ? (c) Draw a stress element oriented at an angle of 45° to the axis of the bar and show all stresses acting on the faces of this element.



Tension test



Elongation = 0.00140 in $E = 30 \times 10^6 \text{ psi}$

Strain $\epsilon = \frac{0.00140 \text{ in}}{2 \text{ in}} = 0.0007$

Using Hooke's Law (steel is stressed below the proportional limit)

$\sigma_x = E \epsilon = 30 \times 10^6 \text{ psi} \times 0.0007$

$\sigma_x = 21,000 \text{ psi}$

a) max. normal stress

σ_x is the max normal stress

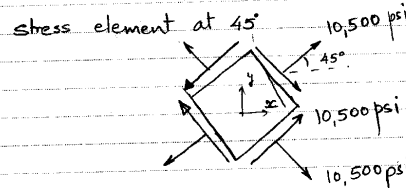
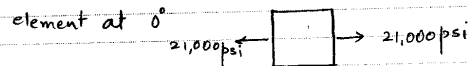
so $\sigma_{\max} = 21,000 \text{ psi}$

b) Max. Shear Stress

max shear stress is on 45° plane and

$\tau_{\max} = \frac{\sigma_x}{2} = \frac{21,000 \text{ psi}}{2} = 10,500 \text{ psi}$

(c) stress Element at $\theta = 45^\circ$



$\tau_{45^\circ} = \tau_{\max} = 10,500 \text{ psi}$

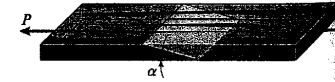
$\sigma_{45^\circ} = \sigma_x \cos^2 45^\circ = \frac{\sigma_x}{2} = 10,500 \text{ psi}$

$\sigma_{45^\circ} = 10,500 \text{ psi}$

Q.3.

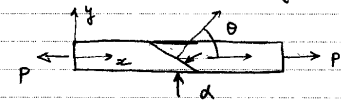
2.6-14 Two boards are joined by gluing along a scarf joint as shown in the figure. For purposes of cutting and gluing the angle α between the plane of the joint and the face of the boards must be between 10° and 40° . Under a tensile load P , the normal stress in the boards is 4.9 MPa .

(a) What are the normal and shear stresses acting on the glued joint if $\alpha = 20^\circ$? (b) If the allowable shear stress on the joint is 2.25 MPa , what is the largest permissible value of the angle α ? (c) For what angle α will the shear stress on the glued joint be numerically equal to twice the normal stress on the joint?



PROB. 2.6-14

Two boards joined by a scarf joint



$10^\circ \leq \alpha \leq 40^\circ$

Due to load P $\sigma_x = 4.9 \text{ MPa}$

a) Stresses on the joint when $\alpha = 20^\circ$

$\theta = 90 - \alpha = 70^\circ$

$\sigma_\theta = \sigma_x \cos^2 \theta$
 $= 4.9 \cos^2 70^\circ$

$\sigma_\theta = 0.57 \text{ MPa}$

$\tau_\theta = -\sigma_x \sin \theta \cos \theta$

$= -4.9 \text{ MPa} \sin(70^\circ) \cos(70^\circ)$

$\tau_\theta = -1.58 \text{ MPa}$

b) Largest angle α if $\tau_{\text{allow}} = 2.25 \text{ MPa}$

$\tau_{\text{allow}} = -\sigma_x \sin \theta \cos \theta$

The shear stress has -ve sign. Its

numerical value can not exceed $\tau_{\text{allow}} = 2.25 \text{ MPa}$

Therefore

$$-2.25 \text{ MPa} = - \frac{(4.9 \text{ MPa})}{2} \sin 2\theta$$

$$\Rightarrow \theta = 33.34^\circ \text{ or } 56.66^\circ$$

$$\alpha = 90 - \theta$$

$$\Rightarrow \alpha = 56.66 \text{ or } 33.34^\circ$$

α must be between 10° and 40° so

$$\alpha = 33.34^\circ$$

Note: if $10 \leq \alpha \leq 33.34^\circ$

$$|\tau_o| \leq 2.25 \text{ MPa}$$

$$33.34^\circ < \alpha \leq 40^\circ$$

$$|\tau_o| \geq 2.25 \text{ MPa}$$

(c) What is α if $\tau_o = 2.60$
take numerical values:

$$\Rightarrow |\tau_o| = \sigma_x \sin \theta \cos \theta$$

$$|\sigma_o| = \sigma_x \cos^2 \theta$$

$$\text{and } \left| \frac{\tau_o}{\sigma_o} \right| = 2$$

$$\Rightarrow \frac{\sigma_x \sin \theta \cos \theta}{\sigma_x \cos^2 \theta} = 2$$

$$\Rightarrow \tan \theta = 2$$

$$\theta = 63.43^\circ$$

$$\Rightarrow \alpha = 90 - \theta = 26.6^\circ$$

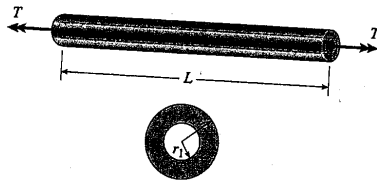
$$\text{so } \alpha = 26.6^\circ$$

Q.4.

3.2-4 A circular steel tube of length $L = 1.25 \text{ m}$ is loaded in torsion by torques T (see figure).

(a) If the inner radius of the tube is $r_1 = 38 \text{ mm}$ and the measured angle of twist between the ends is 0.6° , what is the shear strain γ_1 (in radians) at the inner surface?

(b) If the maximum allowable shear strain is 0.0004 rad and the angle of twist is to be kept at 0.6° by adjusting the torque T , what is the maximum permissible outer radius $(r_2)_{\max}$?



Circular steel tube

$$L = 1.25 \text{ m}, \quad r_1 = 38 \text{ mm}, \quad \phi = 0.6 \left(\frac{\pi}{180} \right) \text{ rad} \\ = 0.01047 \text{ rad}$$

$$\gamma_{\max} = 0.0004 \text{ rad.}$$

a) Shear strain at the inner surface

(from eq. 3.5 b)

$$\gamma_{\min} = \gamma_1 = r_1 \frac{\phi}{L} = \frac{(38 \text{ mm}) \cdot (0.01047 \text{ rad})}{(1250 \text{ mm})}$$

$$\Rightarrow \gamma_1 = 3.18 \times 10^{-6} \text{ rad}$$

b) max outer radius

(from eq. 3.5 a)

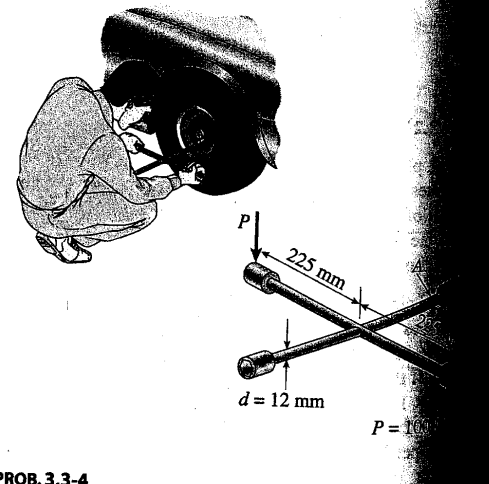
$$\gamma_{\max} = \gamma_2 = r_2 \frac{\phi}{L} \Rightarrow r_2 = \frac{\gamma_{\max} L}{\phi}$$

$$\text{so } (r_2)_{\max} = \frac{(0.0004 \text{ rad}) (1250 \text{ mm})}{0.01047 \text{ rad}}$$

$$r_2 = 47.8 \text{ mm}$$

Q.5 3.3-4 While removing a wheel to change a tire, a lug wrench (see figure). The wrench is made of steel with a modulus of elasticity $G = 78 \text{ GPa}$. Each arm of the wrench is 225 mm long and has a solid circular cross section with a diameter $d = 12 \text{ mm}$.

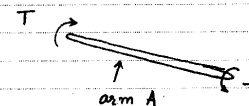
(a) Determine the maximum shear stress in the wrench that is turning the lug nut (arm A). (b) Determine the angle of twist (in degrees) of this same arm.



PROB. 3.3-4

Lug wrench

$$P = 100 \text{ N} \\ L = 225 \text{ mm} \\ d = 12 \text{ mm} \\ G = 78 \text{ GPa}$$



$$T = 2PL \\ = 2(100 \text{ N})(225 \text{ mm}) \\ T = 45 \text{ N}\cdot\text{m}$$

a) Max Shear Stress
(From eq. 3.12)

$$\tau_{\max} = \frac{16T}{\pi d^3} = \frac{16(45 \text{ N}\cdot\text{m})}{\pi (0.012 \text{ m})^3}$$

$$\tau_{\max} = 132.63 \text{ MPa}$$

(b) Angle of twist

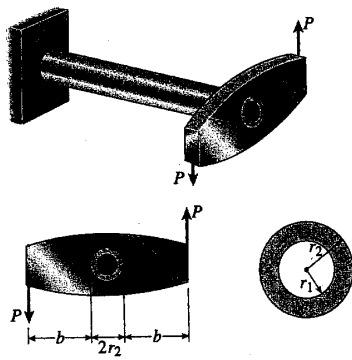
(From eq. 3.15)

$$\phi = \frac{TL}{G I_p} = \frac{(45 \text{ N}\cdot\text{m})(.225 \text{ m})}{(78.4 \text{ Pa}) \left(\frac{\pi}{32}\right) (.012 \text{ m})^4}$$

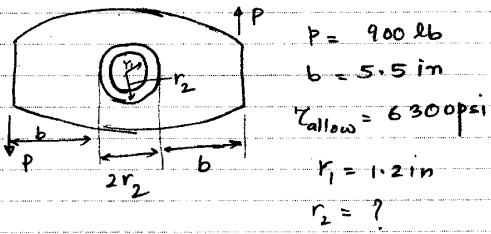
$$\Rightarrow \phi = 0.06376 \text{ rad} = 3.65^\circ$$

Q.6

A tube of inner radius r_1 and outer radius r_2 is subjected to a torque produced by forces $P = 900 \text{ lb}$ applied at a distance b from the center of the tube. The maximum shear stress in the tube is 6300 psi and the minimum permissible stress is 1000 psi. What is the minimum permissible value of r_2 ?



PROB. 3.3-17



Torsion formula

$$T = 2P(b+r_2)$$

$$I_p = \frac{\pi}{2} (r_2^4 - r_1^4)$$

$$\tau_{max} = \frac{T r_2}{I_p} = \frac{4P(b+r_2)r_2}{\pi(r_2^4 - r_1^4)}$$

r_2 is the only unknown in above eq.

$$\Rightarrow (6300 \text{ psi}) = \frac{4(900 \text{ lb})(5.5 \text{ in} + r_2)r_2}{\pi [r_2^4 - (1.2 \text{ in})^4]}$$

$$or. \frac{r_2^4 - 2.07360}{r_2(r_2 + 5.5)} - 0.181891 = 0$$

$$or. r_2^4 - 0.181891 r_2^2 - 1.0004 r_2 - 2.07360 = 0$$

Solve using matlab.

$$r_2 = 1.3988 \text{ in} \approx 1.4 \text{ in}$$

Matlab code for numerical solution.

```

/home/pkp2/f.m
November 6, 2002
% James Gere 3.3-17 Matlab file f.m
% Numerical solution of the equation

function y=f(r)
y=r.^4-0.181891*r.^2-1.0004*r-2.07360

% just write this file f.m in your home
% directory and give command
% "fzero(@f,2)" it will give you zero
% of the function near 2

% Output of the matlab program
    
```

```

>> fzero(@f,2)
ans =
    1.3988
    
```

