TAM202 Spring 2003

HW5 Solution provided by

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3.90, 4.29, 4.43, 4.49, 4.53, 4.62, 4.63 (due 02/18/03)

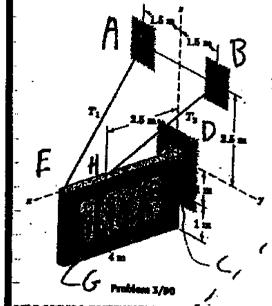
3.90

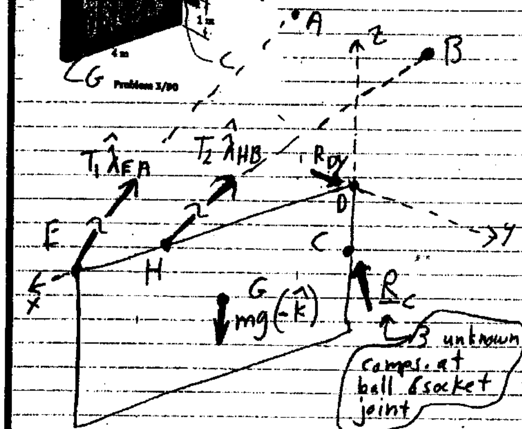
$T_1 = ?,\quad T_2 = ?,\quad R_{Dy} = ?$

$R_{Cx} = ?,\ R_{Cy} = ?,\ R_{Cz} = ?$

Method:

1) Draw FBD
2) Write Force & Moment Balance
3) Solve eqs.





$\sum \underline{F} = \underline{0}$ (3 eqs)

$\sum \underline{M}_{/C} = \underline{0}$ (3 eqs)

6 scalar equations

6 unknowns: $T_1, T_2, R_{Dy}, R_{Cx}, R_{Cy}$, R_{Cz} (don't be thrown off because the book answer only gives $R_C = |\underline{R}_C| = \sqrt{R_{Cx}^2 + R_{Cy}^2 + R_{Cz}^2}$).

Approach 1: Try to be clever.

For example $\sum M_{AB} = 0$ gives $\boxed{R_{Cx} = \frac{2.5}{3.5} mg}$

3.90 (cont'd)

and $\sum M_{EH} = 0 \Rightarrow \boxed{R_{Cy} = 0}$

etc.

Approach 2: "Brute Force"

Write out 6 eqs. in 6 unknowns, put in matrix form, and solve on computer.

Some geometry first:

$$\hat{\lambda}_{EA} = \frac{(-4\hat{\imath} - 1.5\hat{\jmath} + 2.5\hat{k})}{\sqrt{4^2 + 1.5^2 + 2.5^2}}$$

$$\hat{\lambda}_{HB} = \frac{-2.5\hat{\imath} + 1.5\hat{\jmath} + 2.5\hat{k}}{\sqrt{2.5^2 + 1.5^2 + 2.5^2}}$$

$\underline{r}_{DH} = 2.5\,m\,\hat{\imath}$

$\underline{r}_{DE} = 4\,m\,\hat{\imath}$

$\underline{r}_{DG} = 2m\hat{\imath} - 1m\hat{k}$, $\underline{r}_{DC} = -1\,m\,\hat{k}$

Force Balance

$\underline{0} = \sum \underline{F}$

$$\Rightarrow T_1\hat{\lambda}_{EA} + T_2\hat{\lambda}_{HB} + R_{Dy}\hat{\jmath} + \underline{R}_C = mg\hat{k} \quad (1)$$

We can take x, y, z comps (or dot w/ $\hat{\imath}, \hat{\jmath}, \hat{k}$) to get 3 eqs. in 6 unknowns. Note eqn. (1) is re-arranged to put knowns on right & unknowns on left.

Moment Balance

$\underline{0} = \sum \underline{M}_{/D}$

$$\Rightarrow \underline{r}_{DE} \times (T_1\hat{\lambda}_{EA}) + \underline{r}_{DH} \times (T_2\hat{\lambda}_{HB}) + \underline{r}_{DC} \times \underline{R}_C = \underline{r}_{DG} \times (mg\hat{k}) \quad (2)$$

After carrying out cross products we can also break (2) into comps. to get 3 more eqs. for the same 6 unknowns. Fortunately the cross products are pretty sparse.

(2) $\Rightarrow$

$$4\hat{\imath} \times (\lambda_{EAx}\hat{\imath} + \lambda_{EAy}\hat{\jmath} + \lambda_{EAz}\hat{k})T_1$$
$$+ 2.5\hat{\imath} \times (\lambda_{HBx}\hat{\imath} + \lambda_{HBy}\hat{\jmath} + \lambda_{HBz}\hat{k})T_2$$
$$+ -\hat{k} \times (R_{cx}\hat{\imath} + R_{cy}\hat{\jmath} + R_{cz}\hat{k})$$
$$= (2\hat{\imath} - \hat{k}) \times (mg\hat{k})$$

$$\Rightarrow (4\lambda_{EAy}\hat{k} - 4\lambda_{EAz}\hat{\jmath})T_1$$
$$+ (2.5\lambda_{HBy}\hat{k} - 2.5\lambda_{HBz}\hat{\jmath})T_2$$
$$- R_{cx}\hat{\jmath} + R_{cy}\hat{\imath} = -2mg\hat{\jmath} \qquad (2^*)$$

Now break (1) into comps, break (2^*) into comps, & write out all 6 eqs. in an organized way.

$$\lambda_{EAx}T_1 + \lambda_{HBx}T_2 + R_{cx} = 0 \qquad (3)$$
$$\lambda_{EAy}T_1 + \lambda_{HBy}T_2 + R_{cy} + R_{Dy} = 0 \qquad (4)$$
$$\lambda_{EAz}T_1 + \lambda_{HBz}T_2 + R_{cz} = mg \qquad (5)$$
$$R_{cy} = 0 \qquad (6)$$
$$-4\lambda_{EAz}T_1 - 2.5\lambda_{HBz}T_2 - R_{cx} = -2mg \qquad (7)$$
$$4\lambda_{EAy}T_1 + 2.5\lambda_{HBy}T_2 = 0 \qquad (8)$$

Now, eqs 3-8 (the comps. of force & moment balance) are 6 eqs in 6 unknowns.

One could fudge through, but

lets continue in "Brute Force" style. Eqns. 3-8 can be written in matrix form as

$$[A][x] = [y]$$

with

$$[A] = \begin{array}{c} (3)\\(4)\\(5)\\(6)\\(7)\\(8)\\ \\ \end{array}\begin{array}{c}\begin{bmatrix} \lambda_{EAx} & \lambda_{HBx} & 1 & 0 & 0 & 0 \\ \lambda_{EAy} & \lambda_{HBy} & 0 & 1 & 0 & 1 \\ \lambda_{EAz} & \lambda_{HBz} & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -4\lambda_{EAz} & -2.5\lambda_{HBz} & -1 & 0 & 0 & 0 \\ 4\lambda_{EAy} & 2.5\lambda_{HBy} & 0 & 0 & 0 & 0 \end{bmatrix} \\ \begin{matrix} T_1 & T_2 & R_{cx} & R_{cy} & R_{cz} & R_{Dy} \end{matrix}\end{array}$$

$$[x] = \begin{bmatrix} T_1 \\ T_2 \\ R_{cx} \\ R_{cy} \\ R_{cz} \\ R_{Dy} \end{bmatrix} \qquad [y] = \begin{bmatrix} 0 \\ 0 \\ mg \\ 0 \\ -2mg \\ 0 \end{bmatrix}$$

We can now feed this to a calculator or computer to get a soln.

The following soln. uses MATLAB. The key is the backslash (\) command. Given a matrix A & a column vector y

$$x = A\backslash y$$

finds the col. vector x that solves $Ax = y$ (caveats in Math 294!)

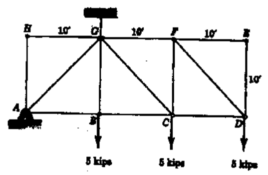

```
% Meriam and Kraige 3.90
% "Solution" by Andy Ruina on 9/11/02
% This is more expansive than need be.

% Using consistent units (meters, kg, Newtons),
% so they don't appear in the calculations below.

m = 100;   g = 9.81;    %let's keep the numbers more round

% Geometry first,  position vectors:
rEA =  [-4   -1.5  2.5];  %position of A relative to E
rHB =  [-2.5  1.5  2.5];  %position of B relative to H

% unit vectors:
lamEA = rEA/norm(rEA); % a unit vector in EA direction
lamHB = rHB/norm(rHB); % a unit vector in HB direction

% define the components of the unit vectors:
lamEAx = lamEA(1); lamEAy = lamEA(2); lamEAz = lamEA(3);
lamHBx = lamHB(1); lamHBy = lamHB(2); lamHBz = lamHB(3);

% set up the matrix A (called [A] in  solution handwork):
A = [  lamEAx       lamHBx      1   0   0   0
       lamEAy       lamHBy      0   1   0   1
       lamEAz       lamHBz      0   0   1   0
         0            0         0   1   0   0
      -4*lamEAz   -2.5*lamHBz  -1   0   0   0
       4*lamEAy    2.5*lamHBy   0   0   0   0];

% Now set up the right hand side, [y]:
y =  [ 0  0 m*g  0  -2*m*g   0]'; % note: ' means transpose

% Now solve the equations. Use the great backslash command:
% All the trouble above, the hand vector calculations and
% the computer work, was to use this one line of code:
x = A\y;

% Now unpack the [x] column vector to our variables:
T1  = x(1)
T2  = x(2)
RCx = x(3)
RCy = x(4)
RCz = x(5)
RDy = x(6)
RC = norm([RCx RCy RCz])

%End of m file
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
T1 =  346.84 N
T2 =  430.58 N

RCx = 560.57 N
RCy =   0    N
RCz = 525.54 N

RDy = -63.06 N

RC =  768.39 N
```

This is the output from the .m file above. Edited in a text editer to save space.

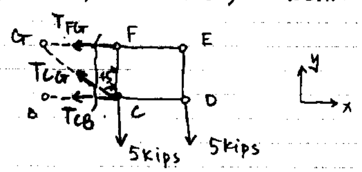
Answers to 3.90

4/29 Determine the force in member CG.
Ans. CG = 14.14 kips T



Problem 4/29

This hw set is about the method of sections. In each problem, you should make "a smart cut" at some section and try to get unknowns directly. For example, in 4.29, if you think about using the method of joints, it may be quite tedious. Instead, a cut as done below, does the job well.

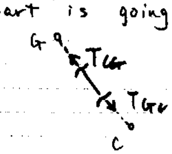


$\Sigma F_y = 0$

$T_{CG} \cdot \cos 45° - 5\text{ kips} - 5\text{ kips} = 0$

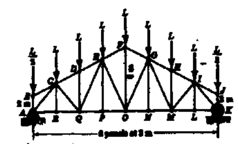
$\underline{T_{CG} = 14.14\text{ kips}}$

Hence, from the law of action and reaction, the force applied on member CG by that part is going in opposite direction.



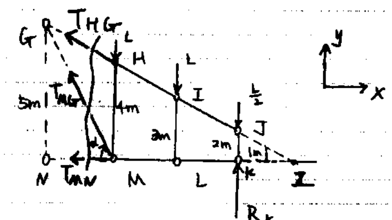
Thus, member CG is under tension.

4/43 Compute the force in member GM of the loaded truss.
Ans. GM = 0



Problem 4/43

If we make a cut crossing member GH, GM and NM, there will be three member forces to find. Fortunately, we notice that, member force in GH and NM will cross each other at some pt. on the right of pt. K. So if we take moment balance about that pt. I, we will have one eqn w/ one unknown.



(no horizontal reaction at K)

We notice the whole truss is symmetrical. Thus we should have the same amount of reaction at pt. A and pt. K. Doing force balance eqn. in y direction gives us:

$$R_K = 8L/2 = 4L$$

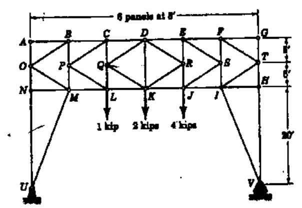
From the geometry given, we know NM = 3 m and GN = 5 m, KI = 6 m.

$$\alpha = \tan^{-1}\frac{5}{3} = 59°$$

$$\Sigma M_I = 0: \quad (\tfrac{L}{2} - 4L)\cdot 6\text{ m} + L \cdot 9\text{m} + L \cdot 12\text{m} - T_{GM}\cdot \sin(59°)\cdot 12\text{ m} = 0$$

$$\underline{T_{GM} = 0}$$

4/49 Determine the force in member DK of the loaded overhead sign truss.
Ans. DK = 1 kip T



Problem 4/49

It seems there is no easy way around. :(

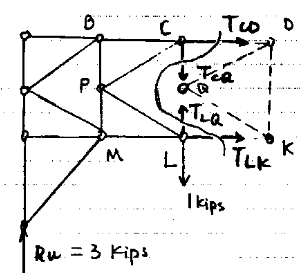
First, find reactions at pt. U. Due to the constrain shown at pt. V, there is no horizontal reactions. Thus, we have only vertical component in reactions at pt. U.

$\Sigma M_V = 0$ (assuming R_u is ↑)

$-R_u \cdot 48' + 1\text{ kips}\cdot 32' + 2\text{ kips}\cdot 24' + 4\text{ kips}\cdot 16' = 0$

$R_u = 3\text{ kips}$ ↑

Make a cut as shown below,



In this way, if we do a moment balance about pt. C, we can find F_{LK} directly.

$\Sigma M_C = 0: \quad T_{LK}\cdot 10' - R_u \cdot 16' = 0$

$T_{LK} = 4.8$ kips (LK is under tension)

Similarly,

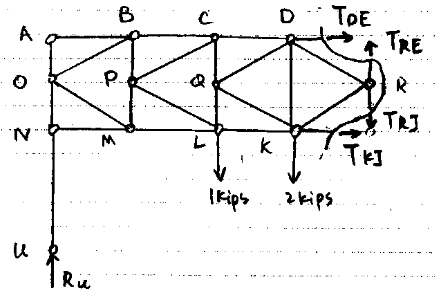
$\Sigma M_L = 0 \quad T_{CD}\cdot 10' - R_u \cdot 16' = 0$

$T_{CD} = -4.8$ kips

∴ CD is under compression.

as assumed.

Make another similar cut shown below,

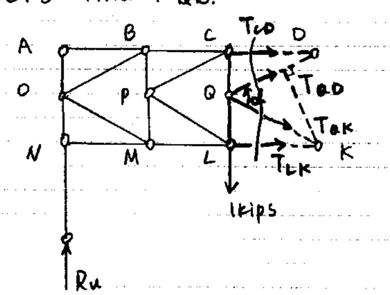


$\circlearrowleft + \Sigma M_J = 0: \quad -R_u \cdot 32' + 1\text{kips} \cdot 16' + 2\text{kips} \cdot 8' - T_{DE} \cdot 10' = 0$

$T_{DE} = -6.4$ kips

DE is under compression.

We want to apply the method of joints at pt. D to find out F_{DK}. Now we know F_{CD} and F_{DE}. So we have to find either F_{QD} or F_{DR}. Let's find F_{QD}.



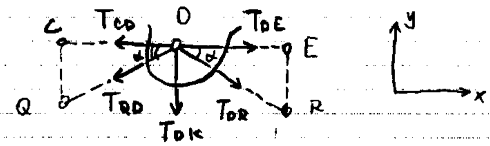
Geometry: $QC = 5'$, $CD = 8'$, $QD = 9.434'$
$\alpha = 2 \cdot \tan^{-1}(5/8) = 64°$

$\circlearrowleft + \Sigma M_K = 0: \quad -R_u \cdot 24' + 1\text{kips} \cdot 8' - T_{CD} \cdot 10' - T_{QD} \cdot (QK \cdot \sin\alpha) = 0$

$T_{QD} = -1.89$ kips

QD is under compression.

Now, we can look at joint D.



$\alpha = \tan^{-1}(5/8) = 32°$

$\Sigma F_x = 0:$

$-T_{CD} + T_{DE} - T_{QD} \cdot \cos\alpha + T_{DR} \cdot \cos\alpha = 0$

$\Sigma F_y = 0:$

$-T_{QD} \cdot \sin\alpha - T_{DR} \cdot \sin\alpha - T_{DK} = 0$

(only two unknowns, F_{DK} and F_{DR})

$T_{DR} = 0$

$T_{DK} = 1$ kips (Tension)

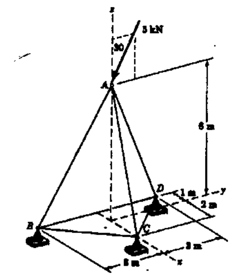
"positive" means F_{DK} is in the direction assumed also.

Thus, from the law of action and reaction, the force applied on member DK is also positive, which means it is under tension.

PROBLEMS

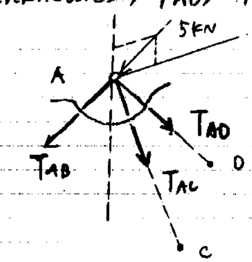
(In the following problems, use plus for tension and minus for compression.)

4/53 Determine the forces in members AB, AC, and AD.
Ans. AB = −4.46 kN; AC = −1.521 kN
AD = 1.194 kN



Problem 4/53

If we do force balance eqns at pt. A, we will have 3 component eqns in 3D and we have exactly 3 unknowns, T_{AB}, T_{AC} and T_{AD}.



Our coordinates are shown above.

$$\vec{T}_{AB} = T_{AB} \cdot \frac{\vec{AB}}{|\vec{AB}|} = T_{AB} \cdot \frac{-\hat{i} - 3\hat{j} - 6\hat{k}}{\sqrt{1^2 + 3^2 + 6^2}}$$

$$\vec{T}_{AC} = T_{AC} \cdot \frac{\vec{AC}}{|\vec{AC}|} = T_{AC} \cdot \frac{2\hat{i} - 6\hat{k}}{\sqrt{2^2 + 6^2}}$$

$$\vec{T}_{AD} = T_{AD} \cdot \frac{\vec{AD}}{|\vec{AD}|} = T_{AD} \cdot \frac{-\hat{i} + 3\hat{j} - 6\hat{k}}{\sqrt{1^2 + 3^2 + 6^2}}$$

#4.53 (Cont'd)

$\Sigma \underline{F} = 0:$

$T_{AB} + T_{AC} + T_{AD} - 5\,\text{kN}\cdot(\cos 30^\circ \underline{k} + \sin 30^\circ \underline{j}) = 0$

$-\frac{1}{\sqrt{46}} T_{AB} + \frac{2}{\sqrt{40}} T_{AC} - \frac{1}{\sqrt{46}} T_{AD} = 0$

$-\frac{3}{\sqrt{46}} T_{AB} + \frac{3}{\sqrt{46}} T_{AD} - 5(\tfrac{1}{2})\ \text{kN} = 0$

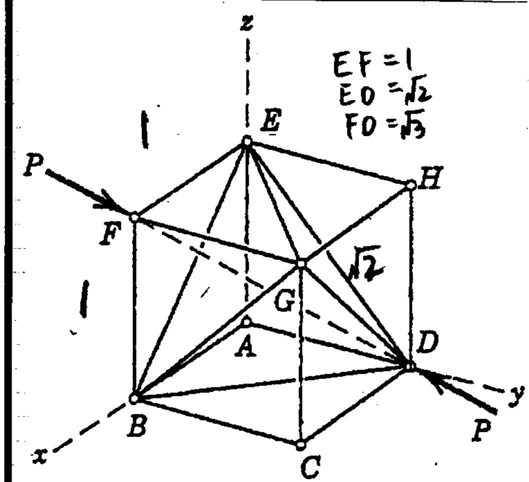
$-\frac{6}{\sqrt{46}} T_{AB} - \frac{6}{\sqrt{40}} T_{AC} - \frac{6}{\sqrt{46}} T_{AD} - 5\cdot\frac{\sqrt{3}}{2}\ \text{kN} = 0$

$$\Rightarrow \begin{cases} T_{AB} = -4.46\ \text{kN} \\ T_{AC} = -1.521\ \text{kN} \\ T_{AD} = 1.194\ \text{kN} \end{cases}$$

Member AB and AC are under compression while AD is under tension.

#4.62

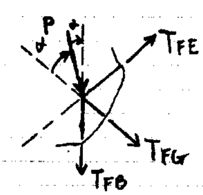
▶4/62 A space truss is constructed in the form of a cube with six diagonal members shown. Verify that the truss is internally stable. If the truss is subjected to the compressive forces P applied at F and D along the diagonal FD, determine the forces in members FE and EG. *Ans.* $F_{FE} = -P/\sqrt{3}$, $F_{EG} = P/\sqrt{6}$



Problem 4/62

The space truss is symmetrical about axis FD.

Thus, $T_{FE} = T_{FG} = T_{FB} = F$



$\alpha = \cos^{-1}(\frac{1}{\sqrt{3}})$

$\Sigma \underline{F} = 0$

$F(-\underline{i} + \underline{j} - \underline{k}) + \frac{P}{\sqrt{3}}\cdot(-\underline{i} + \underline{j} - \underline{k}) = 0$

#4.62 (Cont'd)

$\Rightarrow F = -P/\sqrt{3} \Rightarrow T_{FE} = -P/\sqrt{3}$

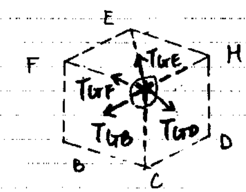
FE is under compression

We apply the method of joints at pt. G to find out T_{EG}. Thus, we have to figure out T_{GH}, T_{GC}, T_{GB}, T_{GD}.

If looking at pt. H, a FBD of that joint tells us, T_{GH}, T_{EH} and T_{DH} are zero-force members.

Similarly, looking at pt. C, we get T_{CG}, T_{CB} and T_{CD} all zeros.

From symmetry, $T_{GD} = T_{ED}$, and $T_{GB} = T_{GE} = R$.



so at joint G, we have 2 unknowns, $T_{GB} = T_{GE} = R$, T_{GD} and $\underline{T}_{GF} = T_{GF}(-\underline{j})$

$\Sigma \underline{F} = 0: \ -\frac{P}{\sqrt{3}}(-\underline{j}) + \frac{T_{GE}}{\sqrt{2}}(-\underline{i} - \underline{j}) + \frac{T_{GB}}{\sqrt{2}}(-\underline{j} - \underline{k}) + \frac{T_{GD}}{\sqrt{2}}(-\underline{i} - \underline{k}) = 0$

collecting $\underline{j}$ terms,

$$\underline{\underline{T_{GE} = P/\sqrt{6}}}$$

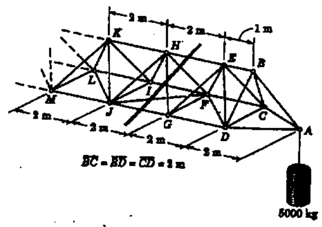
For the truss, no. of members = 18
no. of joints = 8

$(18 + 6 = 24) = (3 \cdot 8 = 24)$

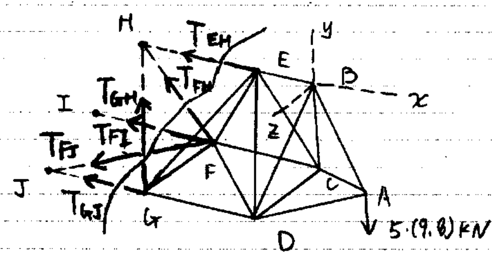
Therefore, internally stable.

4.63.

▸ **4/63 The lengthy boom of an overhead construction crane, a portion of which is shown, is an example of a periodic structure—one which is composed of repeated and identical structural units. Use the method of sections to find the forces in members *FJ* and *GJ*.**
Ans. $FJ = 0$, $GJ = -70.8$ kN



Problem 4/63



All members are assumed in tension and there are six members (EH, FH ~~GH~~, FI, FJ, GJ) cut at the section shown above.
Our coordinates are shown above.
If you take a moment balance eqn. about x axis (along HE), there will only leave one force ($\underline{T}_{FJ}$) which has contribution to the eqn.

Thus, $T_{FJ} = 0$

But since the Q asked us to solve it using the method of section, here we go.

$$\underline{T}_{GJ} = T_{GJ}\,(-\underline{i})$$

$$\underline{T}_{FI} = T_{FI}\,(-\underline{i})$$

$$\underline{T}_{FJ} = T_{FJ}\left(-\frac{\underline{i}+\underline{k}}{\sqrt{2}}\right)$$

4.63 (Cont'd)

$\Sigma \underline{M}_H = 0$:

$$-5\cdot(9.81)\cdot 5\,\text{m}\,\underline{k} + (-2\cos 30^\circ\,\underline{j} + 2\cdot\sin 30^\circ\,\underline{k})\,\text{m} \times \underline{T}_{GJ} + (-2\cos 30^\circ\,\underline{j} - 2\sin 30^\circ\,\underline{k})\,\text{m} \times \underline{T}_{FI} + (\underline{i} - 2\cos 30^\circ\,\underline{j} - \underline{k})\,\text{m} \times \underline{T}_{FJ} = 0$$

taking each component equal to zero

$$\begin{cases} T_{FJ} = 0 \\ T_{FI} = T_{GJ} = -70.8 \text{ kN} \end{cases}$$

Therefore, $T_{GJ} = -70.8$ kN

which means member GJ is under compression