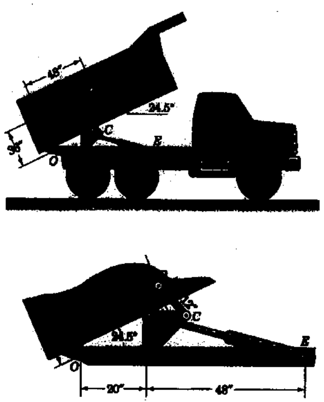
TAM 202 Spring 2003 HW7 Solution provided by Pankaj.

4.115, 5.43, 5.51, 5.67 (due 03/04/03)

▶**4/115** The design of a hoisting mechanism for the dump truck is shown in the enlarged view. Determine the compression P in the hydraulic cylinder BE and the magnitude of the force supported by the pin at A for the particular position shown, where BA is perpendicular to OAE and link DC is perpendicular to AC. The dump and its load together weigh 20,000 lb with center of mass at G. All dimensions for the indicated geometry are given on the figure.
Ans. $P = 26{,}900$ lb, $A = 14{,}600$ lb



Detail of hoisting mechanism

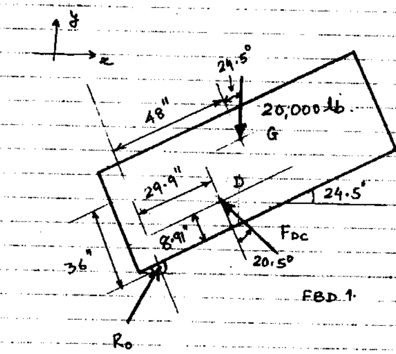
Problem 4/115

Note: 1. Pin at point B connects hydrolic Cylinder and triangular plate, not the truck box.

2. member DC and the hydraulic Cylinder BE are two force members so the direction of force in these members will be along them.

(Continued)

Consider FBD of "Dump"



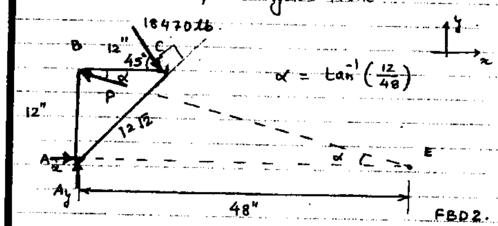
FBD 1

$\Sigma M_O = 0$

$\Rightarrow -20{,}000\,lb(\cos 24.5)\,48 + 20{,}000\,lb(\sin 24.5) * 36 + F_{DC}(\cos 20.5) * 29.9 + F_{DC}(\sin 20.5) * 8.91 = 0$

$\Rightarrow F_{DC} = 18{,}470\,lb$

Consider FBD of "Triangular plate"



$\alpha = \tan^{-1}\left(\frac{12}{48}\right)$

FBD 2.

$\Sigma M_A = 0$

$\Rightarrow P\cos\alpha\, 12 - 18470\,lb\, 12\sqrt{2} = 0$

$\Rightarrow \boxed{P = 26{,}900\ lb}$

$\Sigma F_x = 0 \Rightarrow A_x - 26{,}900\,lb\cos\alpha + 18470\,lb\cos 45 = 0$

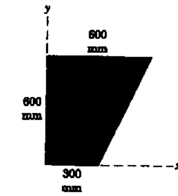
$\Rightarrow A_x = 13060\,lb$

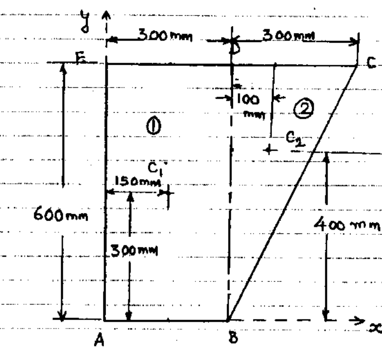
$\Sigma F_y = 0 \Rightarrow A_y + 26{,}900\,lb\sin\alpha - 18470\,lb\sin 45° = 0$

$\Rightarrow A_y = 6530\,lb$

$\boxed{A = \sqrt{A_x^2 + A_y^2} = \sqrt{(13060)^2 + (6530)^2}\,lb = 14600\,lb}$

5/43 Determine the coordinates of the centroid of the trapezoidal area shown.
Ans. $\bar{X} = 233$ mm, $\bar{Y} = 333$ mm





we devide the plate in two parts. ① & ②

For Plate ① ie. ABDE

Area: $A_1 = 600 \times 300\ mm^2 = 18 \times 10^4\ mm^2$

Coordinates of Centroid C_1

$X_1 = 150\ mm \quad Y_1 = 300\ mm$

For Plate ② i.e. BCD

Area: $A_2 = \frac{1}{2}(600 * 300) = 9 \times 10^4\ mm^2$

Coordinates of centroid C_2

$X_2 = (300 + 100)\ mm = 400\ mm$

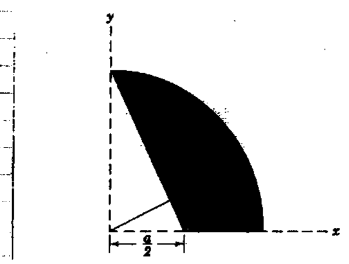
$Y_2 = 400\ mm$

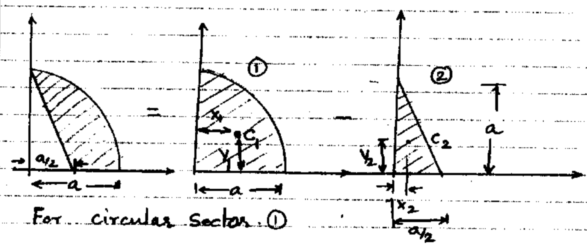
$\bar{X} = \frac{\Sigma AX}{\Sigma A} = \left(\frac{18\times10^4 * 150 + 9\times10^4 * 400}{18\times10^4 + 9\times10^4}\right) mm = 233\ mm$

$\bar{Y} = \frac{\Sigma AY}{\Sigma A} = \left(\frac{18\times10^4 * 300 + 9\times10^4 * 400}{18\times10^4 + 9\times10^4}\right) mm = 333\ mm$

5/51 By the method of this article, determine the x- and y-coordinates of the centroid of the shaded area of Prob. 5/19, repeated here.

Ans. $\bar{X} = \frac{7a}{6(\pi - 1)}$, $\bar{Y} = \frac{a}{\pi - 1}$





For circular Sector ①

Area: $A_1 = \frac{1}{2}(\pi/_2\, a^2)$ & Centroid $x_1 = \frac{4a}{3\pi} = y_1$

For Triangle ②

Area: $A_2 = -\frac{a^2}{4}$ & Centroid $x_2 = \frac{1}{3}\left(\frac{a}{2}\right) = \frac{a}{6}$

$y_2 = \frac{1}{3}a = \frac{a}{3}$

For the Given Section

$$x = \frac{\sum Ax}{\sum A} = \frac{\frac{\pi a^2}{4} \times \frac{4a}{3\pi} - \frac{a^2}{4} \times \frac{a}{6}}{\frac{\pi a^2}{4} - \frac{a^2}{4}}$$

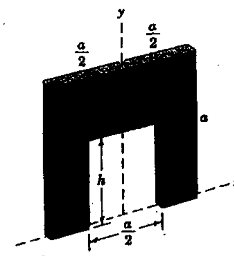
$$\Rightarrow \boxed{x = \frac{7a}{6(\pi-1)}}$$

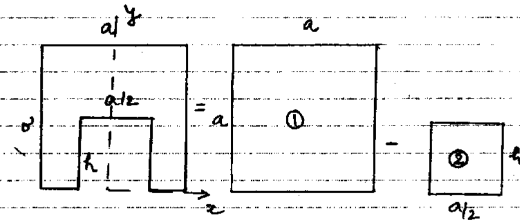
$$y = \frac{\sum Ay}{\sum A} = \frac{\pi a^2/4 \cdot 4a/3\pi - a^2/4 \cdot \frac{a}{3}}{\pi a^2/4 - a^2/4}$$

$$\boxed{y = \frac{a}{\pi - 1}}$$

5/67 Determine the dimension h of the rectangular opening in the square plate which will result in the mass center of the remaining plate being as close to the upper edge as possible.

Ans. $h = 0.586a$





For Plate ①

Area: $A_1 = a^2$, centroid $x_1 = 0$, $y_1 = a/2$

For Plate ②

Area: $A_2 = -\frac{ah}{2}$, centroid $x_2 = 0$, $y_2 = h/2$

For Given plane

$$x = 0 \quad \& \quad y = \frac{a^2 \cdot \frac{a}{2} - \frac{ah}{2}\cdot\frac{h}{2}}{\left(a^2 - \frac{ah}{2}\right)} = \frac{1}{2}\frac{(a^2 - h^2/2)}{(a - h/2)}$$

we want to maximize y so

$$\frac{dy}{dh} = 0 \Rightarrow \frac{1}{2}\frac{(a - h/2)(-h) - (a^2 - h^2/2)(-1/2)}{(a - h/2)^2} = 0$$

$$\Rightarrow h^2/4 - ah + a^2/2 = 0$$

$$\Rightarrow h = a(2 \pm \sqrt{2})$$

h has to be less than a so we discard + sign.

$$\Rightarrow \boxed{h = a(2 - \sqrt{2}) = 0.586a}$$