TAM 202, Spring 2003

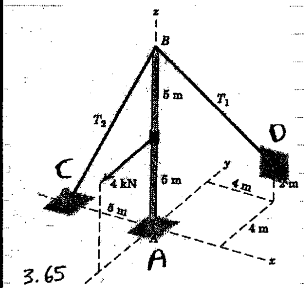
HW4 due 02/11/03

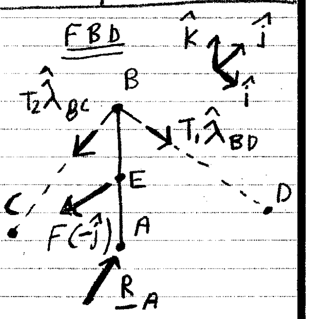
"SOLUTIONS" by Andy Ruina

3.65 Mer. & Kraige

$T_1 = ?$

FBD





$F = 4\,kN$

$\hat{\lambda}_{BD}$ = unit vector pointing from B to D

$= \underline{r}_{BD}/|\underline{r}_{BD}|$ [IRONY: Unit vectors have no units.]

$= (4\hat{i} + 4\hat{j} - 8\hat{k})/\sqrt{4^2+4^2+(-8)^2}$

$= \frac{4}{\sqrt{96}}\hat{i} + \frac{4}{\sqrt{96}}\hat{j} - \frac{8}{\sqrt{96}}\hat{k}$

$\hat{\lambda}_{BC}$ = unit vector from B to C (no need to work out)

$\sum M_{axis\,AC} = 0$ [the key equation!]

$\Rightarrow \left(\sum \underline{M}_{/A}\right) \cdot$ (any vector in $\pm$AC direc.) $= 0$

$\Rightarrow \left(\sum \underline{M}_{/A}\right) \cdot \hat{i} = 0$

$\Rightarrow \left[\underline{r}_{AE} \times (-F\hat{j}) + \underline{r}_{AB} \times (T_1 \hat{\lambda}_{BD})\right] \cdot \hat{i} = 0$

(note line of action of T_2 intersects axis AC, so it drops out. Likewise for the reaction at A.)

$\left[(5\,\cancel{m}\,\hat{k}) \times (-F\hat{j}) + (10\,\cancel{m}\,\hat{k}) \times T_1\left(\frac{4}{\sqrt{96}}\hat{i} + \frac{4}{\sqrt{96}}\hat{j} - \frac{8}{\sqrt{96}}\hat{k}\right)\right] \cdot \hat{i} = 0$

#3.65 (cont'd)

$\Rightarrow \left[5F\hat{i} + \frac{10T_1}{\sqrt{96}}(4\hat{j} - 4\hat{i})\right] \cdot \hat{i} = 0$

$\Rightarrow 5F = \frac{40}{\sqrt{96}} T_1$

$\Rightarrow T_1 = \frac{\sqrt{96}}{8} F$

$= \frac{\sqrt{96}}{8}(4\,kN)$

$$\boxed{T_1 = \frac{\sqrt{96}}{2}\,kN \approx 4.90\,kN}$$ 3.65

Comments on 3.65

One could instead assemble 6 eqs:

$\sum \underline{F} = \underline{0}$ (3 eqs)

$\sum \underline{M}_{/A} = \underline{0}$ (3 eqs)

} 6 scalar equations

↑ A or any other pt.

in 5 unknowns: T_1, T_2 and 3 comps. of $\underline{R}_A$ (R_{Ax}, R_{Ay}, R_{Az}).

For this problem one of these equations is just $0=0$. That is, none of the forces contribute to ΣM_{Az} (no moments about z axis through A). You could solve the 5 eqs. in 5 unknowns and get T_1 as well as T_2, R_{Ax}, R_{Ay} & R_{Az}.

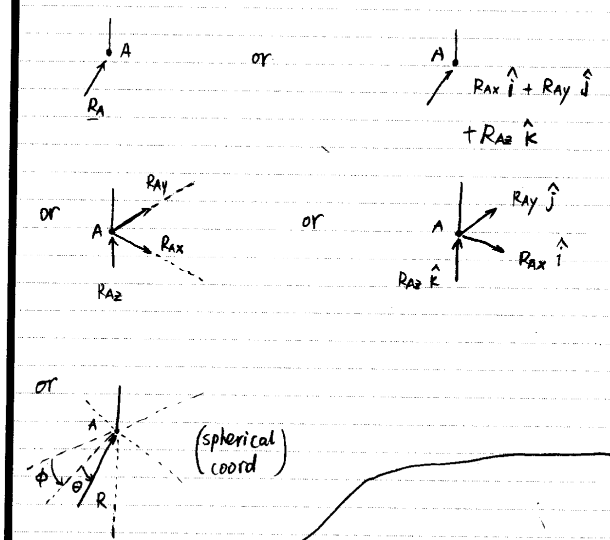
Although you had 5 eqs. in 5 unknowns, if you were alert you might have noticed that one of them, $\Sigma M_{Ax} = 0$, had just one unknown, namely T_1. So you could solve it w/out ever solving the other 4 eqs.

That one eqn. is equivalent to $\Sigma M_{AC} = 0$, the solution presented.

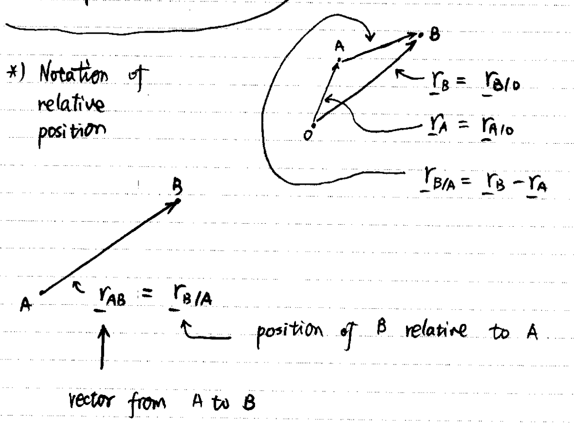
#3.65 Comments (Cont'd)

*) There are various ways to show forces on FBDs. In all cases, the force should be fully defined in terms of knowns and unknowns. All of the following would be OK for the reaction of A.

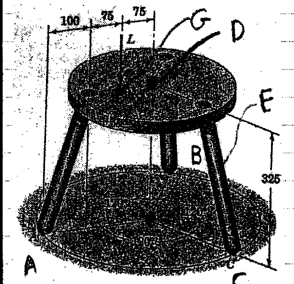
Partial FBDs



*) Notation of relative position



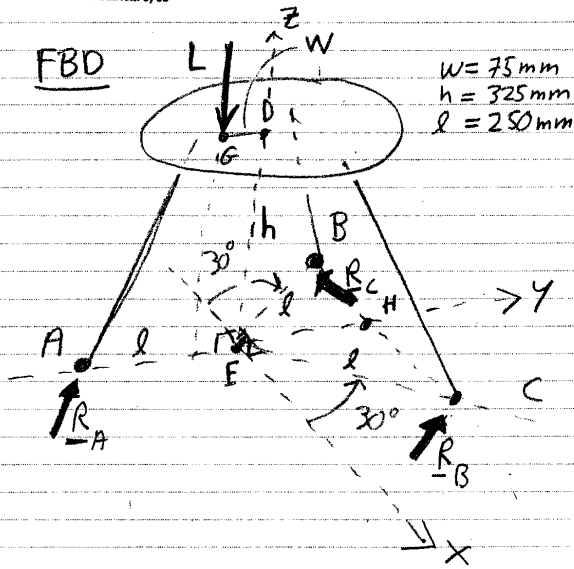
$R_{Az} = ?,\quad R_{Bz} = ?,\quad R_{cz} = ?$



Dimensions in millimeters

Problem 3/69

Neglect weight. No comment on friction at A, B, C! So leave it in & see if we can get an answer.



$w = 75mm$
$h = 325mm$
$l = 250mm$

One FBD in 3D ⇒ 6 scalar eqs. $(\Sigma \underline{F} = 0,\ \Sigma \underline{M}_{/E} = 0)$ ← or any other pt.

We have 9 unknowns (3 comps each for $\underline{R}_A$, $\underline{R}_B$, $\underline{R}_C$ because their directions are unknown).

⇒ Problem is "statically indeterminate" ⇒ We can't find all the unknowns. Can we find the one's we are asked for?

Yes!

$\Sigma M_{BC} = 0 \qquad (1)$

← sum of moments about axis BC.

$\underline{R}_B$ & $\underline{R}_C$ go through axis and don't contribute. Likewise for R_{Ax} & R_{Ay}. So eqn. (1) is one eqn. in one unknown.

$$(1) \Rightarrow \left[\underline{r}_{HG} \times (L(-\hat{k})) + \underline{r}_{HA} \times (R_{Az}\hat{k})\right] \cdot \hat{i} = 0$$

Slide $-L\hat{k}$ down to x-y plane and all vectors are ⊥.

$$\Rightarrow R_{Az}\, l(1 + \underbrace{\sin 30°}_{1/2}) = L(w + l\underbrace{\sin 30°}_{1/2})$$

$$\Rightarrow R_{Az} = \frac{w + l/2}{3l/2} L$$

$$= \frac{75 + 125}{3 \cdot 250/2} L$$

$$\boxed{R_{Az} = \frac{200}{375}L = \frac{8L}{15} \approx .53L}$$

$\Sigma M_{AE} = 0 \qquad (2)$

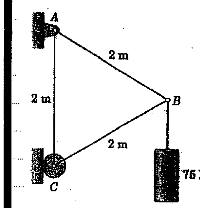
⇒ $R_{Cz} = R_{Bz}$ because they have the same lever arm $l\cos 30°$ about axis AE (the y axis).

We could find R_{Bz} by $\Sigma M_{AB} = 0$ but its easier to use (2) and

$\Sigma F_z = 0 \qquad [(\Sigma \underline{F}) \cdot \hat{k} = 0] \quad (3)$

$\Rightarrow -L + \frac{8}{15}L + 2R_{Cz} = 0$

$$\Rightarrow \boxed{R_{Cz} = \frac{7}{30}L \approx .23L \Rightarrow R_{Bz} \approx .23L}$$

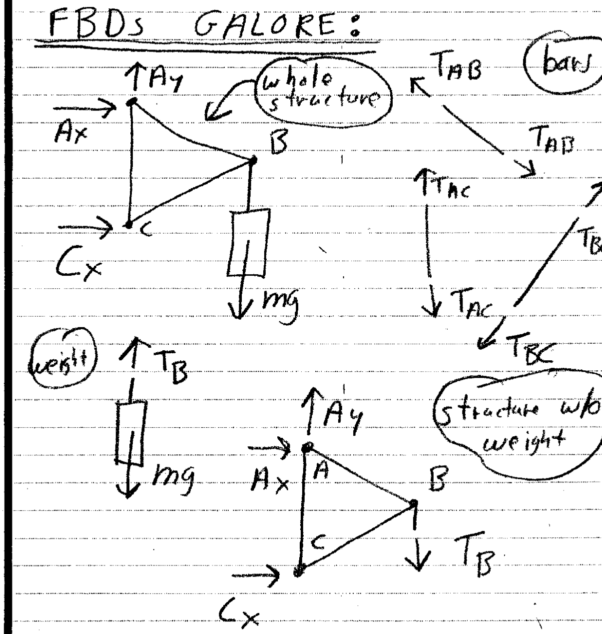


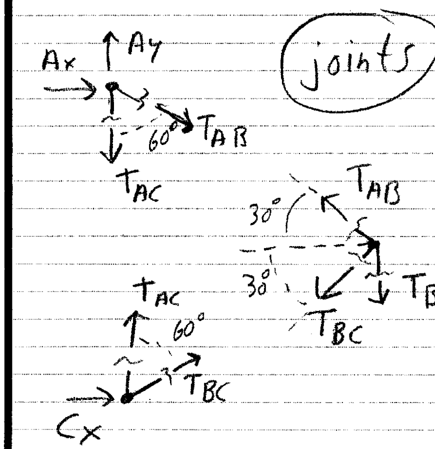
Problem 4/1

Find all tensions.

$\hat{j}$ $\hat{i}$

FBDs GALORE:





FBD of Weight, $\Sigma F_y = 0 \Rightarrow T_B = mg$.

FBD of joint B

$\Sigma \underline{F} = \underline{0} \Rightarrow$ (*)

$$\left\{ T_{AB}(-\cos 30^\circ \hat{i} + \sin 30^\circ \hat{j}) + T_{BC}(-\cos 30^\circ \hat{i} - \sin 30^\circ \hat{j}) - mg\hat{j} = \underline{0} \right\}$$

$\{\ \}\cdot\hat{i}$ (that is, take x-comp of force balance eqn.)

$\Rightarrow T_{AB} = -T_{BC}$ (1)

$\{\ \}\cdot\hat{j}$ (take y comp. of *)

$\Rightarrow T_{AB}\sin(30^\circ) - T_{BC}\sin(30^\circ) - mg = 0$ (2)

(1) & (2) $\Rightarrow T_{AB}(\frac{1}{2}) + T_{AB}\frac{1}{2} = mg$

$$\Rightarrow \boxed{\begin{aligned} T_{AB} &= mg \\ &= 75\,kg \cdot 9.81\,N/kg \\ T_{AB} &\approx 736\,N \end{aligned}}$$

$$\boxed{T_{BC} = -T_{AB} \approx -736N}$$

"Tension in BC is −736N, comp. is 736N."

FBD of joint C, $\Sigma F_y = (\Sigma \underline{F})\cdot\hat{j} = 0$

$\Rightarrow T_{AC} + T_{BC}\cos 60^\circ = 0$

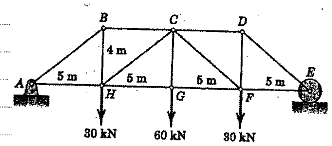
$\Rightarrow T_{AC} = -T_{BC}\cos 60^\circ$

$= -(-736N)\frac{\sqrt{3}}{2}$

$$\boxed{T_{AC} \approx 637N}$$

"tension in AC is 637N"

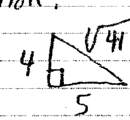
4.7 Find all "bar forces".



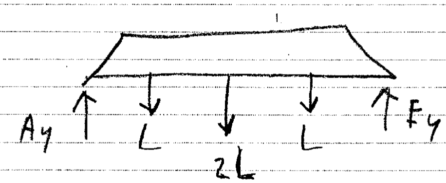
Problem 4/7

$L = 30kN$

note:



FBD of structure

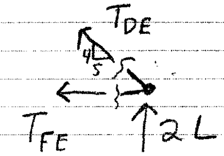


$\Sigma F_y = 0$ & symmetry

$\Rightarrow A_y = E_y = 2L$

Now we start at joint E & work our way down the structure.

joint E:



$\Sigma F_y = 0 \Rightarrow 2L + T_{DE}\frac{4}{\sqrt{41}} = 0$

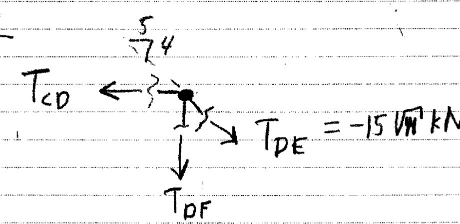
$$\boxed{\begin{aligned} T_{DE} &= \frac{-\sqrt{41}\,L}{2} = \frac{-\sqrt{41}\,30kN}{2} \\ &\approx -96\,kN \\ &\text{also } T_{AB} \text{ by symmetry} \end{aligned}}$$

("tension in DE is −96kN, compression is 96kN")

$\Sigma F_x = 0 \Rightarrow -T_{DE}\frac{5}{\sqrt{41}} - T_{FE} = 0$

$$\Rightarrow \boxed{\begin{aligned} T_{FE} &= \frac{-5\,T_{DE}}{\sqrt{41}} \\ &= \frac{150}{2}\,kN \\ &= 75\,kN \\ &(= T_{AH} \text{ by symmetry}) \end{aligned}}$$

joint D



$\Sigma F_x = 0 \Rightarrow -T_{CD} + T_{DE}\frac{5}{\sqrt{41}} = 0$

$$\boxed{\begin{aligned} T_{CD} &= \frac{5}{\sqrt{41}}\left(-15\sqrt{41}\,kN\right) \\ &= -75\,kN \\ &\text{symmetry} \Rightarrow T_{BC} = -75kN \end{aligned}}$$

$\Sigma F_y = 0 \Rightarrow -T_{DF} - T_{DE}\frac{4}{\sqrt{41}} = 0$

#4.7 (cont'd)

$$\Rightarrow \boxed{T_{DF} = -\frac{4}{\sqrt{41}}\left(-15\sqrt{41}\text{ kN}\right) = 60\text{ kN} \quad (T_{BH} = 60\text{ kN})}$$

joint F

T_{FC} $\quad T_{DF} = 60$ kN

$T_{EF} = 75$ kN

T_{FG}

$L = 30$ kN

$\Sigma F_y = 0 \Rightarrow$

$$T_{FC}\frac{4}{\sqrt{41}} + T_{DF} - L = 0$$

$$\boxed{T_{FC} = \frac{\sqrt{41}}{4}\left[-60\text{ kN} + 30\text{ kN}\right] \approx -48\text{ kN}, \quad (T_{CH} \approx -48\text{ kN})}$$

$\Sigma F_x = 0 \Rightarrow$

$$T_{EF} - T_{FC}\frac{5}{\sqrt{41}} - T_{FG} = 0$$

$$\Rightarrow \boxed{\begin{aligned} T_{FG} &= T_{EF} - \frac{5}{\sqrt{41}}T_{FC} \\ &= 75\text{ kN} - \frac{5}{\sqrt{41}}(-48\text{ kN}) \\ &\approx 112.5\text{ kN} \\ &(\text{symmetry} \Rightarrow T_{HG} \approx 112.5\text{ kN}) \end{aligned}}$$

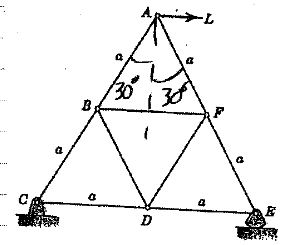
Finally joint G

T_{CG}

60 kN = 2L

$$\Sigma F_y = 0 \Rightarrow \boxed{T_{CG} = 60\text{ kN}} = 2L$$

#4.15



Problem 4/15

Find all bar forces.

FBD

L, C, F_E, F_E

$$\Sigma M_{/C} = 0 \Rightarrow F_E = \frac{\sqrt{3}}{2}L$$

joint A

$\Sigma F_y = 0 \Rightarrow T_{AF} = T_{AB}$

$\Sigma F_x = 0$

T_{AB} $\quad T_{AF} \Rightarrow$

$$-\frac{T_{AB}}{2} + \frac{T_{AF}}{2} + L = 0$$

$$\Rightarrow \boxed{T_{AF} = -L,\ T_{AB} = L}$$

joint E

T_{EF}, T_{DE}, $\frac{\sqrt{3}}{2}L$

$$\Sigma F_y = 0 \Rightarrow T_{EF}\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}L = 0$$

$$\Rightarrow \boxed{T_{EF} = -L}$$

$$\Sigma F_x = 0 \Rightarrow \boxed{T_{DE} = \frac{L}{2}}$$

joint F

$-L$, T_{BF}, $-L$, T_{FD}

$\Sigma F_x = 0,\ \Sigma F_y = 0 \Rightarrow$

$$\boxed{T_{BF} = T_{FD} = 0}$$

joint D

T_{DB}, $T_{DF} = 0$, $T_{DE} = \frac{L}{2}$, T_{CD}

$\Rightarrow \boxed{T_{DB} = 0}$ (zero force member)

$\Rightarrow \boxed{T_{CD} = L/2}$

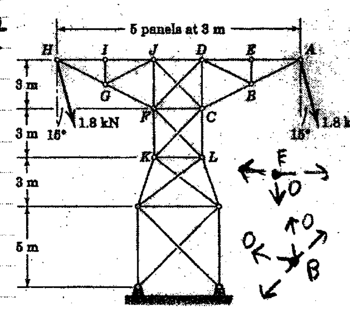
joint B

$T_{AB} = L$, 0, 0, T_{BC}

$\Rightarrow \boxed{T_{BC} = L}$

4.27

!



joint E

$\Rightarrow T_{EB} = 0$

joint B

$\Rightarrow \boxed{T_{DB} = 0}$

E, 0, 0, 0, B

"Zero force members"