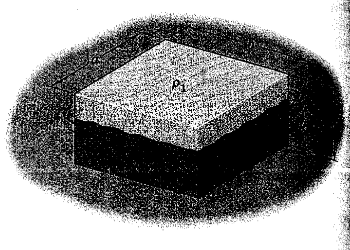
by Vijay. 5.165, 5.181, 5.211, 6.7, 6.13, 6.29 (due 03/11/03)

5/165 A rectangular block of density ρ_1 floats in a liquid of density ρ_2. Determine the ratio $r = h/c$, where h is the submerged depth of block. Evaluate r for an oak block floating in fresh water and for steel floating in mercury.

Ans. $r = \frac{\rho_1}{\rho_2}$, $r = 0.8, 0.577$

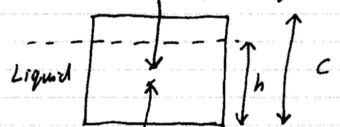


Problem 5/165

Solution:

FBD of the rectangular block

$W = mg = \rho_1 V g = \rho_1 abcg$



$B = \rho_2 V_{sub} g = \rho_2 abhg$

The rectangular block is under the action of two forces, its weight W and the buoyancy force B.

$+\uparrow \Sigma F = 0: \rho_2 abhg - \rho_1 abcg = 0$

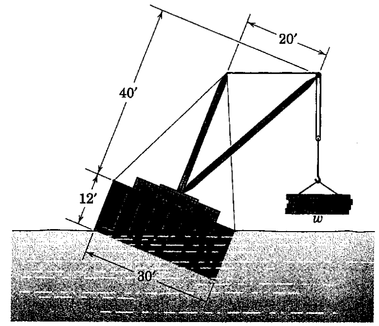
$\Rightarrow h = \frac{\rho_1}{\rho_2} c \Rightarrow \boxed{r = \frac{h}{c} = \frac{\rho_1}{\rho_2}}$

Oak in water: $r = \frac{800}{1000} = 0.8$

Steel in Mercury: $r = \frac{7830}{13570} = 0.577$

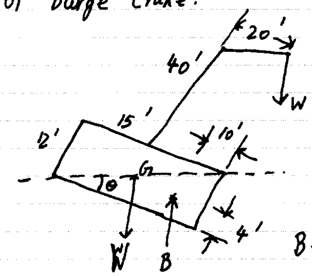
5/181 The barge crane of rectangular proportions has a 12-ft by 30-ft cross section over its entire length of 80 ft. If the maximum permissible submergence and list in sea water are represented by the position shown, determine the corresponding maximum safe load w which the barge can handle at the 20-ft extended position of the boom. Also find the total displacement W in long tons of the unloaded barge (1 long ton equals 2240 lb). The distribution of machinery and ballast places the center of gravity G of the barge, minus the load w, at the center of the hull.

Ans. $w = 100{,}800$ lb, $W = 366$ long tons



Problem 5/181

FBD of barge crane:



B-Total Buoyant Force.

$\theta = \tan^{-1}\left(\frac{12}{30}\right) = 21.8°$

Moment arm of B about G

$= [(15-10)\cos\theta - (6-4)\sin\theta]\,ft = 3.90\,ft$

Moment arm of w about $G = [(40+6)\sin\theta + 20\cos\theta]\,ft = 35.67\,ft$

$B = \rho g V = \left(64 \frac{lb}{ft^3}\right) \times (12 ft) \times (15 ft) \times (80 ft) = 921{,}600\,lb$

$\Sigma M_G = 0: (35.67 ft) w - (3.90 ft)(921{,}600\,lb) = 0$

(Continued)

$\therefore \boxed{w = 100{,}765\,lb}$

$W = B - w = 921{,}600 - 100{,}765$

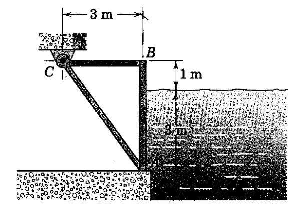
$= 820{,}835\,lb$

or $\boxed{W = \frac{820{,}835\,lb}{2240\,lb/long\,ton} = 366.4 \text{ long tons}}$

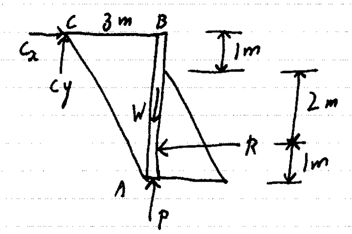
5.211

5/211 The figure shows the cross section of a rectangular gate 4 m high and 6 m long (perpendicular to the paper) which blocks a fresh-water channel. The gate has a mass of 8.5 Mg and is hinged about a horizontal axis through C. Compute the vertical force P exerted by the foundation on the lower edge A of the gate. Neglect the mass of the frame to which the gate is attached.

Ans. $P = 348$ kN



Problem 5/211



Pressure distribution on the gate varies linearly with depth and the resultant is R.

$W = (8.5 \times 10^3\,kg) \times (9.81\,m/s^2) = 83.4\,kN$

$R = (\text{Avg. pressure}) \times \text{Area}$

$\text{Avg. pressure} = \frac{1}{2}\rho g h = \frac{1}{2} \times \left(10^3 \frac{kg}{m^3}\right) \times \left(9.81 \frac{m}{s^2}\right) \times (3m)$

$= 14.72\,kN/m^2$

$R = (14.72\,kN/m^2) \times (3m) \times (6m) = 265\,kN$

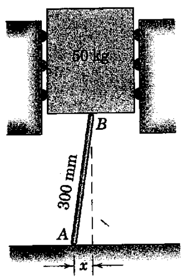
(Continued)

$\Sigma M_C = 0: (265 kN)(3m) + (83.4 kN)(3m) - P(kN)(3m) = 0$

$\Rightarrow \boxed{P = 348.4 kN}$

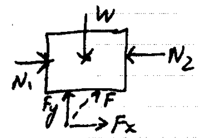
6.7 6/7 The light bar is used to support the 50-kg block in its vertical guides. If the coefficient of static friction is 0.30 at the upper end of the bar and 0.40 at the lower end of the bar, find the friction force acting at each end for x = 75 mm. Also find the maximum value of x for which the bar will not slip.

Ans. $F_A = F_B$ = 126.6 N, x_{max} = 86.2 mm



Problem 6/7

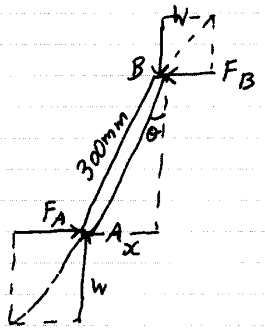
FBD of block:



Vertical force balance $\Rightarrow$ $F_y = W$

$\therefore$ From action & reaction, W is the vertical component of the force of block on rod.

FBD of the light bar:



Given: $\mu_B = 0.3$

$\mu_A = 0.4$

$W = 50 \times 9.81 N$
$= 490.5 N$

a) $x = 75 mm$

$\theta = \cancel{\tan} \sin^{-1}\left(\frac{75}{300}\right) = 14.48°$

Friction angle $\phi = \tan^{-1}(\mu_s)$

At A, $\phi_A = \tan^{-1}(0.4) = 21.8°$

At B, $\phi_B = \tan^{-1}(0.3) = 16.7°$

Since $\theta < \phi_A \,\&\, \phi_B$, bar does not slip.

$\therefore \boxed{F_A = F_B = W \tan\theta = 126.6 N}$

(Continued)

b) To calculate x_{max} when bar will not slip

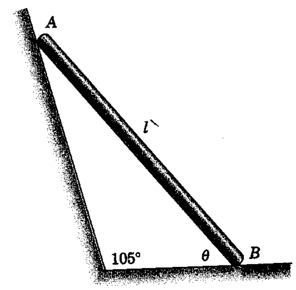
Bar first slips at B when $\theta = \phi_B = 16.7°$

$\therefore x_{max} = 300^{mm}(\sin 16.7°) = 86.2 mm$

$\Rightarrow \boxed{x_{max} = 86.2 mm}$

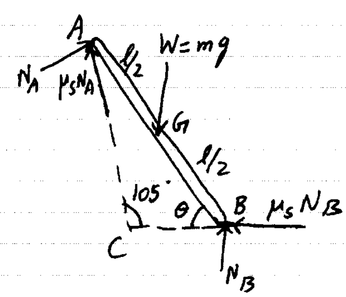
6.13 6/13 The uniform pole of length l and mass m is placed against the supporting surfaces shown. If the coefficient of static friction is μ_s = 0.25 at both A and B, determine the maximum angle θ at which the pole can be placed before it begins to slip.

Ans. θ = 59.9°

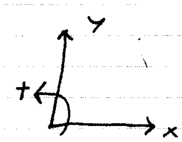


Problem 6/13

FBD of Pole: Note: Assume impending slip at A & B.



$\mu_s = 0.25$



From Sine Law,

$$\frac{\sin 105°}{l} = \frac{\sin\theta}{AC} = \frac{\sin(75°-\theta)}{BC}$$

$$\Rightarrow AC = \frac{l \sin\theta}{\sin 105°}, \quad BC = \frac{l \sin(75°-\theta)}{\sin 105°}$$

$\Sigma F_x = 0: N_A \cos 15° - 0.25 N_A \sin 15° - 0.25 N_B = 0 \rightarrow (1)$

$\Sigma F_y = 0: N_A \sin 15° + 0.25 N_A \cos 15° + N_B - mg = 0 \rightarrow (2)$

(Continued)

Solving ① & ②, we get

$\boxed{N_A = 0.244mg,\ N_B = 0.878mg}$

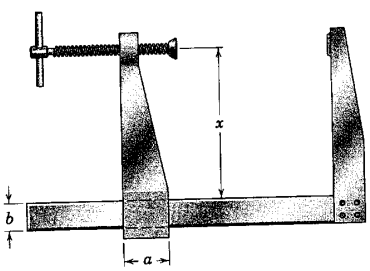
$$\Sigma M_C = 0 : -N_A\left(\frac{l\sin\theta}{\sin 105^\circ}\right) + N_B\left(\frac{l\sin(75^\circ-\theta)}{\sin 105^\circ}\right) - mg\left[\frac{l\sin(75^\circ-\theta)}{\sin 105^\circ} - \frac{l}{2}\cos\theta\right] = 0 \quad \to ③$$

Solving ③, we obtain $\boxed{\theta = 59.9^\circ}$

6.29

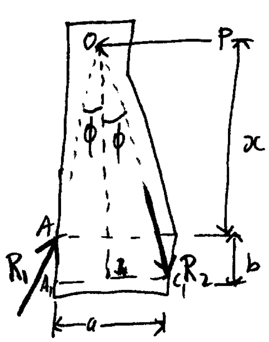
6/29 The movable left-hand jaw of the C-clamp can be slid along the frame to increase the capacity of the clamp. To prevent slipping of the jaw on the frame when the clamp is under load, the dimension x must exceed a certain minimum value. For given values of a and b and a static friction coefficient μ_s, specify this design minimum value of x to prevent slipping of the jaw.

Ans. $x = \dfrac{a - b\mu_s}{2\mu_s}$



Problem 6/29

FBD:



There are only 3 forces acting: Horizontal force P, forces R_1 & R_2 as shown. Since it is a 3-force member, for equilibrium, P, R_1 & R_2 must be concurrent at O.

When the jaw starts to slip, $\phi = \tan^{-1}(\mu_s)$ and $x = x_{min}$.

(Continued)

R_1 & R_2 – Resultant forces at A and C_1 respectively.

From the geometry of the problem,

$$a = A_1B_1 + B_1C_1$$
$$= x\tan\phi + (x+b)\tan\phi$$

$\because \tan\phi = \mu_s$ when the jaw starts to slip,

$$a = x\mu_s + (x+b)\mu_s$$
$$= 2x\mu_s + b\mu_s$$
$$\Rightarrow 2x\mu_s = a - b\mu_s$$
$$\Rightarrow \boxed{x = \frac{a - b\mu_s}{2\mu_s}}$$