TAM 202
Spring 2003

Quiz 0

Solution provided by
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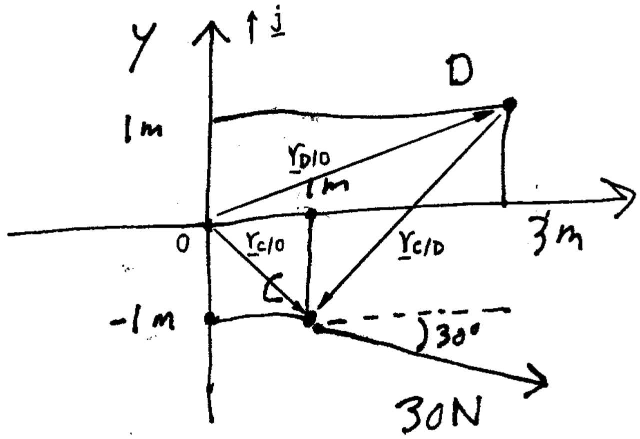
1) (7 pts) Find a force-couple system at D that is equivalent to the single force at C shown.



$\cos 30° = \frac{\sqrt{3}}{2}$

$\sin 30° = \frac{1}{2}$

$\underline{F}_C = (30N)[\cos 30° \underline{i} - \sin 30° \underline{j}] = (15\sqrt{3}N)\underline{i} - (15N)\underline{j}$

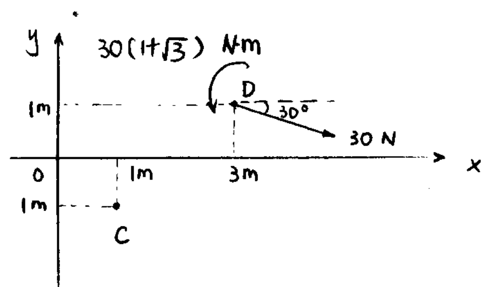
$\underline{F}_D = \underline{F}_C = (15\sqrt{3}\ N)\underline{i} - (15N)\underline{j}$

$\underline{M}_{/D} = \underline{r}_{C/D} \times \underline{F}_D = [\underline{r}_{C/O} - \underline{r}_{D/O}] \times \underline{F}_D$

$= \{[(1m)\underline{i} - (1m)\underline{j}] - [(3m)\underline{i} + (1m)\underline{j}]\} \times [(15\sqrt{3}\ N)\underline{i} - (15N)\underline{j}]$

$= [(-2m)\underline{i} - (2m)\underline{j}] \times [(15\sqrt{3}\ N)\underline{i} - (15N)\underline{j}]$

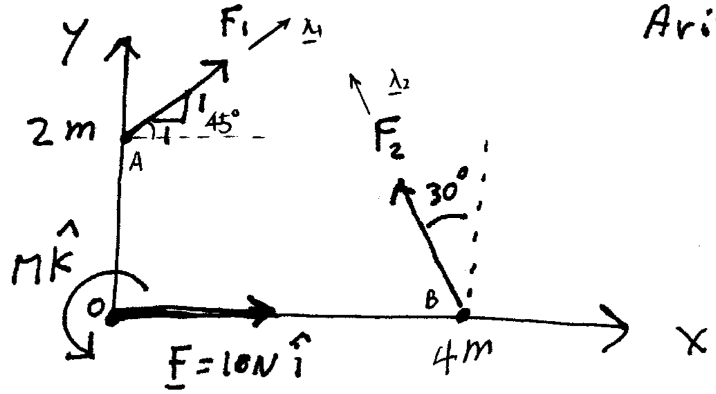
$= 30(1+\sqrt{3})\ N\text{-}m \quad \underline{k}$



force-couple system at D.

$$\boxed{\begin{aligned} \underline{F}_D &= (15\sqrt{3}N)\underline{i} - (15N)\underline{j} \\ \underline{M}_D &= 30(1+\sqrt{3})N\text{-}m\,\hat{k} \end{aligned}}$$

2) (10 pts) The force system ($\underline{F}_1$, $\underline{F}_2$) is equivalent to a for $\underline{F} = 10\,\text{N}\,\hat{i}$ at the origin and a couple $M\hat{k}$. Find M. [You have sufficient information, Arithmetic is not nec. tidy.]



① $\underline{F}_1 = F_1(\cos 45^\circ\,\underline{i} + \sin 45^\circ\,\underline{j}) = F_1\,\underline{\lambda}_1$ $\qquad \underline{F}_2 = F_2(-\sin 30^\circ\,\underline{i} + \cos 30^\circ\,\underline{j}) = F_2\,\underline{\lambda}_2$

$\Rightarrow \quad \underline{F} = \underline{F}_1 + \underline{F}_2 = (F_1\cos 45^\circ - F_2\sin 30^\circ)\,\underline{i} + (F_1\sin 45^\circ + F_2\cos 30^\circ)\,\underline{j}$

$\{\, (10\text{N})\,\underline{i} = (F_1\cos 45^\circ - F_2\sin 30^\circ)\,\underline{i} + (F_1\sin 45^\circ + F_2\cos 30^\circ)\,\underline{j} \,\}$

$\{\ \}\cdot(\underline{i}-\underline{j}) \Rightarrow \quad -F_2(\sin 30^\circ + \cos 30^\circ) = 10\text{ N} \quad \Rightarrow \quad F_2 = -\dfrac{10\text{N}}{\frac{1}{2}+\frac{\sqrt{3}}{2}} = -\dfrac{20\text{N}\,(1-\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})} = -10(\sqrt{3}-1)\text{ N}$

$\{\ \}\cdot\underline{j} \Rightarrow \quad F_1\sin 45^\circ + F_2\cos 30^\circ = 0\text{N} \quad \Rightarrow \quad F_1 = -F_2\,\dfrac{\frac{\sqrt{3}}{2}}{\frac{\sqrt{2}}{2}} = [10(\sqrt{3}-1)\text{N}]\cdot\dfrac{\sqrt{6}}{2} = 5(3\sqrt{2}-\sqrt{6})\text{ N}$

② $\underline{M}_{/O} = \underline{r}_{A/O}\times\underline{F}_1 + \underline{r}_{B/O}\times\underline{F}_2$

$= (2\text{m})\,\underline{j}\times F_1(\cos 45^\circ\,\underline{i} + \sin 45^\circ\,\underline{j}) + (4\text{m})\,\underline{i}\times F_2(-\sin 30^\circ\,\underline{i} + \cos 30^\circ\,\underline{j})$

$= [-(2\text{m})F_1\cos 45^\circ + (4\text{m})F_2\cos 30^\circ]\,\underline{k}$

$= \{-(2\text{m})[5(3\sqrt{2}-\sqrt{6})\text{N}]\frac{\sqrt{2}}{2} + (4\text{m})[-10(\sqrt{3}-1)\text{N}]\frac{\sqrt{3}}{2}\}\,\underline{k}$

$= \{-5(6-2\sqrt{3}) - 20(3-\sqrt{3})\}\text{ N-m }\underline{k} \quad = \quad 30(\sqrt{3}-3)\text{ N-m }\underline{k}$

Note:

(1) $\underline{i}-\underline{j}$ is chosen to be $\perp$ to $\underline{\lambda}_1$ to "kill" $\underline{F}_1$ by dot product

(2) You can also use

$\{\ \}\cdot\underline{i} \Rightarrow F_1\cos 45^\circ - F_2\sin 30^\circ = 10\text{N}$

$\{\ \}\cdot\underline{j} \Rightarrow F_1\sin 45^\circ + F_2\cos 30^\circ = 0\text{N}$

and solve the two eqns

$\boxed{\underline{M} = 30(\sqrt{3}-3)\text{ N-m }\underline{k}}$