Cornell University

ENGRD 202
Professor Andy Ruina
Spring 2003 Lecture Notes

ENGRD/TAM 202 JAN 20, 2003 ①

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see www page (google: ruina)

This Course

STATICS: Force & Moment Balance

key skills: Vectors
Free body diagrams
Solving vector equations

STRENGTH of SOLIDS

How long narrow things stretch, twist & bend.

Concepts: strain & stress

VECTORS: Things represented w/ arrows.

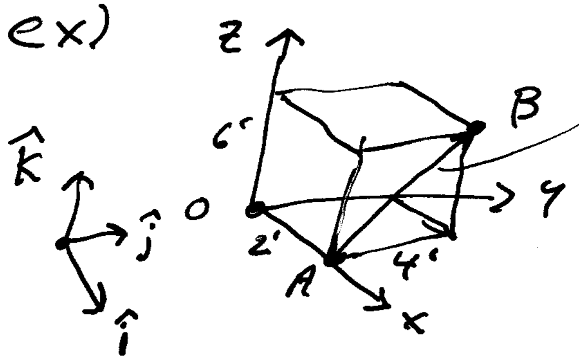
2 basic vectors: 1) relative position $\underline{r}_{A/C}$
2) Force $\underline{F}_D$

All other vectors are derived from these 2.

ADVERTISEMENT:
Read about Vectors, Free Body Diagrams & Statics in on-line book by Ruina & Pratap.

e.g. unit vectors (typically $\underline{r}_D / \underbrace{|\underline{r}_D|}_{\text{magnitude of } \underline{r}_D}$) & moment vectors

ex)



unit vector in dir AB

$= \hat{\lambda}_{AB} = \underline{r}_{AB} / |\underline{r}_{AB}|$

$= \dfrac{4\text{ft}\,\hat{j} + 6\text{ft}\,\hat{k}}{\sqrt{(4\text{ft})^2 + (6\text{ft})^2}}$

$= \dfrac{4}{\sqrt{52}}\hat{j} + \dfrac{6}{\sqrt{52}}\hat{k}$

check: $\underbrace{|\hat{\lambda}_{AB}| = \sqrt{\left(\frac{4}{\sqrt{52}}\right)^2 + \left(\frac{6}{\sqrt{52}}\right)^2} = 1}_{\text{unit vector has magnitude of } 1}$

$\underline{r}_{B/A} \overset{*}{=} \underline{r}_{AB}$

$= \underbrace{4\text{ft}}_{r_x = r_{ABx}}\,\hat{j} + \underbrace{6\text{ft}}_{r_y = r_{ABy}}\,\hat{k}$

$= \underbrace{|\underline{r}_{AB}|}_{\text{mag. of } \underline{r}_{AB}}\ \underbrace{\hat{\lambda}_{AB}}_{\text{unit vector in dir AB}}$

$= \sqrt{52} \cdot \underbrace{\dfrac{4\hat{j} + 6\hat{k}}{\sqrt{52}}}_{\text{ft.}}$

* "$\underline{r}_{B/A}$" reads as "r B with respect to A"

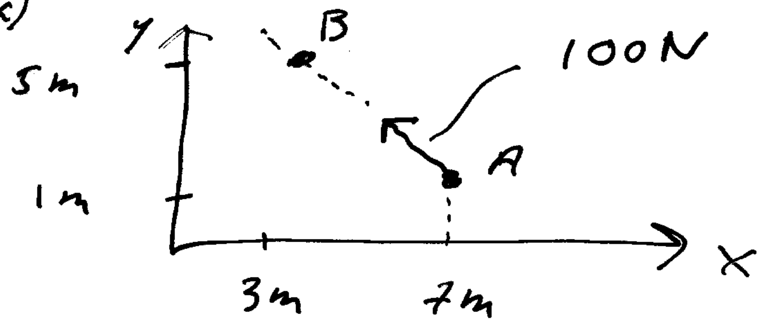
"$\underline{r}_{AB}$" reads as "r A to B"

They mean the same thing $\underline{r}_{B/A} = \underline{r}_{AB}$

But $\underline{r}_{B/A} = -\underline{r}_{A/B}$

$\underline{r}_{AB} = -\underline{r}_{BA}$

ex)



$$\underline{F} = F\,\hat{\lambda} = (100N)\left(\frac{\underline{r}_{AB}}{|\underline{r}_{AB}|}\right) = 100N\,\frac{-\hat{i}+\hat{j}}{\sqrt{2}}$$

$$\underline{F} = F_x\hat{i} + F_y\hat{j} = \underbrace{\frac{-100N}{\sqrt{2}}}_{F_x}\hat{i} + \underbrace{\frac{100N}{\sqrt{2}}}_{F_y}\hat{j}$$

x & y components of $\underline{F}$

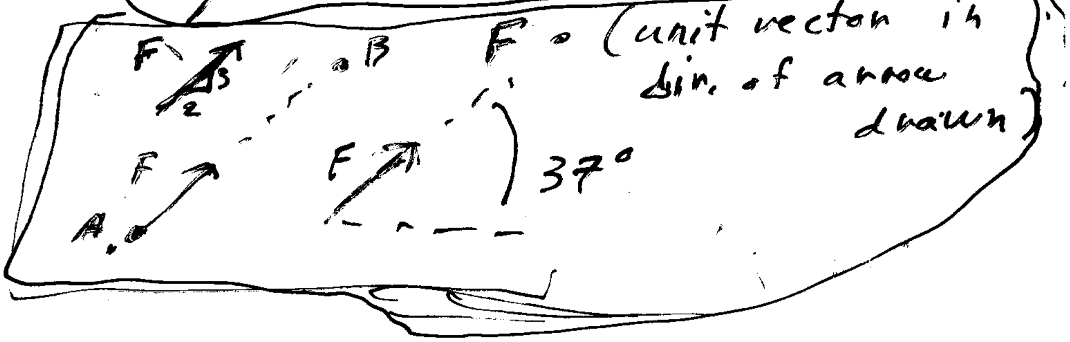
Vector Notation

In equations: $\underline{F}$, $\underline{\underline{F}}$, $\vec{F}$, $\mathbf{F}$, $\mathbb{F}$, (F)

unitvectors: $\hat{i}$, $\hat{j}$, $\underline{i}$, $\underline{j}$ $\hat{\lambda}$, $\underline{\lambda}$

In pictures: $\underline{F}$ ↗ ignore arrow

dir. needs to be clear from picture.

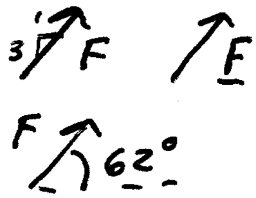
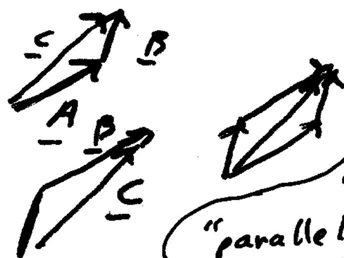
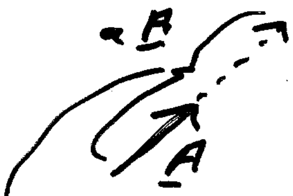


F = (unit vector in dir. of arrow drawn)

by slope, angle or dotted line between pts. w/ known locations

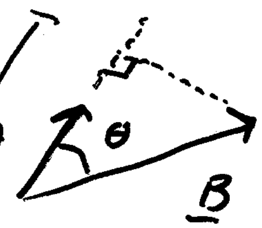
* If you come early please sit towards middle of rows. (Less climbing at start of class)

TODAY: Vectors, Vectors, Vectors — JAN 22 2003

	Picture	abstract/eqn.	Comps
Vector		$\underline{F}$, $\vec{F}$, $\mathbf{F}$, $\underset{\sim}{F}$	$F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$
Addition		$\underline{A} + \underline{B} = \underline{C}$	$C_x = A_x + B_x$ $C_y = A_y + B_y$ $C_z = A_z + B_z$
Multi. by a scalar.	 a times longer than $\underline{A}$	$\underline{C} = a\underline{A}$	$C_x = a A_x$ $C_y = a A_y$ $C_z = a A_z$

"triangle" rule, "parallelogram" rule, "tip to tail" rule

dot product/ inner product/ scalar product



$AB\cos\theta$

$|\underline{B}|$

$|\underline{A}|$

$= A\,(B\cos\theta)$

$= A\cdot$(proj. of B in A dir.)

$= B\cdot$(proj. A in B dir)

$C = \underline{A}\cdot\underline{B}$

a scalar

$C = A_xB_x + A_yB_y + A_zB_z$

$\underline{A}\cdot(\underline{B}+\underline{D})$

$= \underline{A}\cdot\underline{B} + \underline{A}\cdot\underline{D}$

2D: $C = A_xB_x + A_yB_y$

$\hat{i}\cdot\hat{i} = 1$ etc

$\hat{i}\cdot\hat{j} = 0$

ex) $\underline{A} = 2\hat{i} + 4\hat{j}$

$\underline{B} = \hat{i} - 3\hat{j}$

$\underline{A}\cdot\underline{B} = 2\cdot 1 + 4\cdot(-3) = -10$

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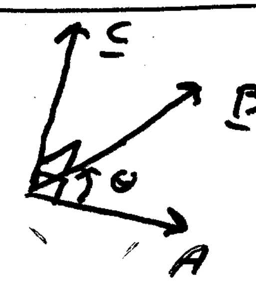
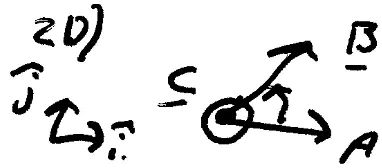
cross product/ vector product

3D)



$|\underline{C}| = AB\sin\theta$

2D)



$\underline{C} = AB\sin\theta\,\hat{k}$

$\underline{C} = \underline{A}\times\underline{B}$

$\hat{i}\times\hat{j} = \hat{k}$ etc

$\hat{i}\times\hat{i} = \underline{0}$

$\underline{A}\times(\underline{B}+\underline{D})$

$= \underline{A}\times\underline{B} + \underline{A}\times\underline{D}$

$\underline{C} = C_x\hat{i} + C_y\hat{j} + C_z\hat{k}$

$C_x = A_yB_z - A_zB_y$

$C_y = -A_xB_z + A_zB_x$

$C_z = A_xB_y - A_yB_x$

ex) $\underline{A} = 3\hat{i} + 4\hat{j} + 6\hat{k}$

$\underline{B} = \hat{i} - 2\hat{j} + 9\hat{k}$

$$\underline{A} \times \underline{B} = (3\hat{i} + 4\hat{j} + 6\hat{k}) \times (\hat{i} - 2\hat{j} + 9\hat{k})$$

$$= (\qquad\qquad)\,\hat{i}$$

$$+ (\qquad\qquad)\,\hat{j}$$

$$+ (3 \cdot -2 - 4 \cdot 1)\,\hat{k}$$

<u>RULES of Vector algebra</u>

All distributive, associative, commutative rules that you know apply to

$\underline{A} + \underline{B}$, $a\underline{A}$, $\underline{B} \cdot \underline{A}$, $\underline{B} \times \underline{A}$ <u>except</u>

$\underline{A} \times \underline{B} = -\underline{B} \times \underline{A}$.

{WATCH OUT: $\frac{\sim}{\underline{A}}$ is nonsense}

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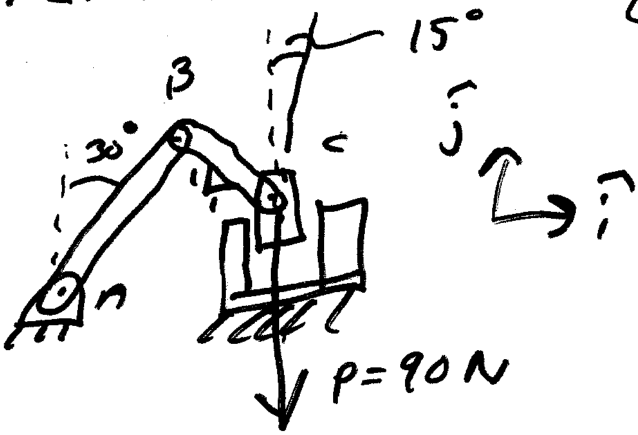
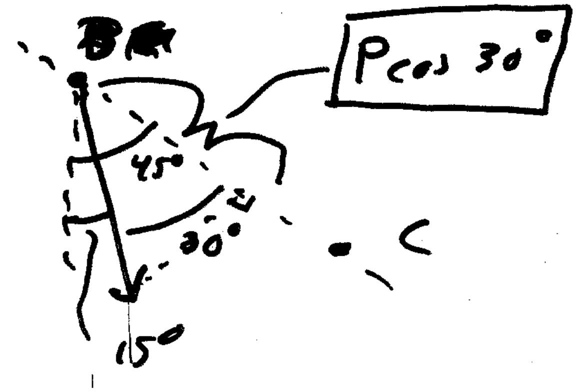
ex) 2.28 from book

Comp. of $\underline{P}$ along AB (1)





Alt. Approach:

$$\left\{\begin{array}{l}\text{Comp. of } \underline{P} \text{ in} \\ \text{dir. BC}\end{array}\right\} = \underline{P} \cdot \hat{\lambda}_{BC}$$

$$= P\left(\cos 15^\circ(-\hat{j}) + \sin 15^\circ \hat{i}\right) \cdot \left(\frac{\hat{i} - \hat{j}}{\sqrt{2}}\right)$$

$$= P\Big(\underbrace{\frac{\sin 15^\circ}{\sqrt{2}} + \frac{\cos 15^\circ}{\sqrt{2}}}_{\cos 30^\circ}\Big)$$

$$= P \cos 30^\circ$$

Trig aside: $\cos 30^\circ = \cos(45^\circ - 15^\circ)$

$$= \cos 45^\circ \cos 15^\circ + \sin 30^\circ \sin 15^\circ$$

$$= \frac{\cos 15^\circ}{\sqrt{2}} + \frac{\sin 15^\circ}{\sqrt{2}}$$

That is: with vectors we effectively derived the trig identity for $\cos(\theta - \phi)$!

Moment of a Force (w.r.t. some pt.)

$$\underline{M}_{/c} = \underline{r}_{A/c} \times \underline{F}$$

$$= \underline{r}_{cA} \times \underline{F}$$

vector

$$|\underline{M}_{/c}| = d\,F$$

Note: moment of a force is dependent on ref. pt.

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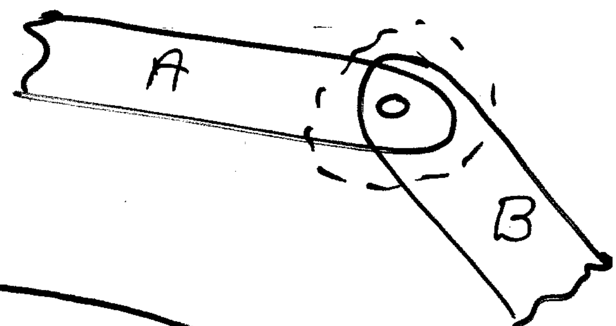
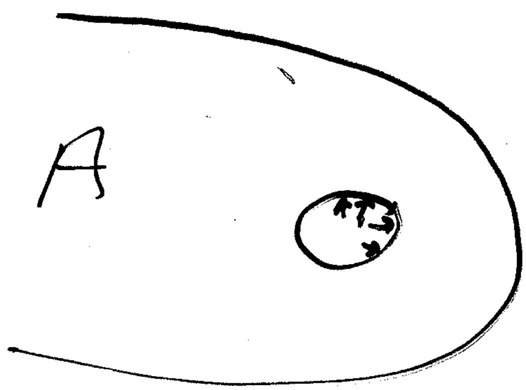
ex)





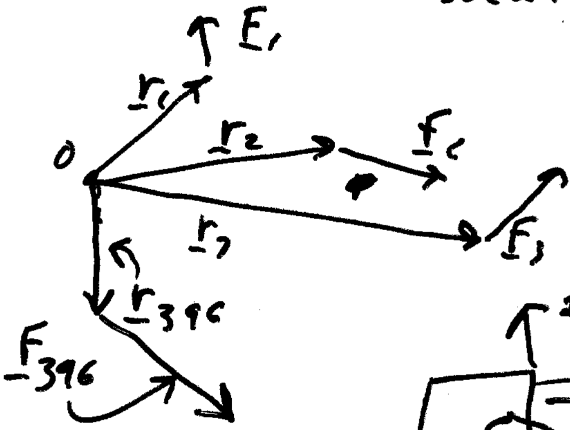
Little arrows show contact forces of B on A.
Usually we don't want to know such detail but instead want the net effect.

SO . . . . . (cont'd next lecture)

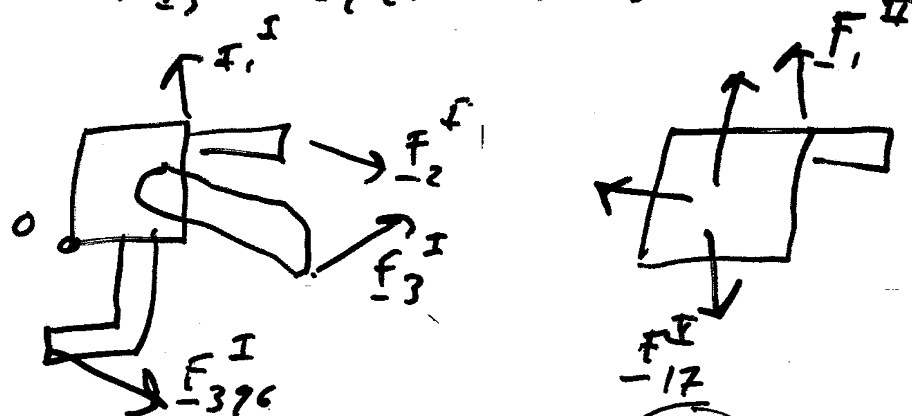
TODAY: Equiv. force systems, couples

(Please read Ruina & Pratap §2.6 online) JAN 24. 2003

Force System: A collection of forces $\underline{F}_i$ & locations $\underline{r}_i = \underline{r}_{i/o} = \underline{r}_{oi}$



Question: When are 2 force systems I & II "equivalent"?



I II

I & II are <u>equivalent</u> iff $\sum \underline{F}_i^I = \sum \underline{F}_i^{II}$

C is any one point.

and $\sum \underline{r}_{i/c} \times \underline{F}_i^I = \sum \underline{r}_{i/c} \times \underline{F}_i^{II}$

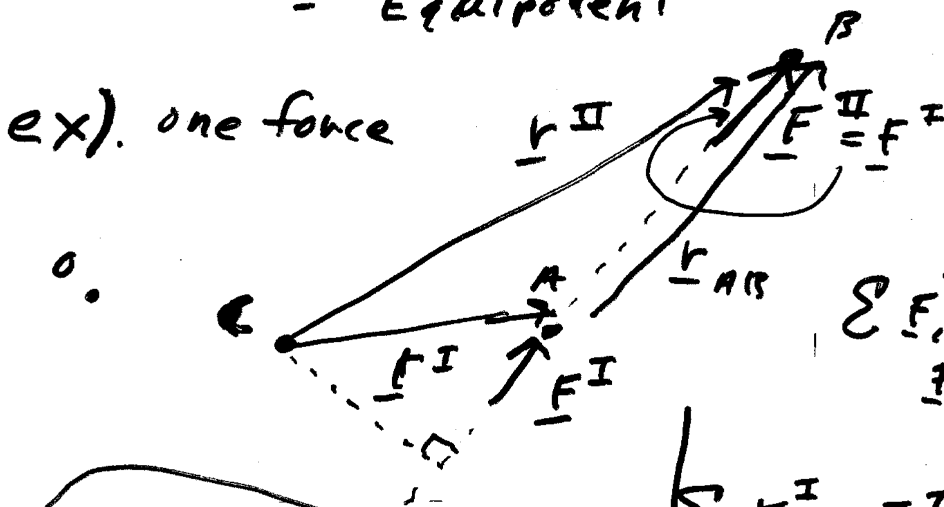
Forces & positions in syst I

Forces & positions in system 2.

Why use this def? Because all that the laws of mechanics know about a force system is the $\sum \underline{F}_i$ & $\sum \underline{r}_{i/o} \times \underline{F}_i$.

"Equivalent" = "statically equiv." = "mechanically equiv."
= "Equipollent"

ex). one force



$\underline{F}_1^{I}$ at $\underline{r}_1^{I}$ is equiv. $\underline{F}_1^{II}$ at $\underline{r}_1^{II}$

$$\sum \underline{F}_i^{I} \overset{?}{=} \sum \underline{F}_i^{II}$$
$$\underline{F}^{I} = \underline{F}^{II}$$

$$\sum \underline{r}_i^{I} \times \underline{F}_i^{I} \overset{?}{=} \sum \underline{r}_i^{II} \times \underline{F}_i^{II}$$

$$\underline{r}^{I} \times \underline{F}^{I} \overset{?}{=} \underline{r}^{II} \times \underline{F}^{I}$$

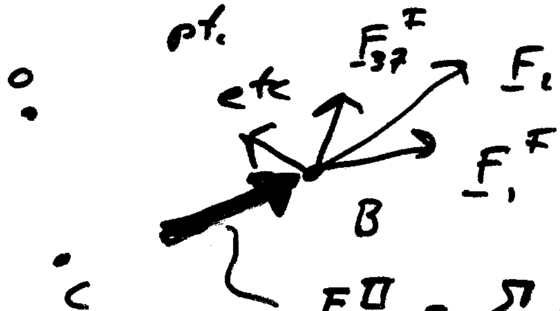
$\underline{r}_{II} = \underline{r}_1 + \underline{r}_{AB}$

$$\underline{r}^{I} \times \underline{F}^{I} \overset{?}{=} (\underline{r}_{I} + \underline{r}_{AB}) \times \underline{F}_1^{I}$$
$$= \underline{r}_{I} \times \underline{F}^{I} + \underbrace{\underline{r}_{AB} \times \underline{F}^{I}}_{0}$$
$$\checkmark = \underline{r}_{I}^{I} \times \underline{F}^{I}$$

$\Rightarrow$ A force is equiv. to same force slid to a new location on its line of action.

$\boxed{\text{Key Fact}}$

ex) A collection of forces acting at one pt.



$F^{II} = \sum F_i^{I}$

is equiv to $\boxed{\text{A single force at that pt.} = \text{sum.}}$

$$\sum F_i^{I} \stackrel{?}{=} \sum F_i^{II}$$

$$\sum F_i^{I} = F^{II}$$

$$\sum M^{I}_{/c} \stackrel{?}{=} \sum M^{II}_{/c}$$

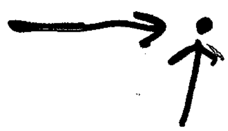
$$\sum r^{I}_{i/c} \times F_i^{I} \stackrel{?}{=} \sum r^{II}_{i/c} \times F^{II}$$

$\boxed{\text{Key Fact}}$

$$r_{B/c} \times \underbrace{\sum F_i^{I}} \stackrel{?}{=} r_{B/c} \times \underbrace{F^{II}}$$

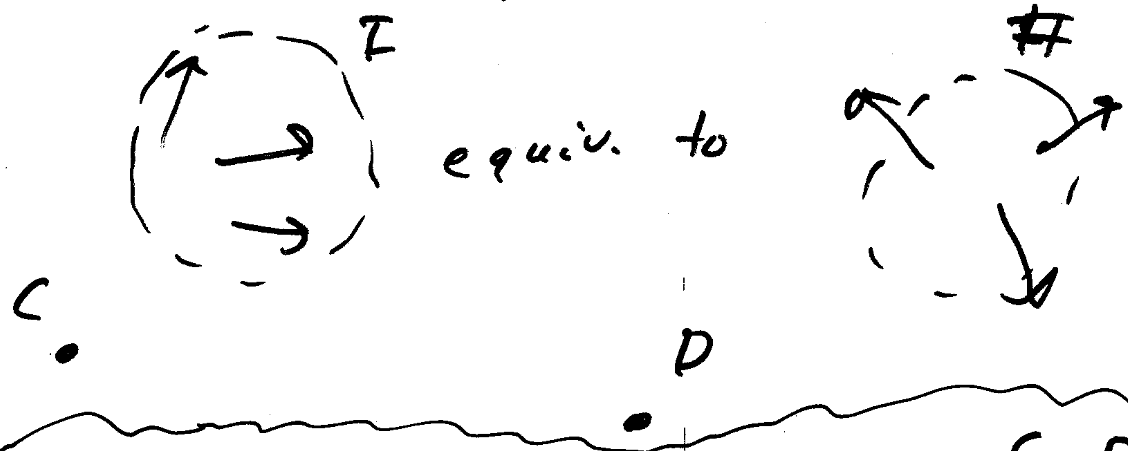
$$\stackrel{\checkmark}{=}$$

subex)



equiv to

<u>FACT</u>: If Syst I is equiv. to Syst II for Ref. pt. C then it is necessarily equiv. for all pts, in the Universe.

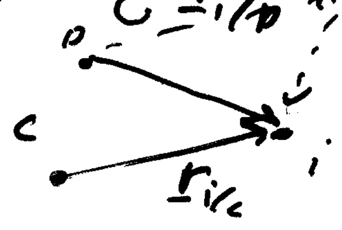


if equiv. for C necessarily equiv. for D

⇒ You only need to check equivalence for one ref. pt. & then equiv. for all ref. pts is assured.

<u>Check</u>: $\sum \underline{F}_i^{I} = \sum \underline{F}_i^{II}$ ind. of ref. pt.

$$\sum \underline{r}_{i/D} \times \underline{F}_i^{I} \stackrel{?}{=} \sum \underline{r}_{i/D} \times \underline{F}_i^{II}$$



$$\underline{r}_{i/D} = \underline{r}_{i/c} + \underline{r}_{c/D}$$

$$\sum \underline{r}_{i/c} \times \underline{F}_i^{I} + \underline{r}_{c/D} \times \sum \underline{F}_i^{I} \stackrel{?}{=} \sum \underline{r}_{i/c} \times \underline{F}_i^{I} + \underline{r}_{c/D} \times \sum \underline{F}_i^{I}$$

= ✓

=

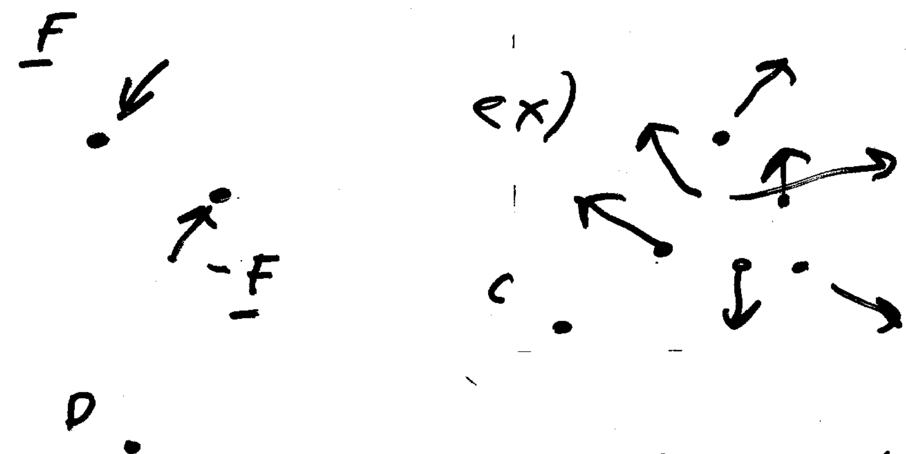
FACT: If a force system has
$\sum \underline{F}_i = \underline{0}$ then

$$\sum \underline{M}_{/C} = \sum \underline{M}_{/D}$$

for all pairs of pts. C & D

ex)

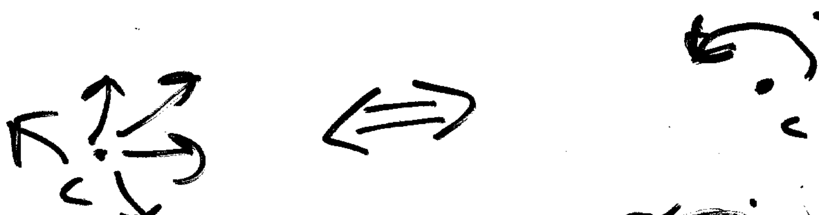
ex)



DEF: A couple is any force distr. that adds to zero.

$\underline{C} = C\hat{k}$

ex)



$$\underline{C} = \sum \underline{r}_{i/D} \times \underline{F}_i$$

FACT: Any force syst. is equiv to a $\underline{F}$ and a $\underline{C}$ at D

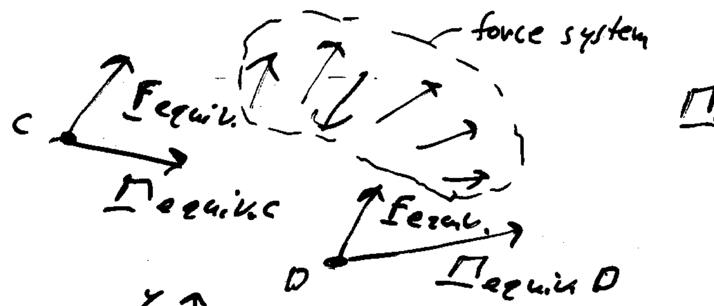
force — couple

TODAY: Couples etc. Cont'd. JAN 27, 2003 (14)

Recall: Any force system is equiv. to a force and a couple: Force acts at C. $\underline{F}_{equiv} = \sum \underline{F}_i$

You pick the pt. C. $\underline{M}_{equiv} = \sum \underline{r}_{i/c} \times \underline{F}_i$

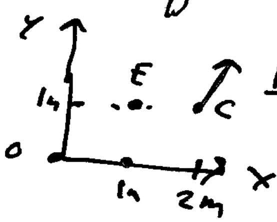
Change the point & you change $\underline{M}_{equiv}$, but not $\underline{F}_{equiv}$.



2D: $\underline{M}_{equiv} = M_{equiv}\hat{k}$

$\underline{M}_{equiv D} = \underline{M}_{equiv C} + \underline{r}_{C/D} \times \underline{F}_{equiv}$

ex)



$\underline{F} = (3\hat{i} + 4\hat{j})$ N can be replaced by a force-couple syst. at origin that is "equivalent".

FACT: in 2D: Every force system is equiv. to either

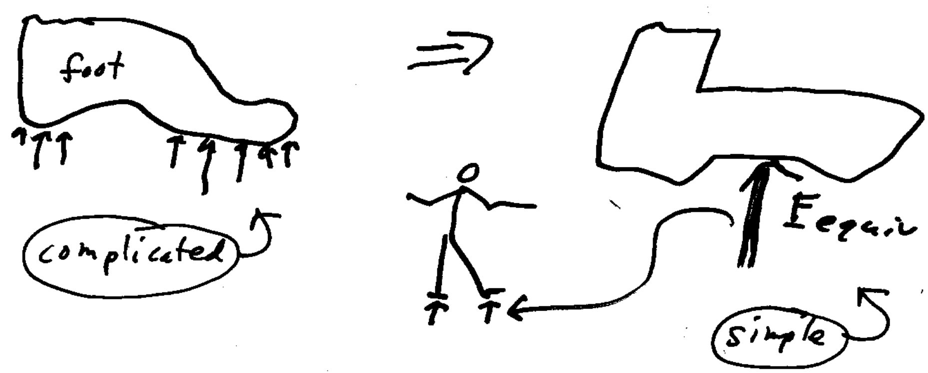
a) just a force at an appropriate location (& that whole line of action) when $\sum \underline{F}_i \neq \underline{0}$

or b) just a couple applied any place when $\sum \underline{F}_i = \underline{0}$

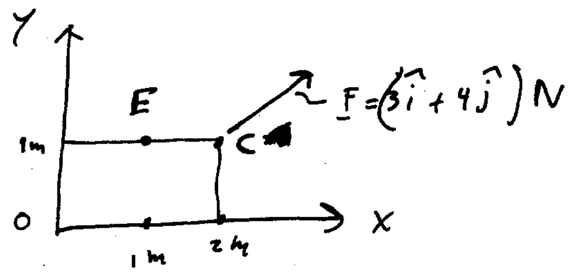
in 3D: Cannot generally reduce a force system to just a force. [can reduce to a "wrench"]

see books

We like this because:



ex)



What is equiv. Force–Couple system at O?

$\underline{F}_{equiv\,O}^{(at)} = \underline{F} = (3\hat{i}+4\hat{j})\,N$ (16b)

$\underline{M}_{equiv\,O}^{(at)} = \sum \underline{M}_{i/O} = \sum \underline{r}_{i/O}\times\underline{F}_i = (2m\,\hat{i}+1m\,\hat{j})\times(3\hat{i}+4\hat{j})\,N$

$= |\underline{r}_{C/O}|\,|\underline{F}|\sin\theta$ (too hard to use in this example)

$= (2m\cdot 3N)\cdot\underline{0} + (2m)\cdot(4N)\,\hat{k} + (1m\cdot 3N)(-\hat{k})$

$+ (1m)(4N)\,\underline{0}$

$\boxed{\underline{M}_{equiv\,O} = 5\,mN\,\hat{k}}$

What is $\underline{M}_{equiv\,E}^{(at)}$?

Use new force-couple system

$\underline{M}_{equiv\,E} = \underline{M}_{equiv\,O} + \underline{r}_{O/E}\times\underline{F}_{equiv}$

$= 5mN\,\hat{k} + [(-\hat{i}-\hat{j})\,m]\times(3\hat{i}+4\hat{j})\,N$

$= [5-4+3]\,mN\,\hat{k}$

$= 4\,mN\,\hat{k}$

Given $\underline{F}_{equiv}$ & $\underline{M}_{equiv}$ is there a place where $\underline{M}_{equiv} = 0$?

Try pt. C:

$\underline{M}_{equiv\,C} = \underline{M}_{equiv\,E} + \underline{r}_{E/C}\times\underline{F}_{equiv}$

$= 4Nm\,\hat{k} + -1m\,\hat{i}\times(3\hat{i}+4\hat{j})\,N$

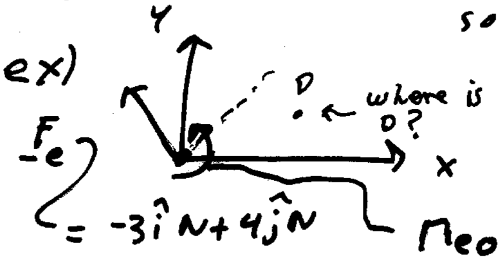
$= [4-4]\,Nm\,\hat{k}$

$= \underline{0}$

{as we knew had to work.}

How to find locations where force system is equiv. to just a force (w/ no couple). Find pt. D so that (17)

ex)



$\underline{0} = \underline{M}_{equiv.} = \underline{M}_{eO} + \underline{r}_{O/D} \times \underline{F}_e$

$= 6Nm\hat{k} + \underline{r}_{O/D} \times (-3N\hat{i} + 4N\hat{j})$

(1 eqn. in 2 unknowns)

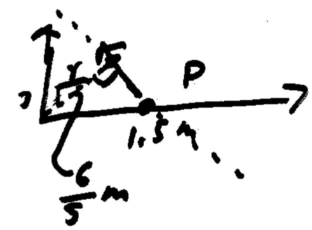
Try to find one point on line of solns.

$\underline{r}_D = x_D\,\hat{i} + 0\,\hat{j}$

$\underline{0} = \underline{M}_{equiv\,D} = 6Nm\hat{k} + -x_D\,\hat{i} \times (-3N\hat{i} + 4N\hat{j})$

$\{\underline{0} = [6\,Nm - x_D \cdot 4N]\,\hat{k}\} \Rightarrow \{\,\}\cdot\hat{k} \Rightarrow x_D = \frac{6Nm}{4N} = 1.5m$

$\Rightarrow$



$\boxed{\text{Ans. one good pt. D is on x axis 1.5 m from origin.}}$

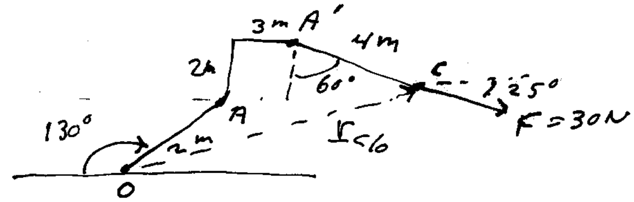
~~TODAY: a) 2 warnings b) FBDs & statics~~ JAN 29, 2003

TODAY: a) 2 warnings b) FBDs & statics

Warning 1: In life, engineering & HW problems geometry what you need is often **not** handed to you on a silver platter.

$\Rightarrow$ Don't let your geometry / trig issues get tangled with your mechanics issues.

ex)



$\underline{M}_{/O} = \underline{r}_{C/O} \times \underline{F}$, $\underline{F} = F\hat{\lambda}$

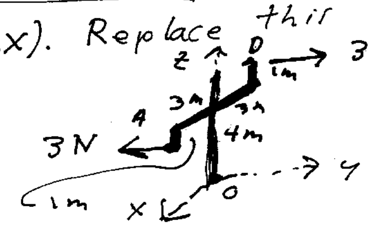
$\underline{r}_{C/O} = \underline{r}_{A/O} + \underline{r}_{A'/A} + \underline{r}_{C/A'}$

$\underline{r}_{A/O} = 2m(\cos 50^\circ\,\hat{\imath} + \sin 50^\circ\,\hat{\jmath})$, $\underline{r}_{A'/A} = (3m\,\hat{\imath} + 2m\,\hat{\jmath})$

$\underline{r}_{C/A'} = 4m(-\cos 60^\circ\,\hat{\jmath} + \sin 60^\circ\,\hat{\imath})$, $\hat{\lambda} = \cos 25^\circ\,\hat{\imath} - \sin 25^\circ\,\hat{\jmath}$

Warning 2: Moment about a pt. in 3D is a 3D vector

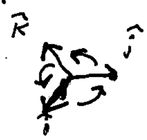
ex). Replace this force system w/ a force & a couple



$\sum \underline{F}_i = \underline{0} \Rightarrow$ equiv force is $\underline{0}$

$\sum \underline{M}_{/O} = \sum \underline{M}_{/A} = \underline{r}_{D/A} \times 3N\,\hat{\jmath}$

system is a couple; $\underline{r}_{D/A} = -6m\,\hat{\imath} + 2m\,\hat{k}$



$\underline{M} = (-6m\,\hat{\imath} + 2m\,\hat{k}) \times 3N\,\hat{\jmath} = \boxed{-18Nm\,\hat{k} + 6Nm\,\hat{\imath}}$

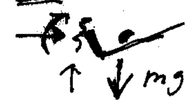
The equiv. couple

FREE BODY DIAGRAMS (FBDs) read Chapter 4 in Ruina/Pratap

(19)

A picture of a collection of material w/ all external forces & couples showing.

FBD of arm



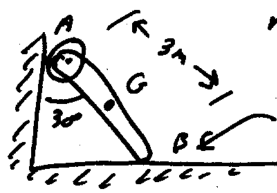
"$\mathcal{I}$ chainsaw or scalpel"

Laws of Mechanics : Apply to any FBD of any system in static equilibrium: $\sum_{\text{external}} \underline{F}_i = \underline{0}$, $\sum_{\text{external}} \underline{M}_{/C} = \underline{0}$

For every pt. C

Generally: 2D ⇒ 3 indep. scalar eqs., 3D) ⇒ 6 ind. scalar eqs

ex)

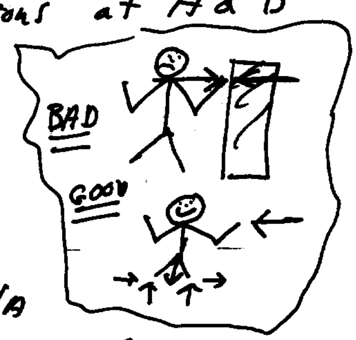


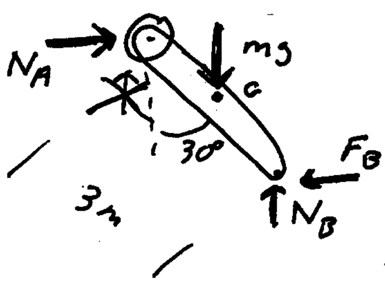
$m = 10\,kg$

$\downarrow g = 10\,N/kg$

assume no slip at B

Reactions at A & B





2D ⇒ 3 eqs.

3 unknowns: N_B, F_B, N_A

$\underline{0} = \sum \underline{M}_{/B} = \underline{r}_{G/B} \times (-mg\,\hat{j})$

$+ \underline{r}_{A/B} \times N_A\,\hat{i}$

$\{\underline{0} = \frac{3}{4} m \cdot 100N\,\hat{k} + \frac{3\sqrt{3}}{2} m N_A (-\hat{k})\}$

(20)

$\{\ \} \cdot \hat{k} \Rightarrow \quad 0 = 75\,Nm - \frac{3\sqrt{3}}{2} N_A\,m$

$$\boxed{N_A = \frac{50}{\sqrt{3}}\,N}$$

N_A = force comp.

N = 1 Newton.

(cont'd next lecture)

TODAY: 1) FBDs & Statics (cont'd), 2) Q&A, 3) Quiz 0 practice & H.W. (1/31/0?) (21)

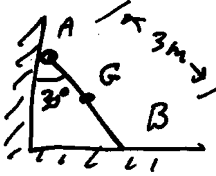
FBD: Pic. of system & all forces on it.

Mechanics: $\sum \underline{F}_i = \underline{0}$, $\sum \underline{M}_{/c} = \underline{0}$

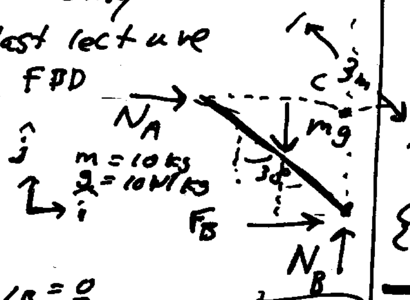
sum over all forces/couples on FBD and no others.

Note: "Chainsaw", ⇒ External forces only

ex/ Cont'd from last lecture



⇒



Last class: $\sum \underline{M}_{/B} = \underline{0}$

$\quad$ ↳ $(\quad)\hat{k} = \underline{0} \Rightarrow N_A = \frac{2\sqrt{3}}{3}\,100N$

Going on...

Method 1: $\{\sum \underline{F}_i = \underline{0}\}$

$\{\ \}\cdot\hat{i}$, $\{\ \}\cdot\hat{j} \Rightarrow$ 2 eqs for F_B & N_B

$\{N_A\hat{i} + N_B\hat{j} + F_B\hat{i} - mg\hat{j} = \underline{0}\}$

$\{\ \}\cdot\hat{i} \Rightarrow N_A + F_B = 0$

$\{\ \}\cdot\hat{j} \Rightarrow N_B - mg = 0$ $\Big\}$ 2 eqs for F_B & N_B

we know N_A already

Method 2: Can we find F_B w/out having found N_A first?

$\sum \underline{M}_{/C} = \underline{0}$ ("kills" N_B & N_A)

$\Rightarrow \{(\frac{3}{4}m)\,mg\,\hat{k} + (3m\frac{\sqrt{3}}{2})F_B\,\hat{k} = \underline{0}\}$

$\{\ \}\cdot\hat{k} \Rightarrow F_B = \frac{-3/4}{3\sqrt{3}/2}\,100N$

Q&A

QUIZ 0

Feb 3, 2003 | TODAY: 3D Statics ①

Recall FBD = picture of isolated system and all external forces & couples that act on it.

Laws of Mechanics (statics)

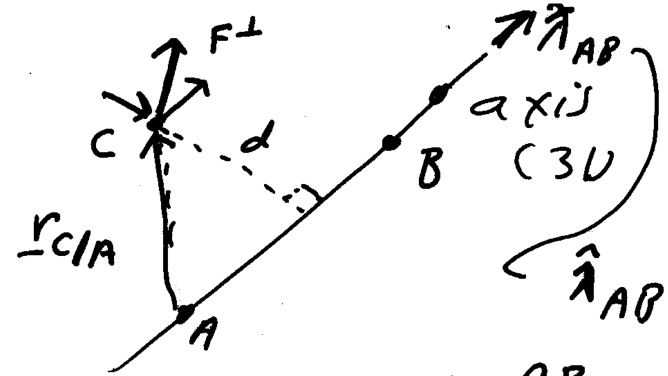
For all forces/couples on a FBD

$$\underbrace{\sum \underline{F}_i = \underline{0} \quad \text{and} \quad \sum \underline{M}_{/C} = \underline{0}}_{\text{generally 6 ind. scalar eqs in 3D}}$$

(C: any pt. you like)

Moment about an axis

tendency of forces (& couples) to cause rotation about an axis (given force syst. has different moment about diff. axes)



Moment about axis AB

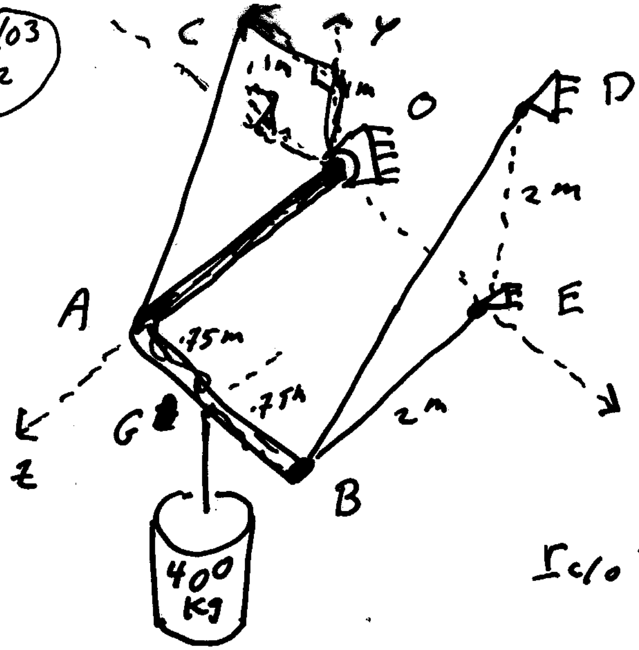
$= \underbrace{dF^\perp}_{\text{hard to evaluate from geometry (often)}}$

$= (\underbrace{\underline{r}_{C/A} \times \underline{F}}_{\underline{M}_{/A}}) \cdot \hat{\lambda}_{AB}$

$= (\underline{r}_{C/B} \times \underline{F}) \cdot \hat{\lambda}_{AB}$

ex) 3.67 from Meriam

2/3/03
pg 2

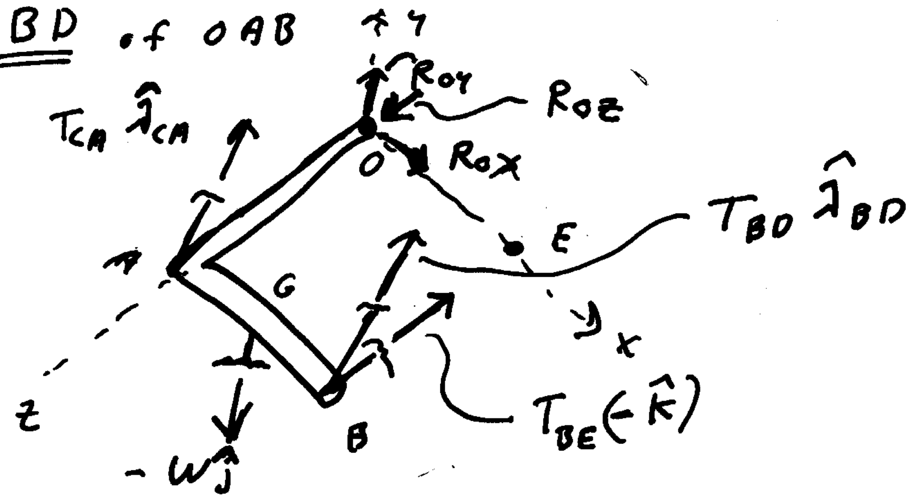


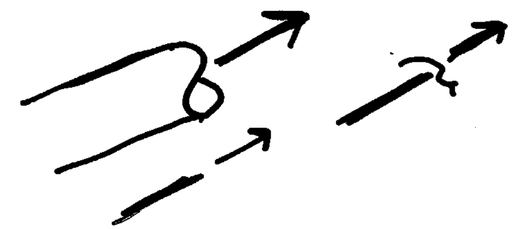
O, C, D, E are co-planar.

Bent bar OAB. wires AC, BD, rod BE. Ball & socket joints at O, A, B, C, D, E, G

$\underline{r}_{C/O} = -1m\,\hat{\imath} + 1m\,\hat{\jmath}$

FBD of OAB





Find unknown forces & reactions: T_{BE}, T_{BD}, T_{CA}, R_{Ox}, R_{Oy}, R_{Oz} (6 unknowns)

$$\underbrace{\sum \underline{F}_i = \underline{0}, \quad \sum \underline{M}_{/H} = \underline{0}}_{6\text{ eqs.}} \quad (1)$$

Method 1: Pick any pt H you like. Write eqs. 1. Break into comps. solve 6 eqs for 6 unknowns.

Method 2 :

(2/3/03 pg 3)

Try to get 1 eq. in 1 unknown.

$\sum M_{axis AB} = 0$

only R_{oy} contributes

$\Rightarrow \boxed{R_{oy} = 0}$

$\sum M_{AXIS\ OB} = 0$

only T_{CA} & W contribute

$$\left(\sum \underline{M}_{/B}\right) \cdot \underline{\lambda}_{OB} = 0$$

$\underline{\lambda}_{OB} = \underline{r}_{OB}/|\underline{r}_{OB}|$

$$\left(\underline{r}_{G/B} \times W(-\hat{j}) + \underline{r}_{A/B} \times T_{CA}\,\hat{\lambda}_{AC}\right) \cdot \hat{\underline{\lambda}}_{OB} = 0$$

$\underline{r}_{G/B} = -\frac{3}{4}\,m\,\hat{i}$, $\underline{r}_{A/B} = -1.5\,m\,\hat{i}$

$$\hat{\lambda}_{AC} = \frac{\underline{r}_{AC}}{|\underline{r}_{AC}|} = \frac{-2\hat{k} - \hat{i} + \hat{j}}{\sqrt{6}}$$

$$\hat{\lambda}_{OB} = (1.5\,m\,\hat{i} + 2\,m\,\hat{k}) / \sqrt{(1.5\,m)^2 + (2\,m)^2}$$

Plug in, crank $\Rightarrow T_{AC} = \ldots$

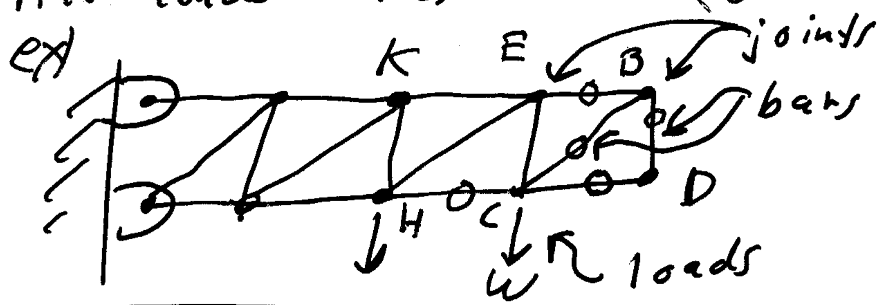
$\sum M_{axis\ OA} \Rightarrow T_{BD} = \ldots\ldots$

Feb. 5, 2007 ①

TRUSSES:

Truss: A collection of "bars" connected at ends by hinges (2D), or ball & socket joints (3D). All loads must be at "joints".

ex



$\boxed{\text{TRUSSES are GOOD.}}$

a) useful

b) agreeable

Primary Goal: Given geometry, loads find $\underbrace{\text{tensions}}_{\text{"bar forces"}}$ in bars, & reactions.

Method:

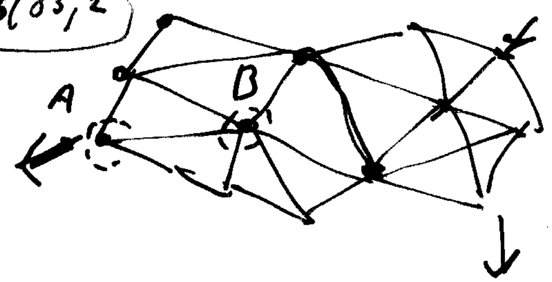
FBDs of a) joints

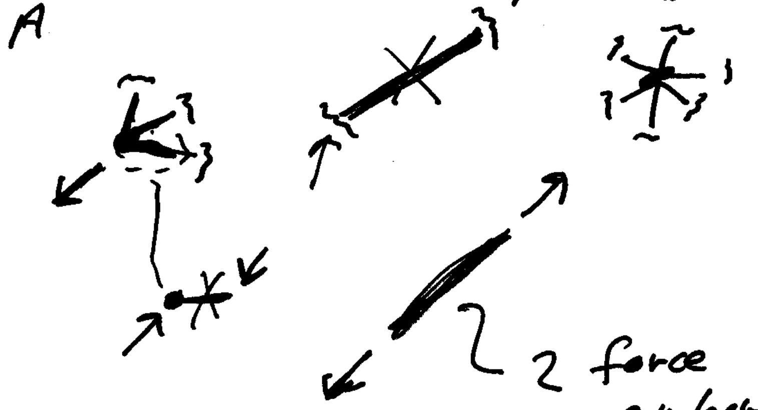
b) part of truss

$\sum \underline{F}_i = \underline{0}$, $\{\sum \underline{M}_{/c} = \underline{0}\}$

Key idea: bars are "2 force members".

E13(03,2

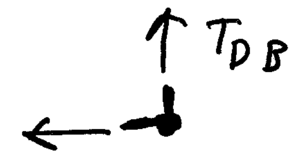




2 force member

If only two forces act on a body in static equil.
$\Rightarrow$ equal in magnitude
opposite in dir.
point along the line connecting the 2 pts
of action.

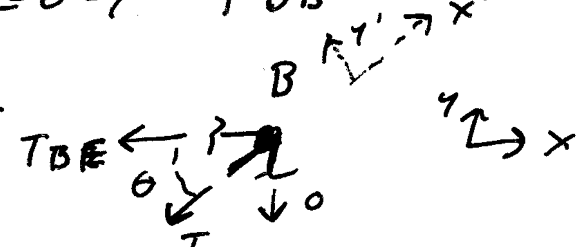
FBD of joint D from ex



$\sum F_x = 0 \Rightarrow T_{DC} = 0$

$\sum F_y = 0 \Rightarrow T_{DB} = 0$

joint B



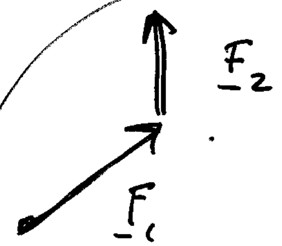
$\sum F_{y'} = 0 \Rightarrow T_{BE} \sin\theta = 0$

$T_{BE} = 0$

$\sum F_y = 0 \Rightarrow T_{BC} \sin\theta = 0$

$T_{BC} = 0$

2/5/03 pg 3



$\underline{F}_1 + \underline{F}_2 = \underline{0}$

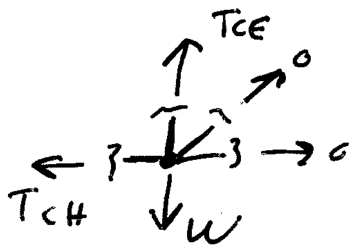
$\underline{F}_1 \not\parallel \underline{F}_2$

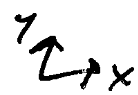
$\Rightarrow \underline{F}_1 = \underline{0}$

$\underline{F}_2 = \underline{0}$

How to get zero-force members.

joint C

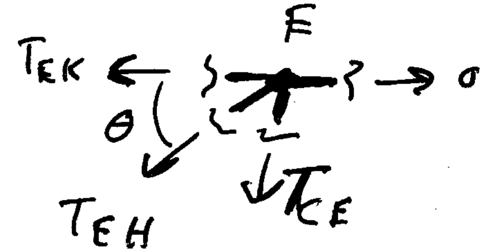




$\sum F_x = 0 \Rightarrow T_{CH} = 0$

$\sum F_y = 0 \Rightarrow T_{CE} = W$

joint E



$\sum \underline{F}_i = \underline{0} \Rightarrow$

$\left.\begin{aligned} \sum F_x = 0 &\Rightarrow -T_{EK} - T_{EH}\cos\theta = 0 \\ \sum F_y = 0 &\Rightarrow -T_{CE} - T_{EH}\sin\theta = 0 \end{aligned}\right] *$

* is 2 eqs. in 2 unknowns

$\cancel{T_{CE} = -T_{EH}\sin\theta}$

$\cancel{\boxed{T_{CE} = -W\sin\theta}}$

$T_{EH} = -T_{CE}/\sin\theta$

$\boxed{T_{EH} = -W/\sin\theta}$

"bar EH is in compression"

Feb. 10

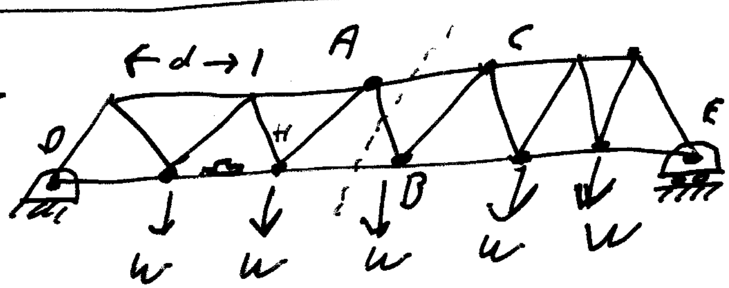
TODAY: Trusses (cont'd)

METHOD (most of time)

1. Draw FBD of whole structure (use to find reactions if possible)
2. Find 0-force bars by inspection.
3. Draw FBDs of joints or sections as appropriate.

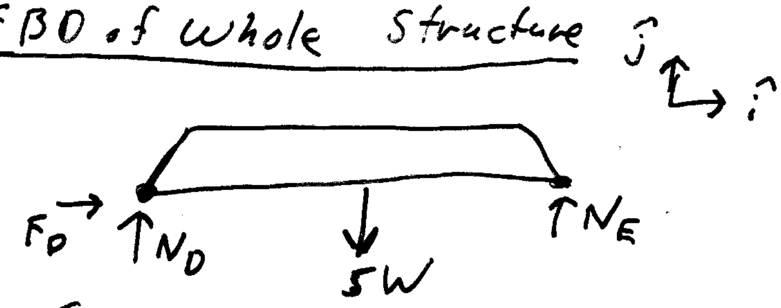
Method of "Sections"

ex/



$T_{AB} = ?$

FBD of whole structure

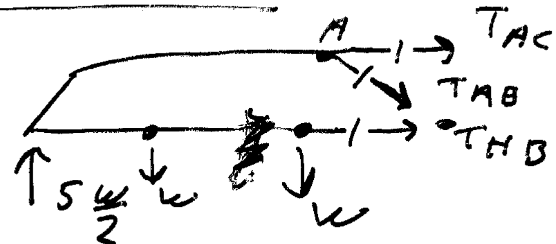


$\{\sum \underline{F}_i = \underline{0}\} \cdot \hat{i} \Rightarrow F_D = 0$

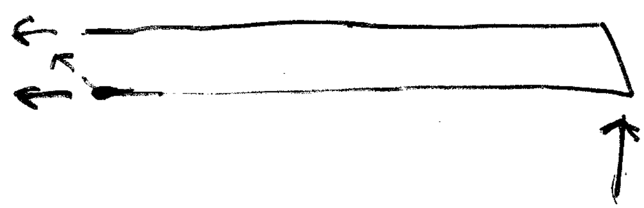
$\sum \underline{M}_{/D} = \underline{0} \Rightarrow N_E = \frac{5w}{2}$

$\sum \underline{M}_{/E} = \underline{0} \Rightarrow N_D = 5w/2$

FBD of section



$\sum F_y = 0 \Rightarrow \boxed{T_{AB}}$



$T_{AC} = ?$

$\Sigma \underline{M}_{/B} = \underline{0}$

$-(T_{AC} \cdot h)\hat{k} + (w \text{ terms}) \cdot (\text{horizontal distances}) = \underline{0}$

$h \ll$ horiz. distances

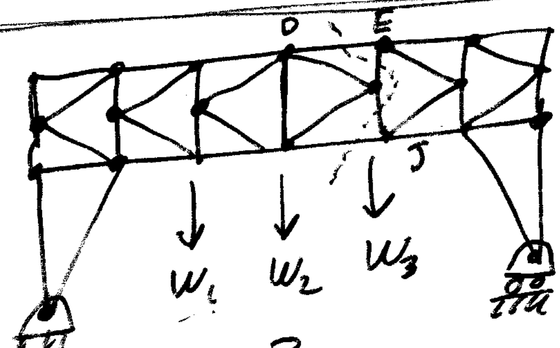
$\Rightarrow |T_{AC}| \gg$ weight

top in compression
bottom in tension

$\overset{\curvearrowleft}{+} \Sigma M_{/B} = 0$ ✓

ex)

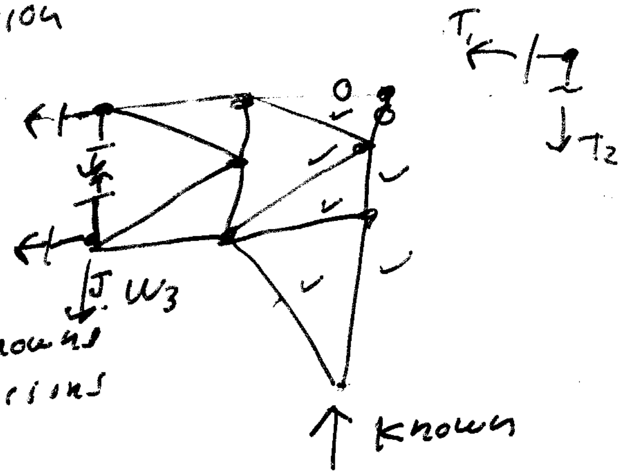
geometry given



$T_{DE} = ?$

FBD of structure ⇒ reactions

section



4 unknowns tensions

$\overset{\curvearrowleft}{+} \Sigma M_{/J} = 0 \Rightarrow T_{DE}$

Sometimes
a) luck
b) no luck
⇒ multiple section cuts.
~~solve~~ set up sets of eqs.

$\left(\Sigma \underline{r}_{i/C} \times \underline{F}_i\right) \cdot \hat{\lambda}_{CD} = 0$

Feb 12, 2003

TODAY: 3D TRUSSES

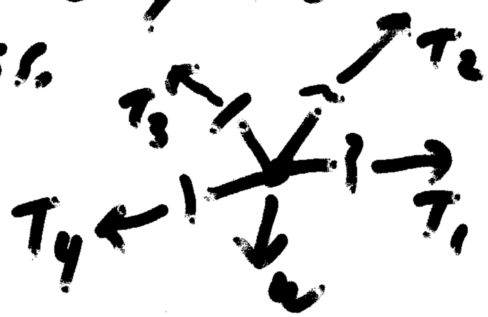
Methods (just like 2D)

* **joints**

Each FBD $\Rightarrow \underbrace{\sum \underline{F}_i = \underline{0}}_{\text{3 scalar eqs.}}$

one joint at a time

work way around truss.



* **Sections**

Each FBD $\Rightarrow$

$\sum \underline{F}_i = \underline{0}$, $\sum \underline{M}_{/D} = \underline{0}$

or

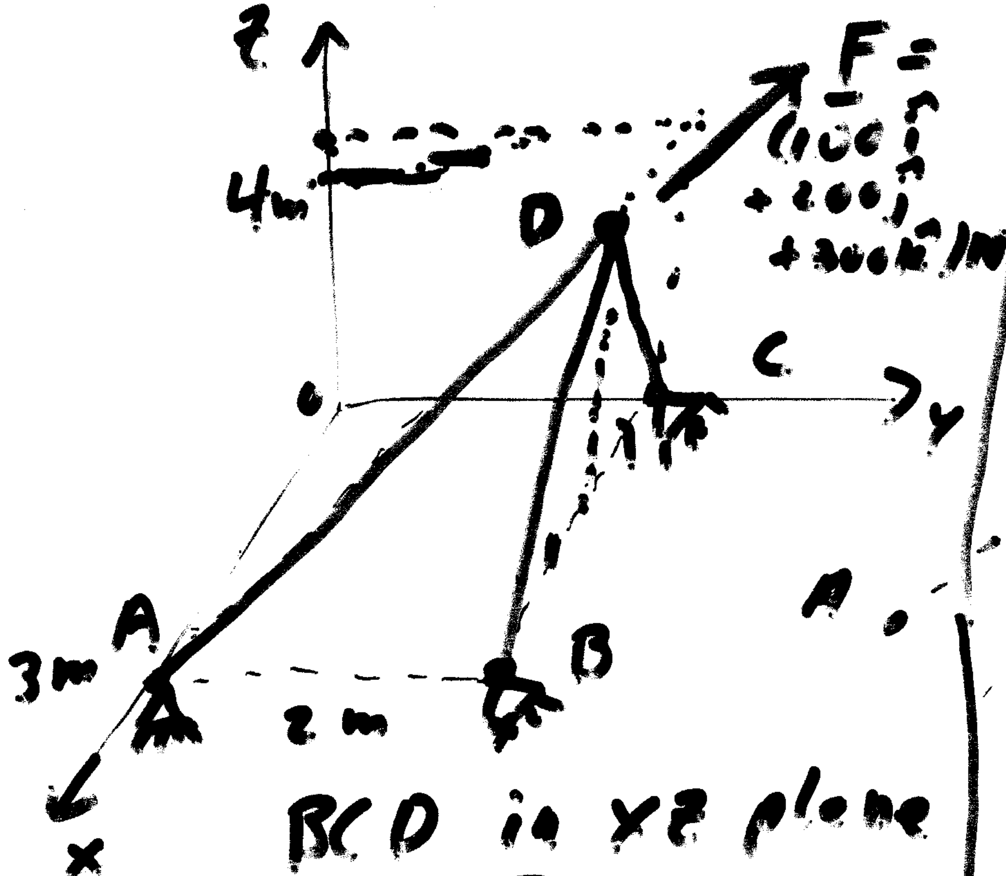
$\sum M$ / 6 diff. axes

or

~~~

6 ind. scalar eqs.
~~~

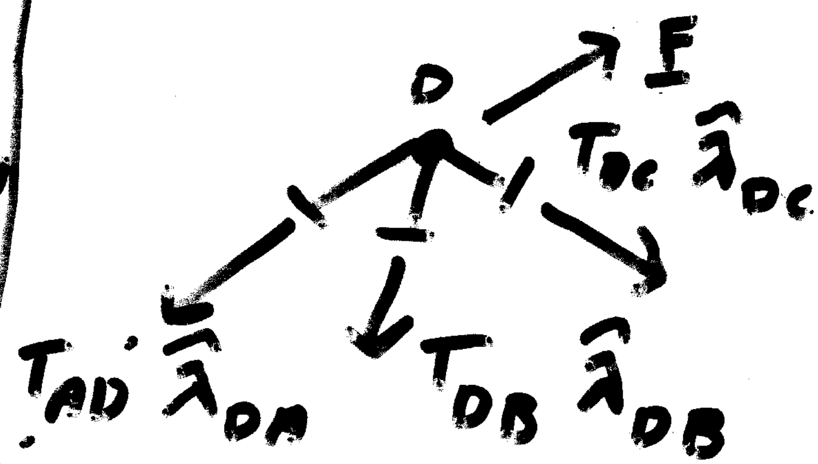
ex) 2/12 pg 2



BCD in xz plane

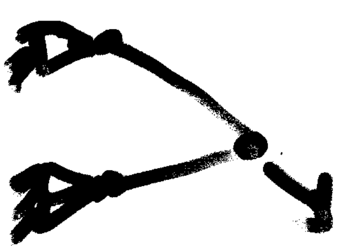
$T_{DB} = ?$

FBD joint D

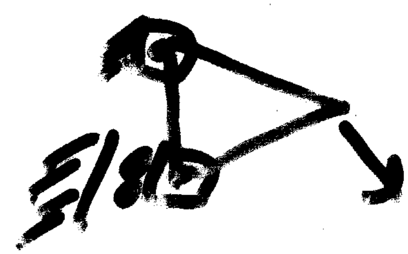


$$\Sigma \underline{F}_i = \underline{0} \qquad (1)$$

$$T_{AD}\,\hat{\lambda}_{DA} + T_{DB}\,\hat{\lambda}_{DB} + T_{DC}\,\hat{\lambda}_{DC} + \underline{F} = \underline{0}$$



or



Side geometry: 2/12 p73

$\hat{\lambda}_{DA} = \dfrac{\underline{r}_{DA}}{|\underline{r}_{DA}|} =$

$= \dfrac{1.5\hat{i} - 2\hat{j} - 4\hat{k}}{\sqrt{1.5^2 + 2^2 + 4^2}}$

$= \lambda_{DAx}\hat{i} + \lambda_{DAy}\hat{j} + \lambda_{DAz}\hat{k}$

$\hat{\lambda}_{DC} = \ldots$

$\hat{\lambda}_{DB} = \ldots$

$\{①\}\cdot\hat{i},\ \{①\}\cdot\hat{j},\ \{①\}\cdot\hat{k}$

3 eqs. in 3 unknowns.

How to solve?

Write in matrix form

$\lambda_{DAx} T_{DA} + \lambda_{DBx} T_{DB} + \lambda_{DCx} T_{CD} = -F_x$

$\lambda_{DAy} T_{DA} + \ldots\ldots$

$\lambda_{DAz} T_{DA} + \ldots\ldots$

$\Rightarrow$

$$\underbrace{\begin{bmatrix} \hat{\lambda}_{DAx} & \hat{\lambda}_{DBx} & \hat{\lambda}_{DCx} \\ \hat{\lambda}_{DAy} & \hat{\lambda}_{DBy} & \hat{\lambda}_{DCy} \\ \hat{\lambda}_{DAz} & \hat{\lambda}_{DBz} & \hat{\lambda}_{DCz} \end{bmatrix} \begin{bmatrix} T_{DA} \\ T_{DB} \\ T_{DC} \end{bmatrix} = \begin{bmatrix} -F_x \\ -F_y \\ -F_z \end{bmatrix}}_{\text{ready for computer soln. / calculator}}$$

$\Rightarrow \quad T_{BD} = \ldots$

MATLAB Soln.

```
rDA = [1.5  -2  -4]';
rDB = [1.5  0  -4]';
rDC = [-1.5  0  -4]';
lamDA = rDA/norm(rDA);
lamDB = rDB/norm(rDB);
lamDC = rDC/norm(rDC);
F = [100  200  300]';
A = [lamDA  lamDB  lamDC];
Tensions =      A\(-F)
```

backslash

done

2/12 pg 6

__Short cut__

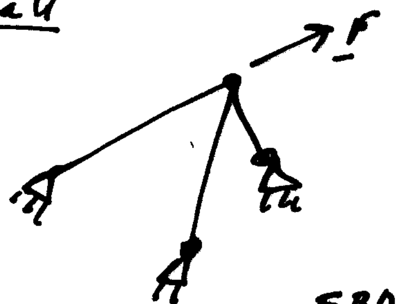
$$\{①\} \cdot (\underline{r}_{DA} \times \underline{r}_{DC} \quad)$$

$\Rightarrow$ 1 eq. for T_{DB}

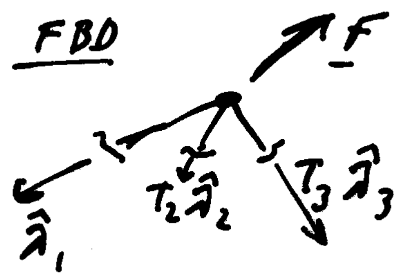
Feb. 17, 2003

TODAY: * 3D trusses (con't, review)
* Machines & Structures
(First of a few lectures)

Recall



$\Rightarrow$ FBD



$$\Sigma \underline{F}_i = \underline{0} \Rightarrow \underset{①}{\{} T_1 \hat{\lambda}_1 + T_2 \hat{\lambda}_2 + T_3 \hat{\lambda}_3 + \underline{F} = \underline{0} \}$$

In matrix form

$$\underbrace{\begin{bmatrix} [\hat{\lambda}_1] & [\hat{\lambda}_2] & [\hat{\lambda}_3] \end{bmatrix}}_{\text{cols. are the comps of unit vectors}} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = [\underline{F}]$$

Solve by brute force (computer) or ...

$$\{①\} \cdot \underbrace{\hat{\lambda}_1 \times \hat{\lambda}_3}$$

is $\perp$ to $\hat{\lambda}_1$

" $\perp$ to $\hat{\lambda}_3$

$$T_2 \, \hat{\lambda}_2 \cdot (\hat{\lambda}_1 \times \hat{\lambda}_3) = \underline{F} \cdot (\hat{\lambda}_1 \times \hat{\lambda}_3)$$

$$\boxed{T_2 = \frac{\underline{F} \cdot \hat{\lambda}_1 \times \hat{\lambda}_3}{\hat{\lambda}_2 \cdot \hat{\lambda}_1 \times \hat{\lambda}_3}}$$

Math Aside

2/17 pg 2

$\underline{A} \cdot \underline{B} \times \underline{C}$ is called mixed triple product

(used also for moment about an axis: $M_{\underline{\lambda}} = \underline{\lambda} \cdot \underline{r} \times \underline{F}$)

Some Facts:

$$\underbrace{\underline{A} \cdot \underline{B} \times \underline{C} = \underline{A} \times \underline{B} \cdot \underline{C}}$$

why? a) mult. out & check

b) Parallelapiped volume

Also: $\underline{A} \cdot \underline{B} \times \underline{C} = \det \begin{bmatrix} [\underline{A}]' \\ [\underline{B}]' \\ [\underline{C}]' \end{bmatrix}$

$= \det \left[[\underline{A}] \,|\, [\underline{B}] \,|\, [\underline{C}] \right]$

$\underline{F} \cdot \hat{\lambda}_1 \times \hat{\lambda}_3 = \underline{F} \times \hat{\underline{\lambda}}_1 \cdot \hat{\lambda}_3$

$= - \hat{\lambda}_1 \times \bar{F} \cdot \hat{\lambda}_3$

$\hat{\lambda}_2 \cdot \hat{\lambda}_1 \times \hat{\lambda}_3 = - \hat{\lambda}_1 \times \hat{\lambda}_2 \cdot \hat{\lambda}_3$

$$\Rightarrow T_2 = \frac{\det \left[[\hat{\lambda}_1] \,|\, [\underline{F}] \,|\, \hat{\lambda}_3 \right]}{\det \left[[\hat{\lambda}_1] \,|\, [\hat{\lambda}_2] \,|\, [\hat{\lambda}_3] \right]}$$

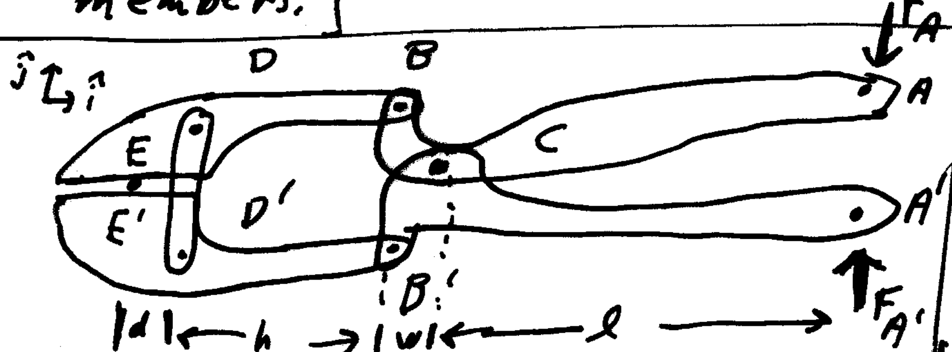
Kramers rule (we've derived using vectors)

Machines: 2/17 pg 3

(& Structures)

Collections of objects connected in simple ways.

[Different from trusses in that some of objects are not 2-force members.]

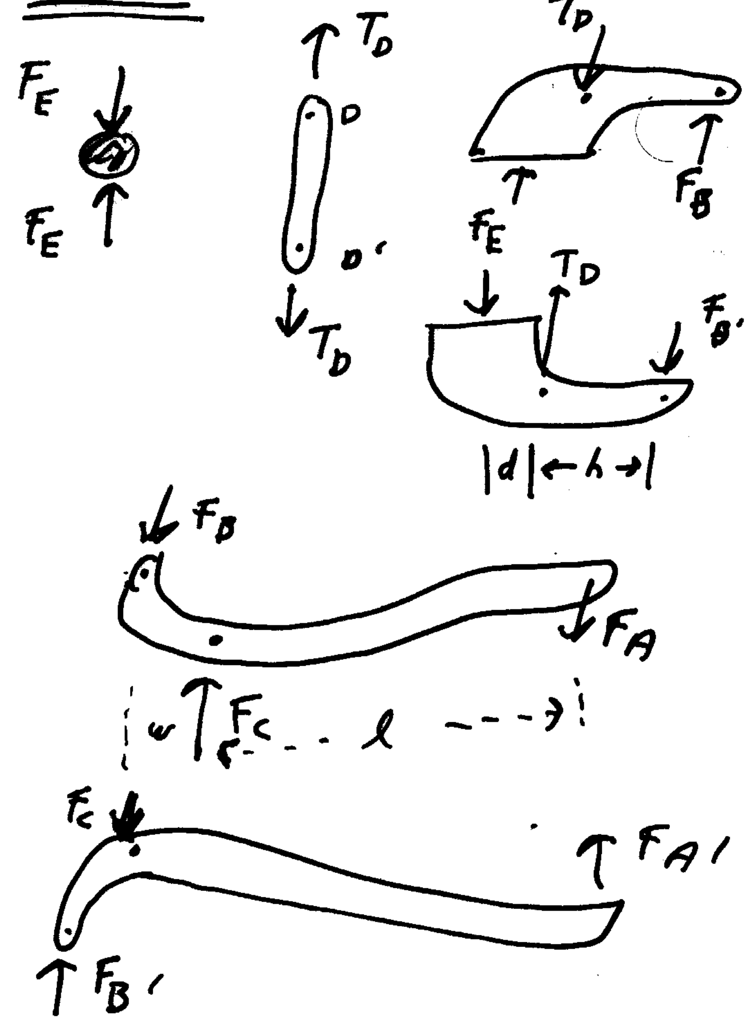


Assume top-bottom symmetric

above is FBD of bolt-cutter & bolt.

Neglect Weight

FBDs



$\sum \underline{M}_{/C} = \underline{0}$ (upper handle)

$\Rightarrow \quad -\ell F_A + F_B w = 0$

$$\boxed{F_B = \frac{\ell}{w} F_A} \quad ①$$

$\sum \underline{M}_{/D} = \underline{0}$ (cutter piece)

$-h F_B + d F_E = 0$

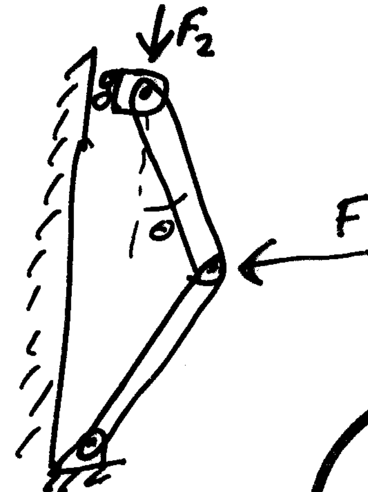
$$\boxed{F_E = \frac{h}{d} F_B} \quad ②$$

$$① \,\&\, ② \Rightarrow \boxed{F_E = \frac{h\ell}{wd} F_A}$$

ex) $\frac{\ell}{w} \approx 40$, $\frac{h}{D} \approx 4$

$$\Rightarrow \boxed{F_E \approx 160 F_A \,!}$$

Note: "toggle mechanism"



θ is small

$\Rightarrow F_2 \gg F_1$

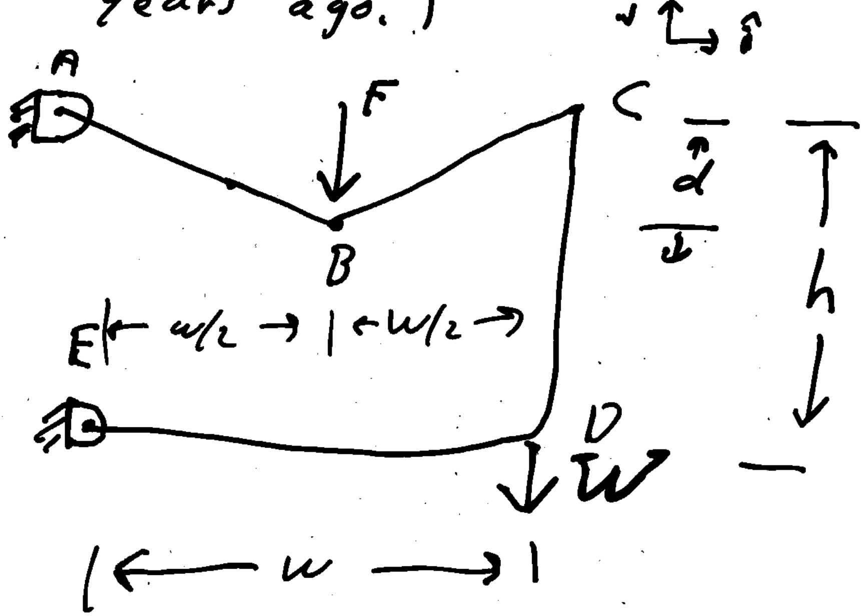
$$\boxed{\frac{F_1}{F_2} = \tan\theta}$$

2/17
pg 4

Feb. 19, 2003

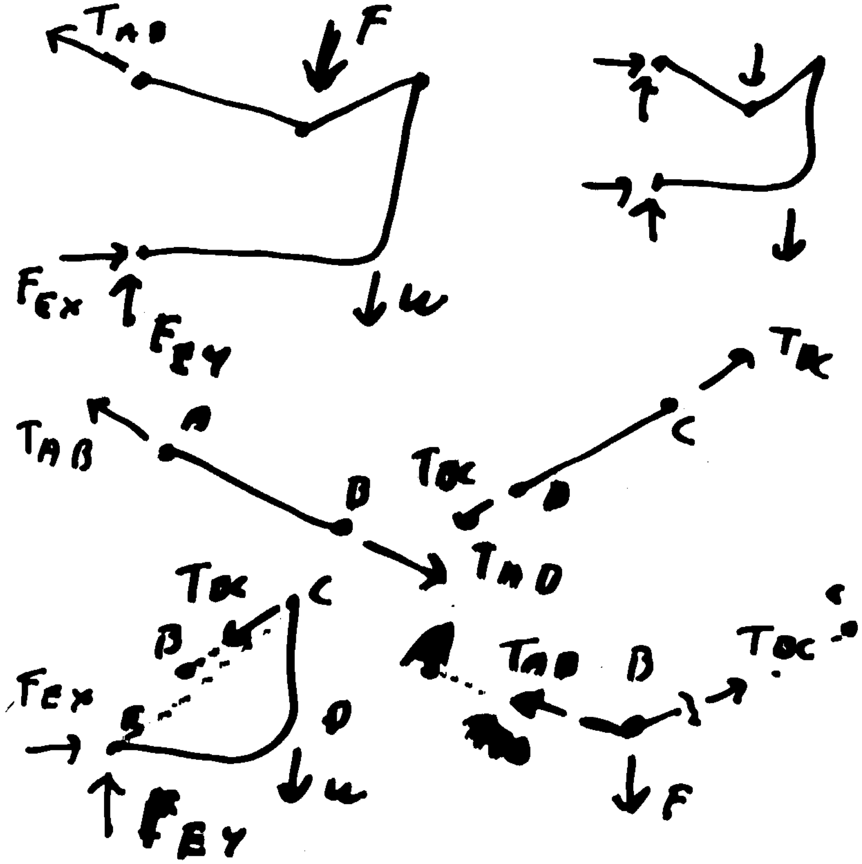
TODAY : Struct & Machine (cont'd)

ex) (from final exam a couple years ago.)



Given w, d, h, W find F, T_{BC}.

FBDs



Body EDC (2/17 pg. 2)

$\sum \underline{M}_{/E} = \underline{0}$

$$-wW\hat{k} + \underline{r}_{C/E} \times \left(T_{BC}\,\hat{\lambda}_{CB}\right) = \underline{0} \quad ①$$

$$\underline{r}_{C/E} = \underline{r}_{EC} = w\hat{i} + h\hat{j}$$

$$\hat{\lambda}_{CB} = \frac{-w/2\,\hat{i} - d\hat{j}}{\sqrt{w^2/4 + d^2}}$$

$$\underline{r}_{C/E} \times \underline{\lambda}_{CB} = (w\hat{i} + h\hat{j}) \times \frac{\left(-\frac{w}{2}\hat{i} - d\hat{j}\right)}{\sqrt{\cdots}}$$

$$= \frac{(-wd + \frac{hw}{2})\hat{k}}{\sqrt{\;}}$$

① ⇒

$$\left\{-wW\hat{k} + \frac{(-wd + \frac{hw}{2})}{\sqrt{\;}} T_{BC}\hat{k} = \underline{0}\right\}$$

$$\{\;\}\cdot\hat{k} \Rightarrow \boxed{T_{BC} = W\frac{\cancel{w}\sqrt{\cdots}}{\left(\frac{h\cancel{w}}{2} - \cancel{w}d\right)}}$$

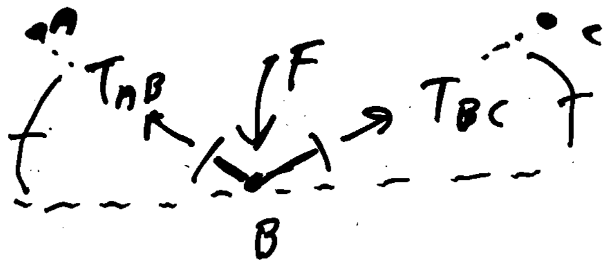
$$\boxed{T_{BC} = \frac{W\sqrt{\;}}{\left(\frac{h}{2} - d\right)}}$$

$T_{BC} > 0$ if $d < h/2$

$T_{BC} < 0$ if $d > h/2$

$T_{BC} \to \infty$ $d \to h/2$

$F = ?$



2/11 PS2

$\sum \underline{F} = \underline{0}$

$\{ T_{AB} \hat{\lambda}_{BA} + T_{BC} \hat{\lambda}_{BC} - F \hat{j} = \underline{0} \}$

take x & y comps $(\{\} \cdot \hat{i}, \{\} \cdot \hat{j})$

Symmetry on $\sum F_x = 0$

$\Rightarrow T_{AB} = T_{BC}$

$\sum F_y = 0 \Rightarrow T_{BC} = \dots$

$\sum \underline{M}_{/A} = \underline{0} \Rightarrow \underbrace{F \qquad \text{w/out } T_{AB}}$

$\sum \underline{M}_{/\text{any pt on line AB except B}}$ ''

$\{ \} \cdot \hat{\lambda}_{\perp \text{ to } AB}$ ''

$(\{ \} \times \hat{\lambda}_{AB}) \cdot \hat{k}$

Feb 21, 2003

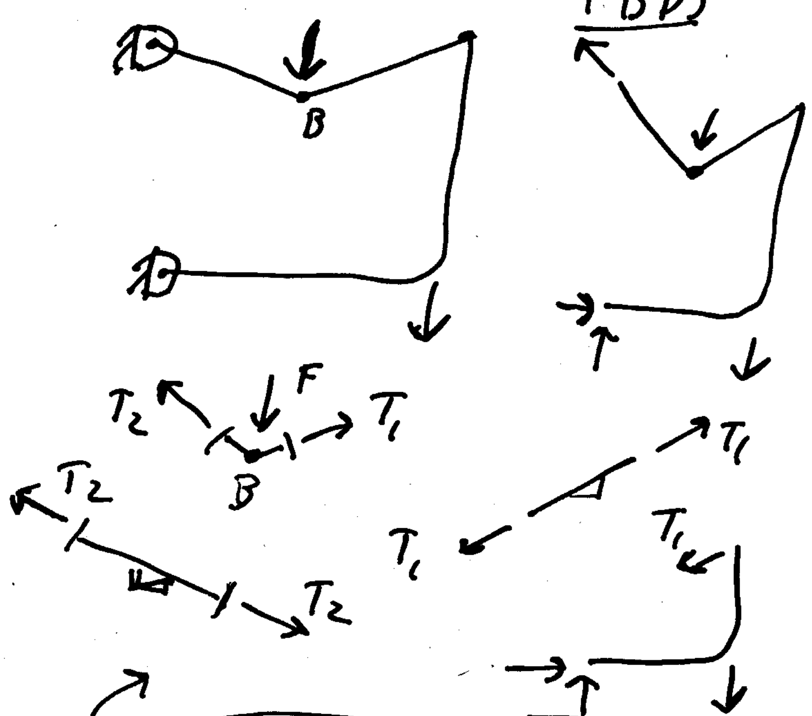
TODAY: Machines (cont'd)

a) last lecture example again.

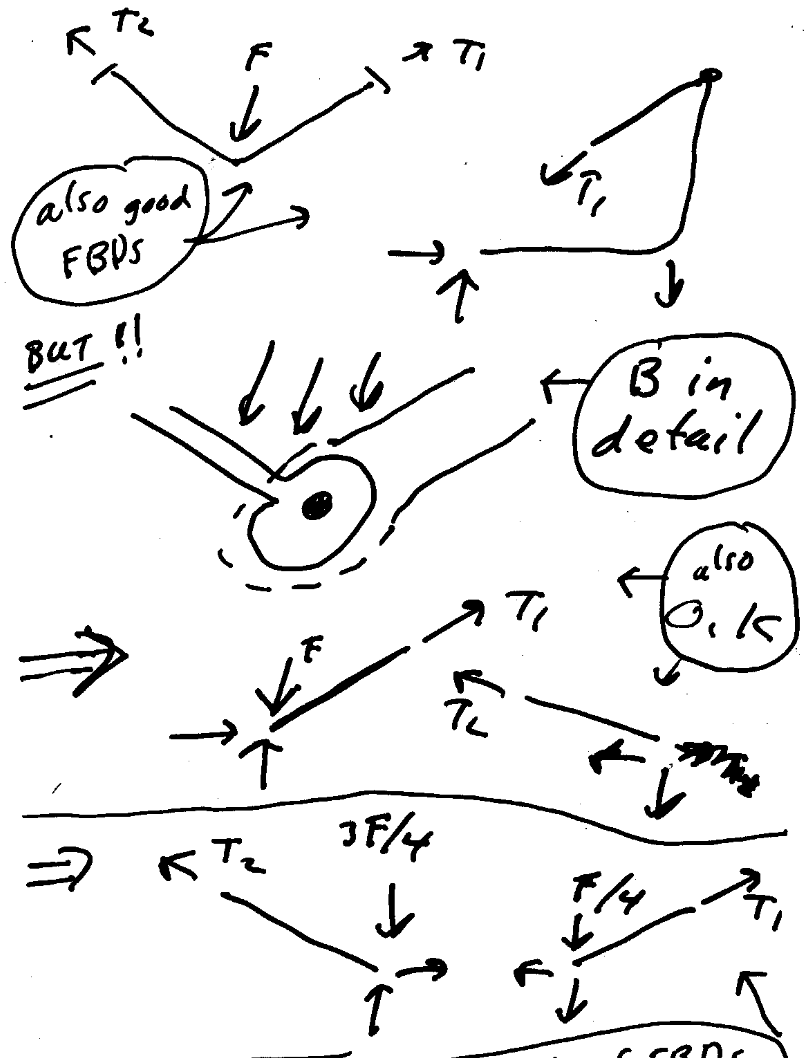
b) force amplification

c) An old Prelim question

last lecture



easiest & simplest FBDs



another OK set of FBDs

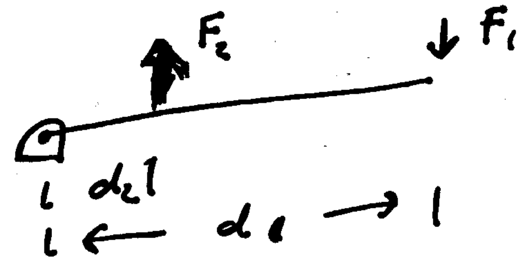
[The ultimate values of T_1 & T_2 will not depend on whether F is applied to the pin at B, or bar 1, or both bars.]

2/2? page 2

How to get big forces from small forces?

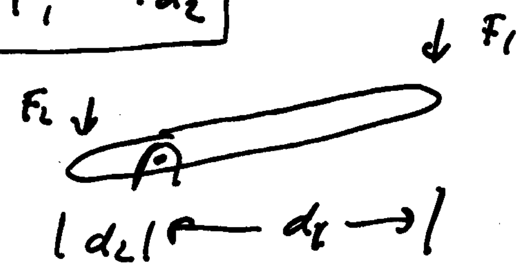
ex) lever

a)



$$\boxed{F_2 = F_1 \, d_1/d_2}$$

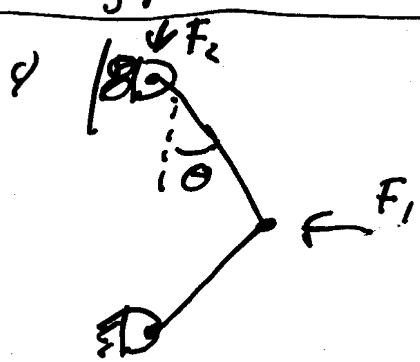
b)



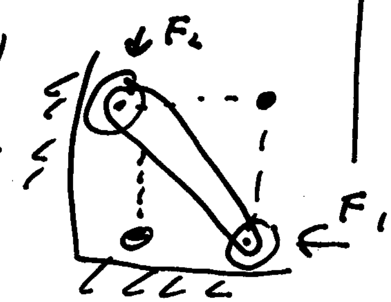
$\frac{d_1}{d_2}$ big $\Rightarrow$ F_2 big

ex) Toggle mechanism

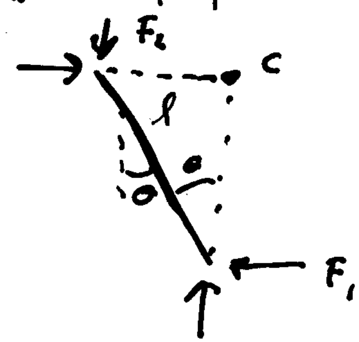
c)



d) or



FBD for (d)



$\sum \underline{M}_{/c} = \underline{0}$

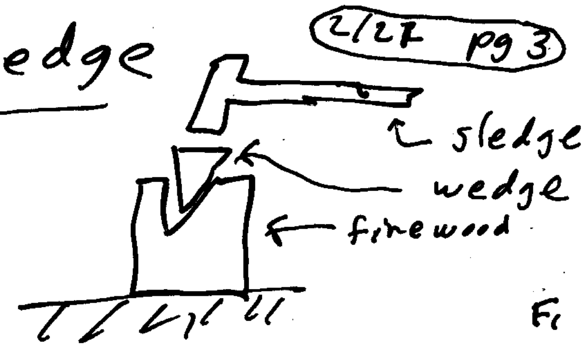
$-F_1 l \cos\theta + F_2 l \sin\theta = 0$

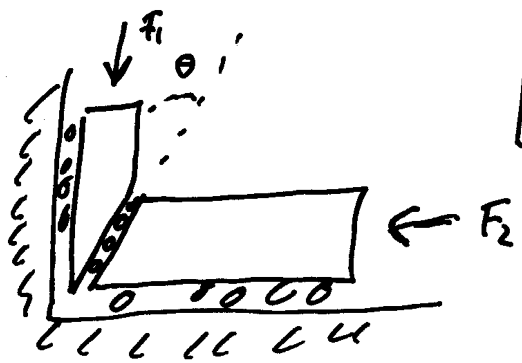
$$\boxed{F_2 = F_1/\tan\theta}$$

θ small $\Rightarrow$ F_2 big

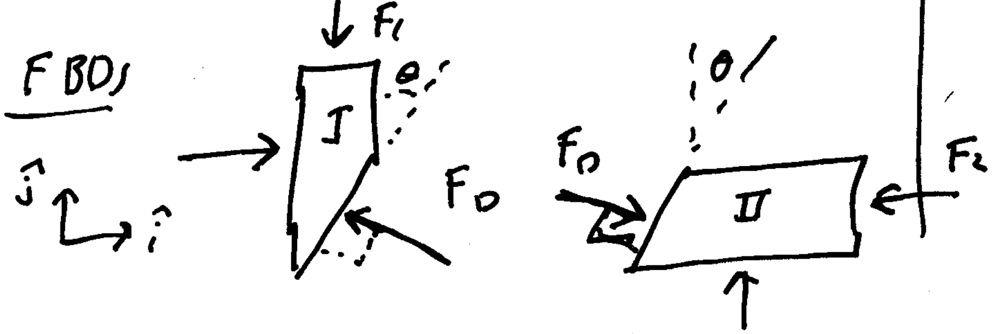
ex) Wedge

2/27 pg 3



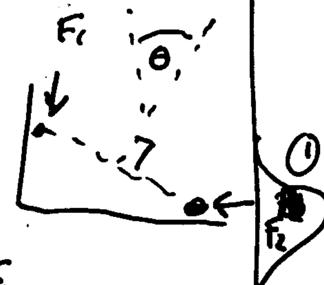


FBDs



FBD I: $\sum F_y = 0$

$\Rightarrow F_1 = F_D \sin\theta$ (1)



FBD II: $\sum F_x = 0$

$F_2 = F_D \cos\theta$ (2)

(1) & (2) $\boxed{F_2 = F_1 / \tan\theta}$

small $\theta \Rightarrow$ big F_2

Morals: For frictionless passive machine

Energy balance

$\Rightarrow$ Net work done on machine $= 0$

$|\delta_1 F_1| = |\delta_2 F_2|$

δ_1 & δ_2 are displ. of F_1 & F_2

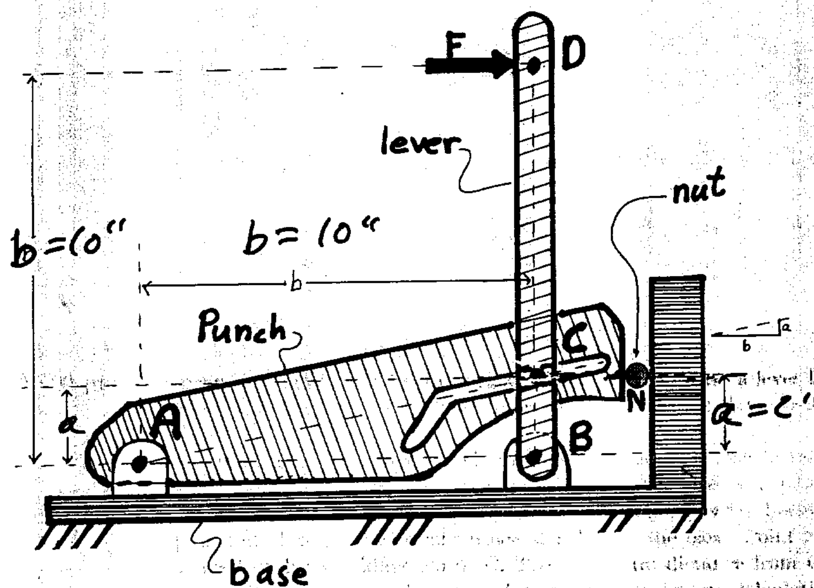
$\boxed{\text{Force amplifiers are motion attenuators}}$

Prelim 1, Fall 02   Feb 21 lecture pg 4

3) (30 pts) The proposed nutcracker design consists of two moving parts: a lever hinged to the fixed base at B and a punch hinged to the fixed base at A. All joints and slots are assumed to have negligible friction.

Mechanism and geometry clarifications: The vertical lever has a pin at C and a horizontal force F applied at D. The punch has a slot in which the lever pin slides at C. The slot is parallel to the line AC. The spherical nut is cracked by being squeezed between the vertical surface of the punch at N and the vertical surface attached to the base. Point N at the left edge of of the nut is level with the sliding pin at C. The horizontal distance from C to N does not enter the solution, but assume it is c if you need it for an intermediate calculation.

Quantities: $F = 10\,\text{lb}$, $a = 2\,\text{in}$, $b = 10\,\text{in}$.



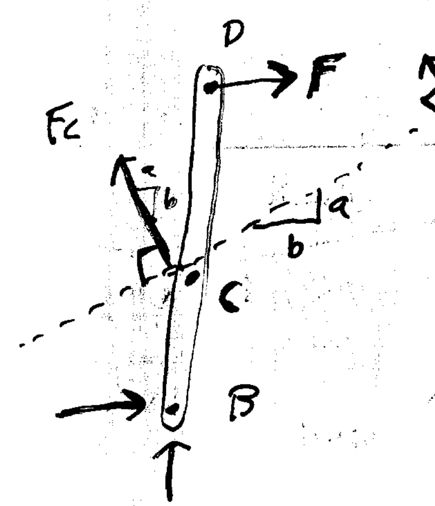
a) **(25 pts)** Find the force acting on the nut at N. A number is desired (i.e., so many lb force). [Hint: Only substitute in numbers when you have a formula for your answer in terms of a, b and F.]

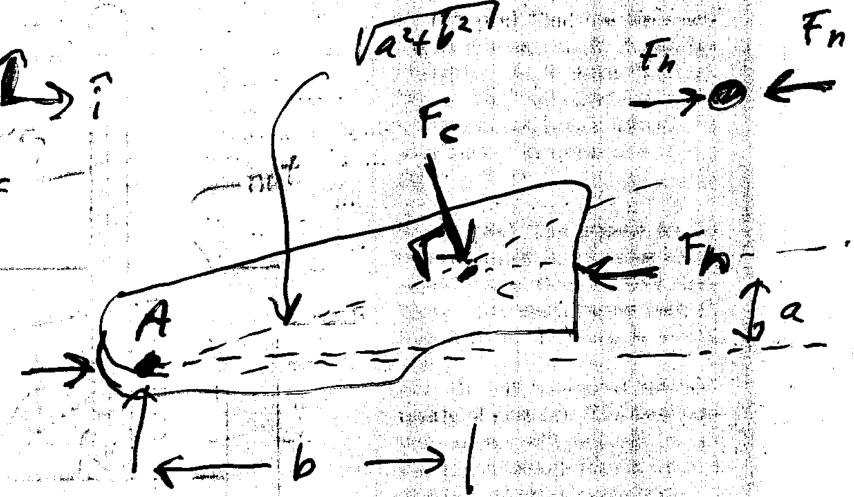
b) **(5 pts)** The answer to (a) is conspicuous in its being either much smaller than F, very similar to F, or much bigger than F. Which is it? Explain, in words, why. Part (b) will be graded independently of part (a). The best possible answer will generate an approximate formula for the force at N using next-to-no equations.

$F = 10\,\text{lbf}$ F_N = force on nut = ?

FBDs

$\hat{j}$ $\hat{i}$ $\vec{\lambda}_c$





(cont'd next lecture)

~~start a page~~ Feb 24 (pg 1) ex (cont'd)

lever : $\sum \underline{M}_{/B} = \underline{0}$

$$-Fb\hat{k} + \underline{r}_{c/B} \times (F_c \hat{\lambda}_c) = \underline{0}$$

$$\underline{r}_{c/B} = a\hat{j}$$

$$\hat{\lambda}_c = \frac{-a\hat{i} + b\hat{j}}{\sqrt{a^2+b^2}}$$

$$-Fb\hat{k} + (a\hat{j}) \times \left(F_c\left(\frac{-a\hat{i}+b\hat{j}}{\sqrt{a^2+b^2}}\right)\right) = \underline{0}$$

$$\left\{-Fb\hat{k} + \frac{a^2}{\sqrt{a^2+b^2}} F_c \hat{k} = \underline{0}\right\} \quad ①$$

$$\{\} \cdot \hat{k} \Rightarrow \boxed{F_c = \frac{b\sqrt{a^2+b^2}}{a^2} F}$$

Punch $\sum \underline{M}_{/A} = \underline{0}$

$$\underline{r}_{c/A} \times (F_c(-\hat{\lambda}_c)) + \underline{r}_{N/A} \times F_n(-\hat{i}) = \underline{0}$$

$$\left\{-\sqrt{a^2+b^2}\, F_c \hat{k} + a F_n \hat{k} = \underline{0}\right\}$$

$$\{\} \cdot \hat{k} \Rightarrow \boxed{F_N = \frac{\sqrt{a^2+b^2}}{a} F_c} \quad ②$$

$$① \,\&\, ② \Rightarrow F_N = \frac{b(a^2+b^2)}{a^3} F$$

$b = 10''$, $a = 2''$, $F = 10\,\text{lb}$

$$F_N = \frac{10(104)}{8} 10\,\text{lb.}$$

$$\boxed{F_N = 130 \cdot 10\,\text{lb} = 1300\,\text{lb.}}$$

This lecture ~~picks~~ picks up w/ the example (nut cracker) from last lecture.

2/24/03 Pg. 2

CENTER OF MASS, CENTROID

(center of gravity)

$\underline{r}_G$ = average position of the mass

position of C.O.M. w.r.t. O

Analogy: test scores

$$T_A = \frac{T_1 \cdot n_1 + T_2 \cdot n_2 + T_3 \cdot n_3}{n_1 + n_2 + n_3}$$

test score average

$$= \frac{65 \cdot 3 + 75 \cdot 5 + 95 \cdot 2}{10}$$

$$= \text{----}$$

average = { (value 1)(amount of stuff w/ value 1)

+ (value 2)(amount of stuff w/ value 2)

⋮

} / (total amount of stuff)

$$\underline{r}_G = \frac{\sum \underline{r}_i m_i}{m_{tot}}$$

$m_{tot} = \sum m_i$

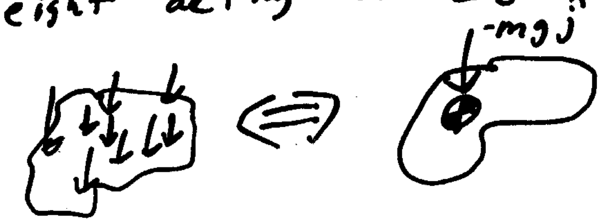
$$\boxed{m_{tot}\,\underline{r}_G = \sum \underline{r}_i m_i}$$

$$m_{tot} X_G = x_1 m_1 + x_2 m_2 + x_3 m_3 + \dots x_n m_n$$

Why be interested in $\underline{r}_G$? (2/29 p73)

1) The gravity forces on a system (for near-earth gravity: $-g\hat{j}$; constant g downwards) are "equivalent" to the total weight acting at $\underline{r}_G$.

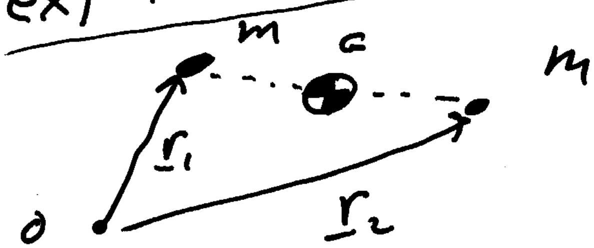
FACT



2) Centroid is important for understanding beam cross sections (as you will see in a month or so).

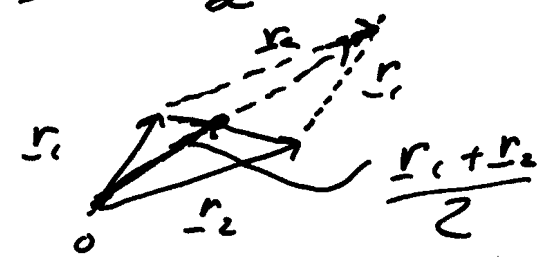
$$m_{tot}\underline{r}_G = \begin{cases} \sum \underline{r}_i m_i & \text{discrete} \\ \int \underline{r}\, dm & \text{continuous} \end{cases}$$

ex) Two equal pt. masses



$$2m\underline{r}_G = m\underline{r}_1 + m\underline{r}_2$$

$$\underline{r}_G = \frac{\underline{r}_1 + \underline{r}_2}{2}$$



G = Com is at midpoint.

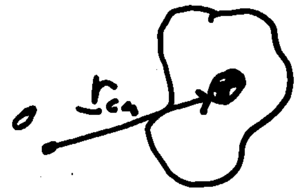
[Note: "objective". Location is independent of position or orientation of coord. syst. used for calculation.]

Feb. 26, 2007 (p. 1)

TODAY: 1) C.O.M. (cont'd) 2) Q&A

Center of mass

$$m_{tot}\underline{r}_{cm} = \begin{cases} \sum_i \underline{r}_i m_i \\ \int \underline{r}\, dm \end{cases}$$

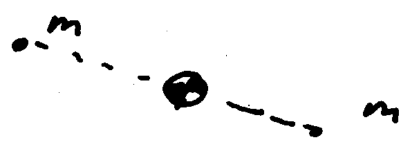


$$dm = \begin{cases} \rho\, dV \\ \rho\, dA \\ \rho\, ds \end{cases}$$

ρ = mass per unit
- volume
- area
- length

as appropriate

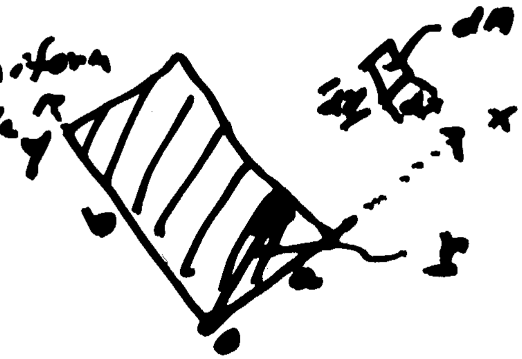
ex) two equal masses (again)



C.O.M. is "objective", location has a well defined place w.r.t. object $\Rightarrow$

You can use any coord. syst. you like to find C.O.M.

ex) uniform rectangle density ρ



$$m_{tot}\underline{r}_{cm} = \int \underline{r}\, dm$$

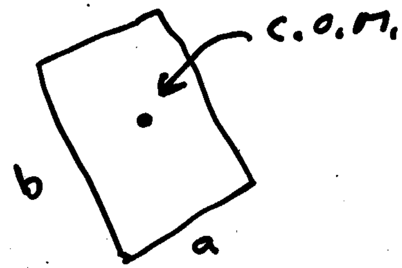
$$\rho a b\, \underline{r}_{cm} = \int_0^b\int_0^a (x\hat{i} + y\hat{j})\rho\, dx\, dy$$

$$= \int_0^b \left(\frac{a^2}{2}\hat{i} + ya\hat{j}\right)\rho\, dy$$

1/26 pg 2

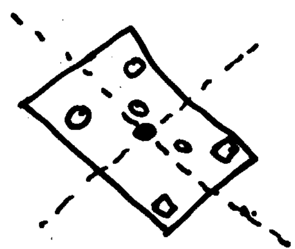
$$= \left(\frac{a^2 b}{2}\,\hat{i} + \frac{a b^2}{2}\,\hat{j}\right)\rho$$

$$\Rightarrow \underline{r}_{cm} = \frac{a}{2}\,\hat{i} + \frac{b}{2}\,\hat{j}$$



Would have got same location w/ this coord syst. but w/ much more calculation.

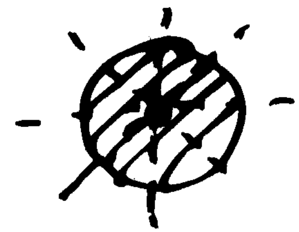
Note: C.O.M. respects symmetry of object



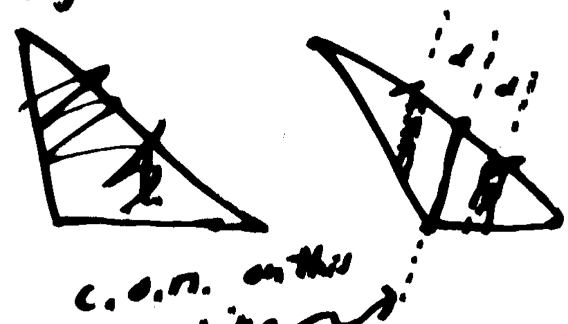
ex)

c.o.m on symmetry line

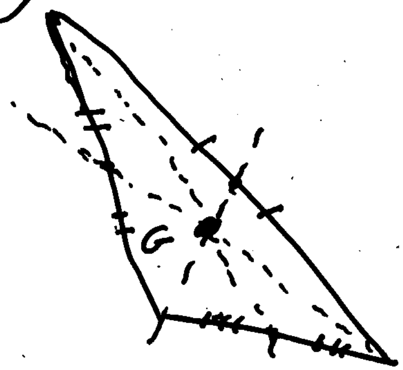
ex)



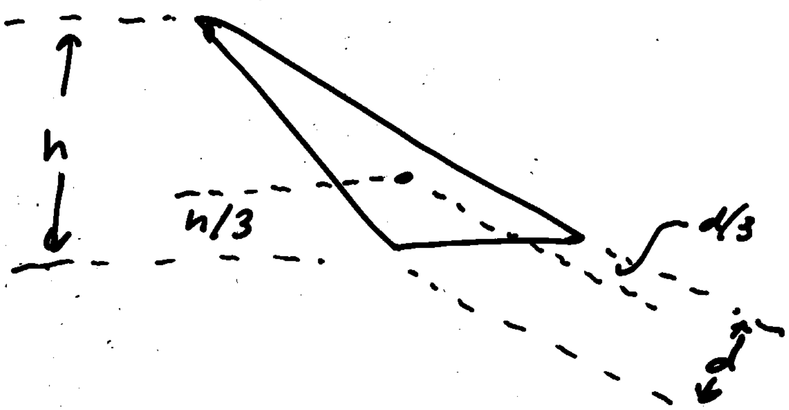
ex) triangle



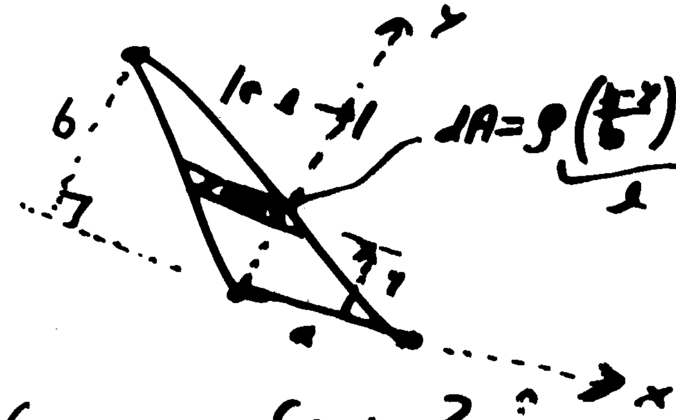
repeat for all 3 side bisectors . . .



Geometry fact: G is 1/3 of the way up from each base



ex) triangle using calculus



$$dA = \rho\underbrace{\left(\frac{b-y}{b}\right)a}_{\ell}\,dy$$

$$\{m\,\underline{r}_{cm} = \int \underline{r}\,dm\}\cdot\hat{j}$$

$$m\,y_{cm} = \int y\,\underset{\hookrightarrow \rho\,dA}{dm}$$

$$= \int_0^b y\,\underbrace{\rho\,\frac{b-y}{b}\,a\,dy}_{dm}$$

$$(\rho ab/2)\,y_{cm} = \rho\left(\frac{y^2}{2}\Big|_0^b a - \frac{y^3}{3}\frac{a}{b}\Big|_0^b\right)$$

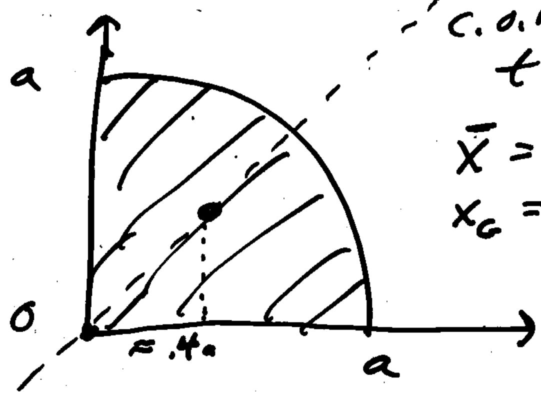
$$= \rho\left(\frac{b^2 a}{2} - \frac{b^2 a}{3}\right) = \frac{1}{6}\rho b^2$$

$$\boxed{y_{cm} = b/3}$$

2/26 pg 9)

(uniform)

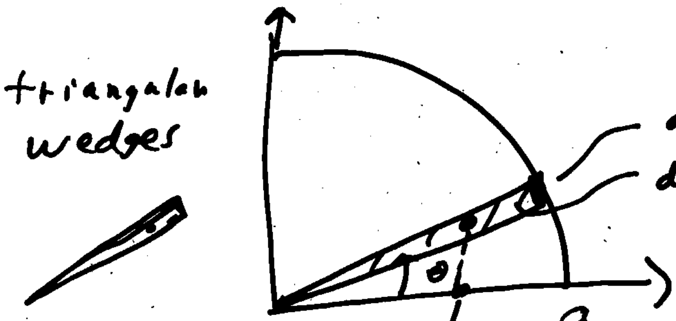
ex) quarter circle



$\bar{X} = \bar{Y}$

$x_G = y_G$

triangular wedges



$ds = a\, d\theta$

$dm = \rho \underbrace{\dfrac{a^2\, d\theta}{2}}_{dA}$

$\frac{2}{3}(\cos\theta)\, a$

$$m_{tot}\, x_{cm} = \int x\, dm$$

$$= \int_0^{\pi/2} \underbrace{\left(\tfrac{2}{3}(\cos\theta)\, a\right)}_{x} \underbrace{\left[\rho\, \frac{a^2}{2}\right] d\theta}_{dm}$$

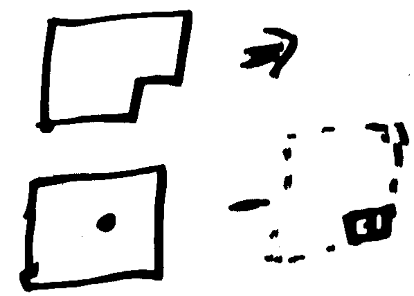
$$= \frac{1}{3}\rho a^3 \underbrace{\int_0^{\pi/2} \cos\theta\, d\theta}_{1}$$

$$\frac{\pi a^2}{4}\cancel{\rho}\, x_{cm} = \frac{1}{3}\cancel{\rho}\, a^3$$

$$x_{cm} = \frac{4}{3\pi}\, a \approx .4a$$

$$\underline{r}_{cm} = \frac{4}{3\pi}\, a\, (\hat{i} + \hat{j})$$

ex) composite object



$$m_{tot}\, \underline{r}_{cm} = m_1 \underline{r}_1 + m_2 \underline{r}_2$$

$$(m_1 - m_2)\, \underline{r}_{cm} = m_1 \underline{r}_1 - m_2 \underline{r}_2$$

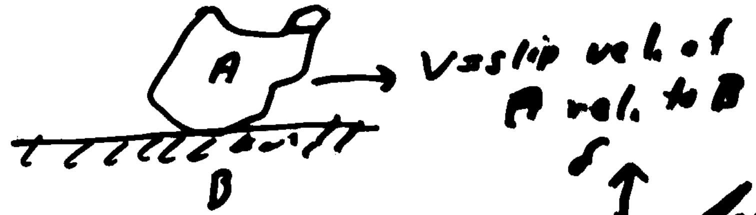
March 3, 2003 page 1

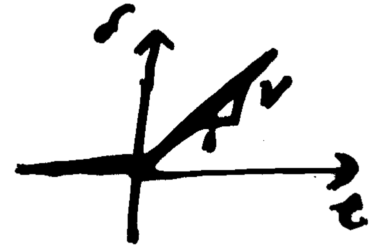
TODAY: Friction

Friction = force which resists or prevents slipping between contacting solids.

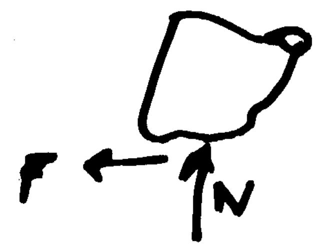
Some books: "smooth" = no friction
"rough" = ∞ friction

(actually friction strength is not well correlated to surface roughness unless surface is lubricated = wet w/ water or oil)

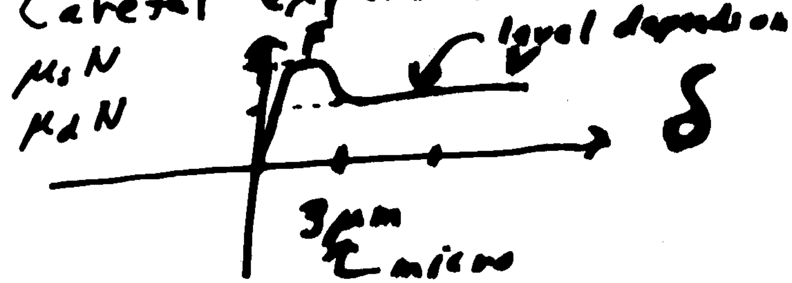




Partial FBD



Careful experiment N = const.



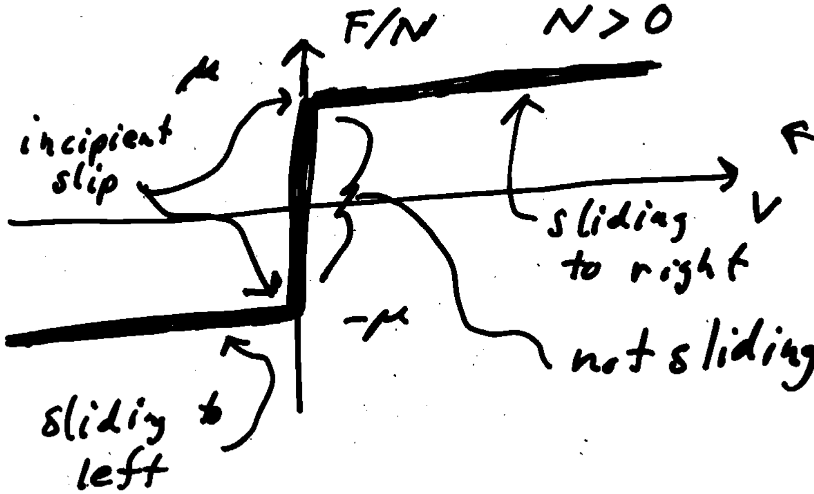
μ_s = static or stationary coeff. of friction

$\mu_d = \mu_K$ = dynamic or kinetic coeff. of friction

This is too complicated ⇒ use a simple model ⇒

$\underbrace{\mu = \mu_s = \mu_k = \mu_d}$

on coeff. of friction



this curve is the constitutive law for dry friction which we will use

3/3 pg. 2

friction law called

$$\left[\begin{array}{l}\text{Coulomb's law/} \\ \text{Amonton's law/} \\ \text{Davinci law}\end{array}\right] \text{ for friction}$$

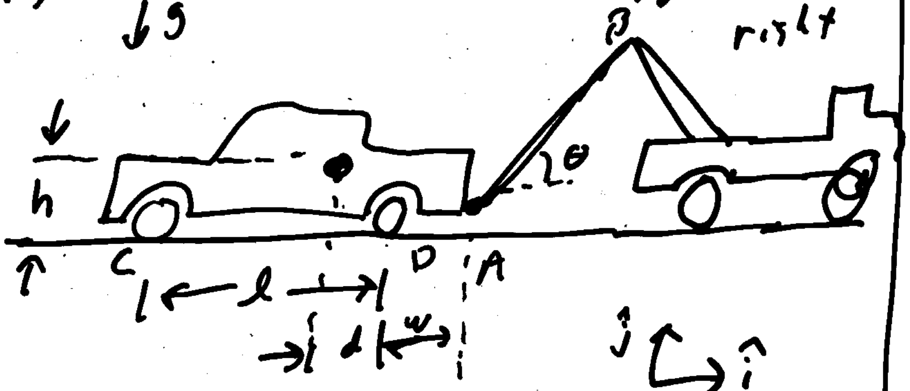
$$F = \begin{cases} \mu N & \text{for } v > 0 \\ -\mu N & \text{for } v < 0 \end{cases}$$

$\underbrace{|F| \leq \mu N}$ for $v = 0$

$-\mu N \leq F \leq \mu N$

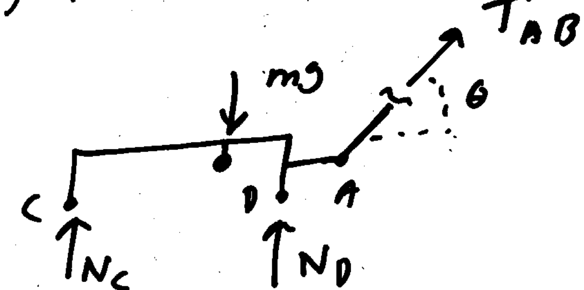
⇒ Try various cases

ex) m = mass   dragged to right



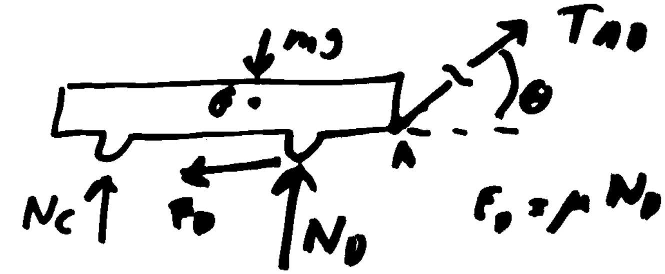
$T_{AB} = ?$

subex) No brakes on:



$\sum F_x = 0 \Rightarrow \boxed{T_{AB} = 0}$

subex) Front brakes on (3/3 pg. 3)



$F_D = \mu N_D$

Brute approach:

$\sum F_x = 0 \Rightarrow -\mu N_D + T_{AB}\cos\theta = 0$

$\sum F_y = 0 \Rightarrow N_C + N_D + T_{AB}\sin\theta - mg = 0$

$\sum \underline{M}_{/D} = \underline{0} \Rightarrow \underline{r}_{C/D} \times N_C\hat{j} + \underline{r}_{G/D} \times (mg\hat{j}) + \underline{r}_{A/D} \times T_{AB}(\cos\theta\hat{i} + \sin\theta\hat{j}) = \underline{0}$

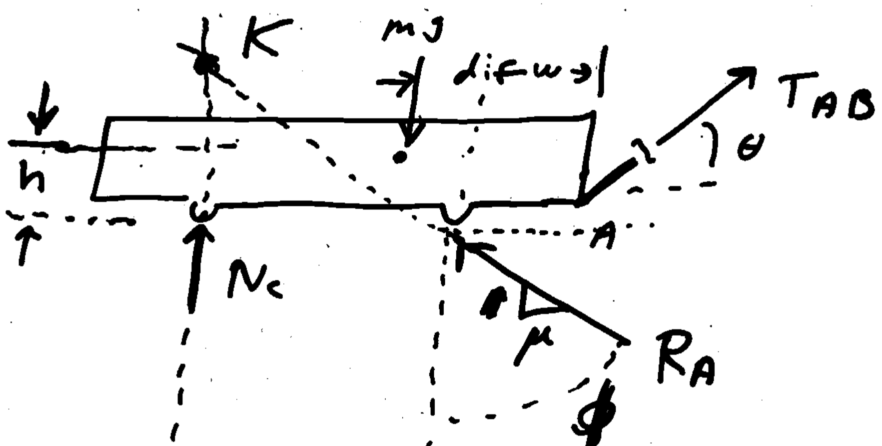
3 eqs for N_C, N_D, T_{AB}

$\Rightarrow F_D = \mu N_D$

9/3 pg 4

Short cut

Draw FBD again



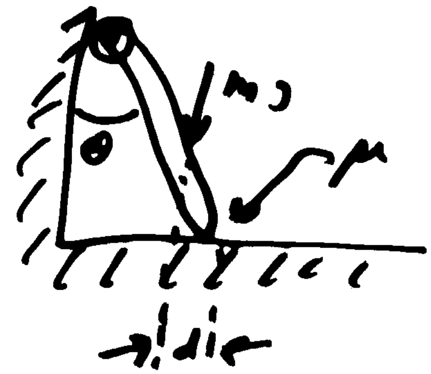
$\sum \underline{M}/_K = \underline{0} \Rightarrow$ one eqn. for on unknown T_{AB}

Aside: Alt. description for friction: friction angle ϕ

$\tan \phi \equiv \mu$

↑ angle between net contact force & normal at slip on incipient slip.

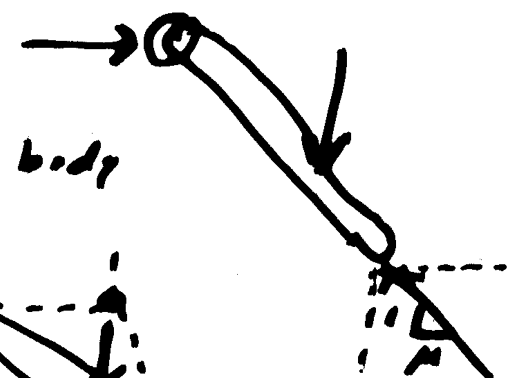
ex)



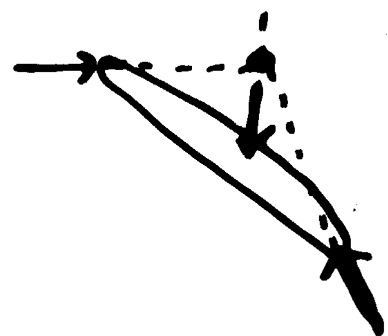
given μ, θ, m, g how big can d get for person not to fall.

Assume incipient slip

FBD:



3 force body

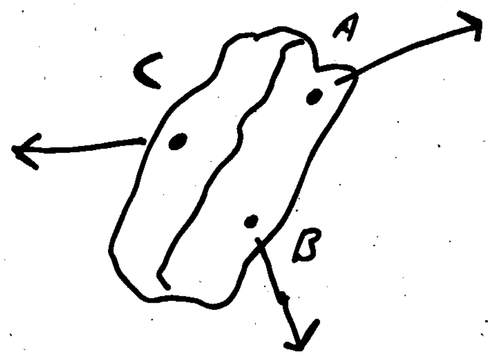


3/5/03 pg. 1

TODAY: 3 force bodies, friction, hydrostatics (lot of z).

3 force body

A body (in 2D or 3D) w/ 3 forces acting on it.



$\sum M_{/AB} = 0$, Force at C only contributes ⇒ contrib. $= 0$ ⇒ force at C in plane ABC.

Likewise for $\sum M_{/AC} = 0$, $\sum M_{/BC} = 0$

⇒ All forces co-planar in ABC plane.

⇒ 2D problem in plane ABC.

H is pt. of intersection of lines of action of forces at A & B.

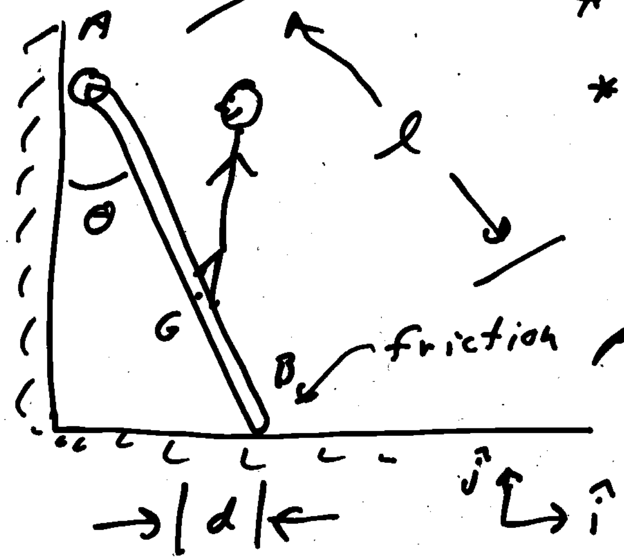
$\sum \underline{M}_{/H} = 0$

⇒ line of action of force at C intersects H.

⇒ 3 force body all lines of action intersect at one pt. in plane ABC (that pt can be at ∞; all 3 lines $\parallel$).

[3/5/04 pg 2]

ex) repeat from end of last class



* ladder has no mass.
* person m

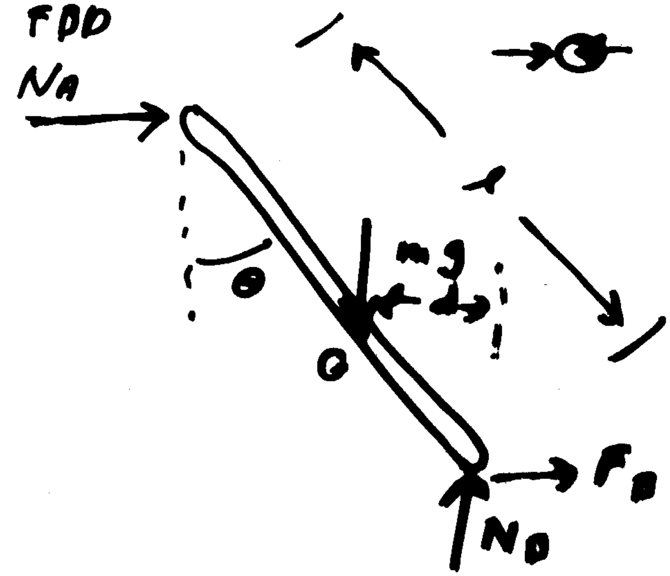
$\downarrow g$

What is biggest d so ladder does not fall?

2 approaches

1) Assume no slip at B. Find reaction at B. make sure $F_{fB} \leq \mu N_B$.

FBD



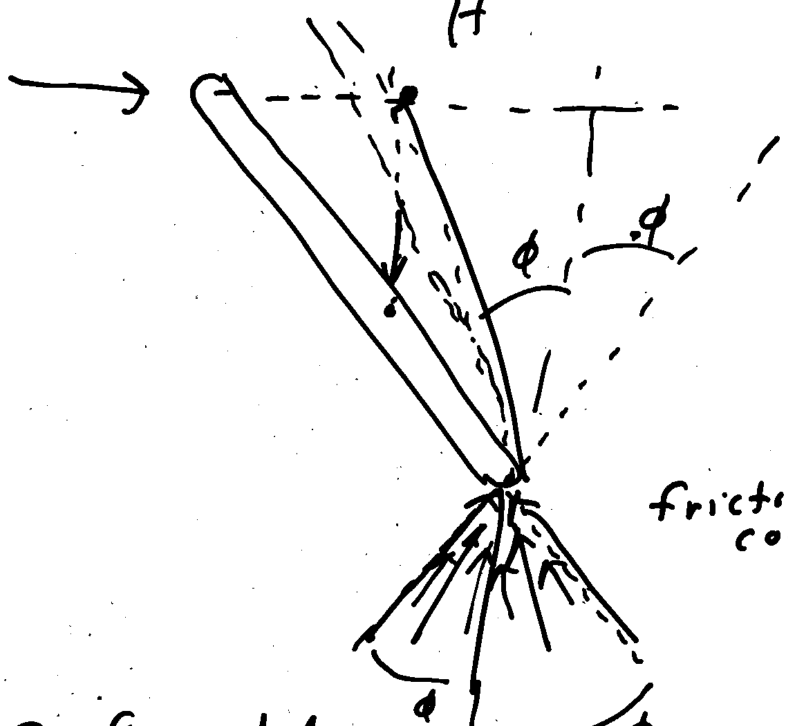
Solve for N_A, F_B, N_B as usual ($\Sigma \underline{F} = \underline{0}$, $\Sigma \underline{M} = \underline{0}$)
Answer in terms of θ, ℓ, m, g, d.

$$\frac{|F_B|}{N_B} \leq \mu$$

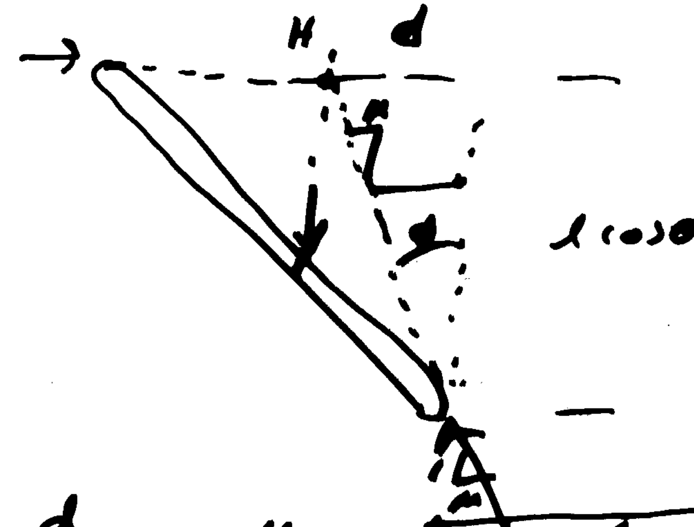
gives an inequality for d.
[Will get answer for all d if $\theta < \phi$]

2/5/02 — 3

2) More tricky

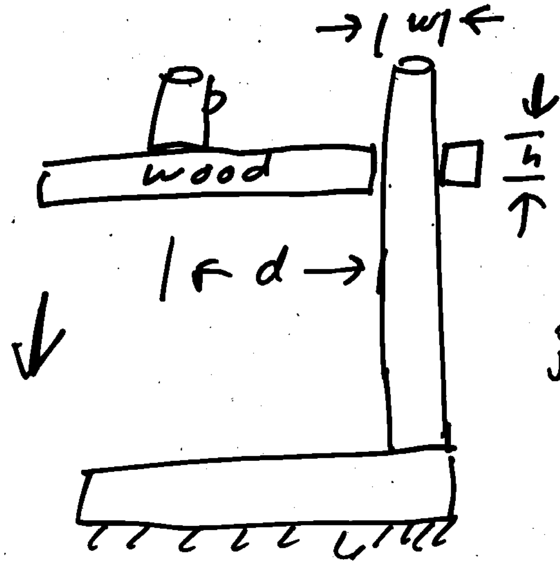


3 force body
$\Rightarrow$ pt. H in friction cone associated w/ B.
Worst case $\Rightarrow$ H at edge of cone



$\frac{d}{l\cos\theta} = \mu \Rightarrow \boxed{d = \mu l \cos\theta}$

pg [illegible], 2003



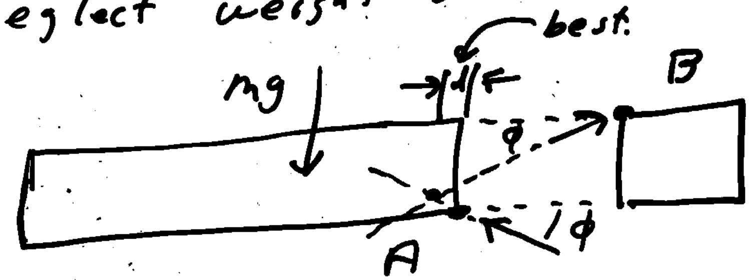
"coffee stand"

min. d to hold coffee.

$\hat{j}$ $\hat{i}$

μ given

neglect weight of wood.



How close can d get & still sat. equilib.

$\boxed{\tan\phi = \mu}$

March 3rd, 2003
Lecture given by Professor Alan Zehnder

Topic: Hydrostatics

Special case of distributed loads in which:
1) Force is perpendicular to surface it acts on
2) Pressure increases linearly with depth

$$p = \underbrace{\rho}_{\text{density of Fluid}} \underbrace{g}_{\text{acceleration of gravity}} \underbrace{h}_{\text{depth}}$$

Let's just check units.

$$p = \frac{kg}{m^3} \cdot \frac{m}{s^2} \cdot m$$

$$= \frac{kg\,m}{s^2} \cdot \frac{1}{m^2} = \frac{N}{m^2} \checkmark$$

For reference,

$$1 \text{ atm} = 10^5 \frac{N}{m^2}$$

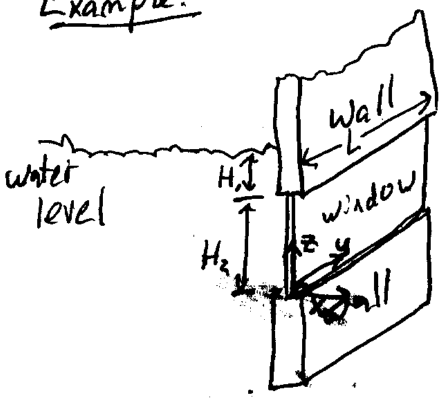
At the bottom of Cayuga Lake

$$p \approx \underset{(\rho)}{10^3} \quad \underset{(g)}{10} \quad \underset{(h)}{(100)} = 10^6 \, Pa = 10 \text{ atm}$$

Start from fundamentals
- Force + moment due to pressure on small area
- Integrate to get total $\underline{F}$ + $\underline{M}$

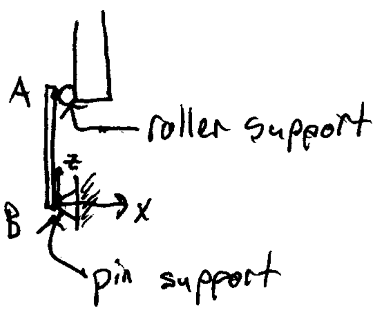
Build Shortcuts
- Replace distributed force by a single force acting at centroid of diagram of force distribution.

Example:



- Find force & moment due to water pushing on window
- Find reaction forces of wall pushing on windows

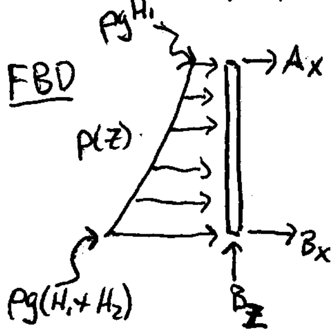
- Idealize problem as 2D
- Idealize as pinned on bottom supports, roller on top



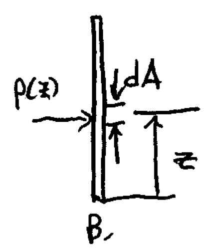
Note: In this coordinate system:

$$p = p(z) = \rho g(H_1 + H_2 - z)$$

FBD



Force due to $p(z)$ acting on dA



$$d\underline{R} = p(z)\, dA\, \hat{\imath}$$
$$= p(z)\hat{\imath}\, dx\, dy$$

- Moment (about B) due to $p(z)$ acting on dA

$$dM_B = -p(z)\, z\, dy\, dz$$

- Integrate to get $\underline{R}$ & M_B

$$\underline{R} = \int_A d\underline{R} = \int_0^L \int_0^{H_2} \rho g (H_1 + H_2 - z)\, dz\, dy\, \hat{\imath}$$

total force of water pushing on window

$$= \rho g L \left(H_1 H_2 + \frac{H_2^2}{2}\right)\hat{\imath}$$

$$M_B = \int_A dM_B = \int_0^L \int_0^{H_2} -\rho g (H_1 + H_2 - z) z\, dz\, dy$$

$$= -\rho g L \frac{H_2^2}{2}\left(H_1 + \frac{H_2}{3}\right)$$

- Determine Reaction Forces.

$$\Sigma \underline{F} = \underline{0} = \rho g L \left(H_1 H_2 + \frac{H_2^2}{2}\right)\hat{\imath} + A_x\hat{\imath} + B_x\hat{\imath} + B_z\hat{k}$$

$$\Sigma M_B = 0 = -\rho g L \frac{H_2^2}{2}\left(H_1 + \frac{H_2}{3}\right) - A_x \cdot H_2$$

Solving, we find that:

$$B_z = 0$$

$$A_x = \frac{-\rho g L \frac{H_2^2}{2}\left(H_1 + \frac{H_2}{3}\right)}{H_2} = -\rho g L \frac{H_2}{2}\left(H_1 + \frac{H_2}{3}\right)$$

$$B_x = -\rho g L \left(H_1 H_2 + \frac{H_2^2}{2}\right) - A_x = -\rho g L H_2\left(\frac{H_1}{2} + \frac{H_2}{3}\right)$$

Let's try some numbers

Say, $H_1 = 0.5\,m$

$H_2 = 1\,m$

$L = 5\,m$

Approximate $g = 10\,m/s^2$

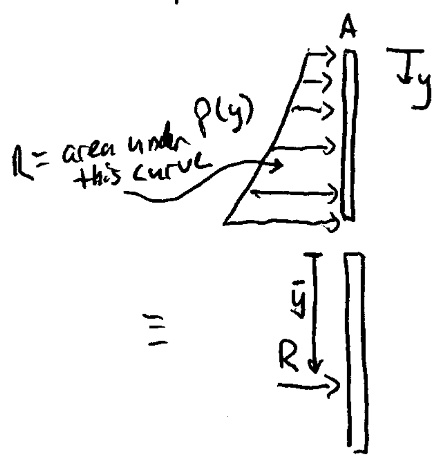
$\Rightarrow \rho g = 10^4$

$$A_x = -10^4(5)\left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}+\tfrac{1}{3}\right) = -\tfrac{25}{12} \times 10^4 = -2.08 \times 10^4\,N \approx -4000\text{ lbs}$$

$$B_x = -2.92 \times 10^4\,N$$

Check if $R + A_x + B_x = 0$ ✓

Replace distributed force by single force at centroid of pressure diagram



Force due to pressure

$$R = \int_A p(y)\,dA = L \int p(y)\,dy$$

depth into board

$= L \cdot$ (area under curve of $p(y)$ diagram

Note: $\bar{y} \equiv$ centroid of area under $p(y)$ curve

Moment around A

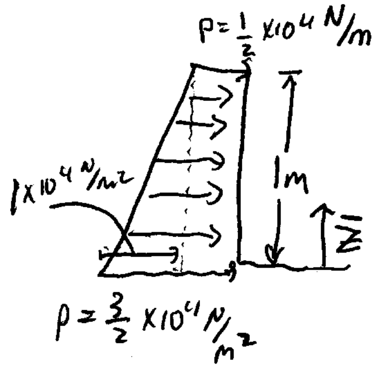
$$M_A = \int_A p(y)\,y\,dA = L \int y\,p(y)\,dy$$

$$M_A = L \cdot \bar{y} \cdot R$$

Centroid of area under p(y) curve

$$\bar{y} = \frac{\int y\, p(y)\, dy}{\int p(y)\, dy}$$

Previous Example:



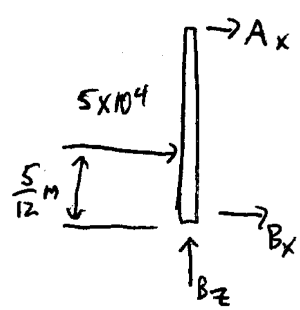
Note: $\rho g \approx 10^4$

$L = 5$

$$R = 1m\left(\tfrac{1}{2}\times 10^4 + \tfrac{1}{2}(1\times 10^4)\right)5$$

$$= 5\times 10^4\,N = 50{,}000\,N$$

$$\bar{z} = \frac{\left[\underbrace{\tfrac{1}{2}\left(1\times\tfrac{1}{2}\times 10^4\right)}_{\text{(rectangular area)}} + \tfrac{1}{3}\left(1\times\tfrac{1}{2}\left(1\times 10^4\right)\right)\right]5}{\text{Total Area}} = \frac{5\left(\tfrac{1}{4}+\tfrac{1}{6}\right)\times\cancel{10^4}}{5\times\cancel{10^4}} = \frac{5}{12}\,m$$



3/10/03 (1)

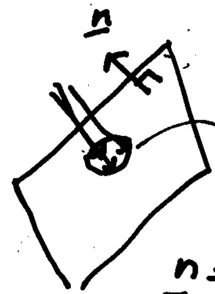
This week:

MON: Hydrostatics (cont'd)
WED: Quiz
FRI: Statics Capstone

Hydrostatics

$d\underline{F} = -p\, dA\, \underline{n}$

p = pressure



under water surface

dA = bit of area

$\underline{n}$ = unit normal of surface pointing towards water.

OF INTEREST:

$$\underline{F}_{TOT} = \int d\underline{F} = \int_S p(-\underline{n})\, dS$$

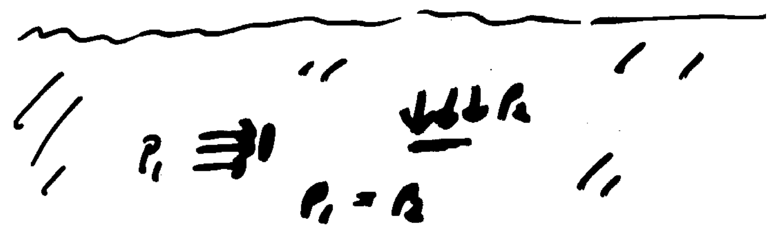
$$\underline{M}_{/c\ tot} = \int \underline{r}_{/c} \times d\underline{F}$$

$$= \int_S \underline{r}_{/c} \times (-p\underline{n})\, dS$$

dS is like dA bit of surface

Key facts:

no shear $\Rightarrow$ p is the same for all $\underline{n}$ at a given pt. in space



Why? FBD of Δ

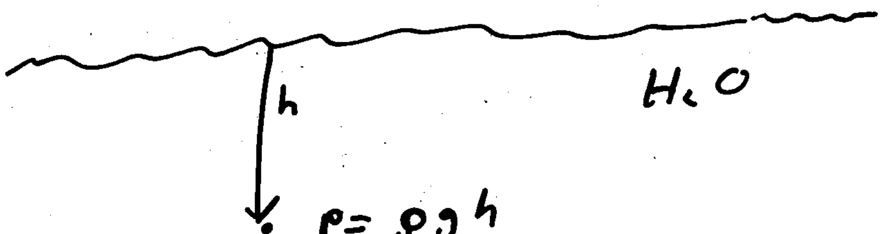
$\sum F_x = 0 \Rightarrow p_x = p_y$
$\sum F_y = 0 \quad\quad = p$

3/10/03

<u>if</u> ρ = density of fluid = const.

[good model for water almost all the time

[bad model for air over length scales of kms.



$p = \rho g h$
↳ "gauge" pressure

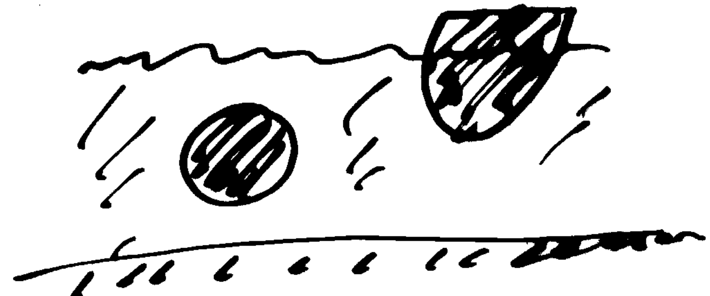
$P_{real} = P_{gauge} + P_{atm.}$

Beginner advice: ignore atmospheric pressure.

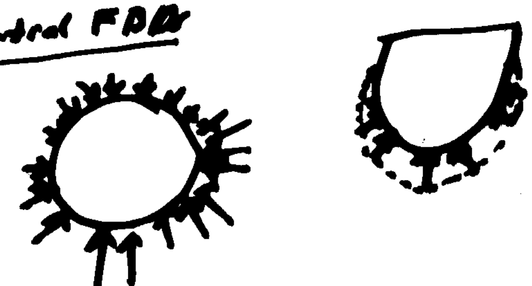
Archimedes Principle

"Aha!" Force & moment from H_2O pressure on submerged object.

I.



pressure FBDs



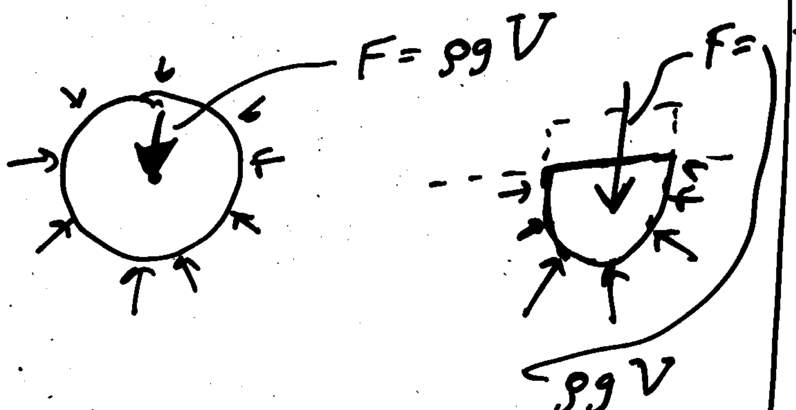
Pressure at each pt. in space same as it would have been if objects replaced w/ water

3/10/03 ③

Force on object from H_2O

= Force that would have acted on water w/ same shape.

FBD of water



Because water in equilibrium:

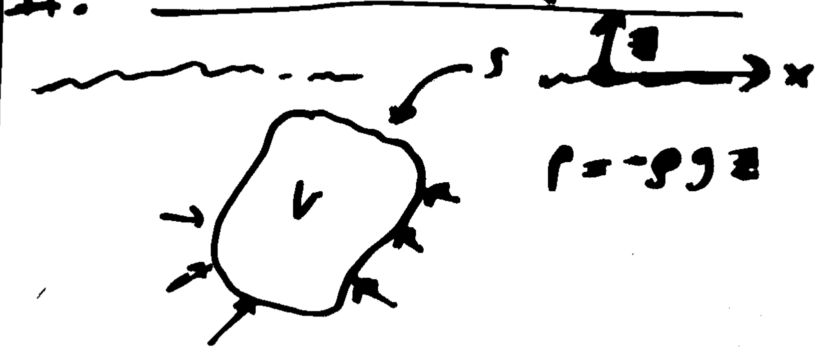
$\Sigma \underline{F} = \underline{0}$, $\Sigma \underline{M}|_0 = \underline{0}$

$\Rightarrow$ net effect of pressure

= a force at centroid pointing up w/ magnitude = weight of displaced fluid

($\equiv$ stat. Equiv. to)

II. Modern Calculus approach



$$\underline{F} = \int d\underline{F} = \int_S -\rho g z \, \underline{n} \, dS$$

Div. Thm. $\int_V \nabla \cdot \underline{F} \, dV = \int \underline{F} \cdot \underline{n} \, dS$

Simpler version of div. thm.: Gauss's Thm

$$\int_V \frac{\partial F_1(x,y,z)}{\partial x} \, dV = \int_S F_1 n_x \, dS'$$

$$\int_V \frac{\partial F_2}{\partial y} \, dV = \int_S F_2 n_y \, dS'$$

$$\int_V \frac{\partial F_3}{\partial z} \, dV = \int_S F_3 n_z \, dS'$$

(Gauss Thm. is like one "component" of div. thm.

For hydrostatics $F_1 = F_2 = F_3$ = pressure

$$\int_S -p \, (n_x \hat{i} + n_y \hat{j} + n_z \hat{k}) \, dS'$$

$+\rho g z = F = F_1 = F_2 = F_3$

$$= \int_V 0 + 0 + +\rho g \hat{k} \, dV$$

$$= \boxed{+\rho g V \hat{k} = F_{tot}}$$

3/14/03 (pg. 1)

TODAY : (Statics review)

All of Statics

0. You can draw FBD of any system or subsystem.

FBD = picture of syst. w/ all external forces showing (& no internal forces or "inertial" forces).

Laws of statics (for syst. in static equilib:

I A. $\sum \underline{F}_i = 0$ ← all ext.

B. $\sum \underline{M}_{i/c} = \underline{0}$ ← any pt. (all pts)

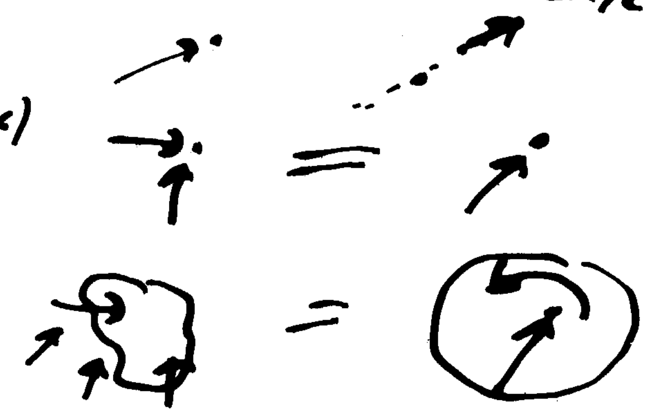
$\underline{M}_{i/c} = \underline{r}_{i/c} \times \underline{F}_i$

Observations:

Can, on one FBD replace a collection of forces w/ "equivalent" system

"equivalent" ≡ same $\sum \underline{F}_i$, same $\sum \underline{M}_{i/c}$

ex)



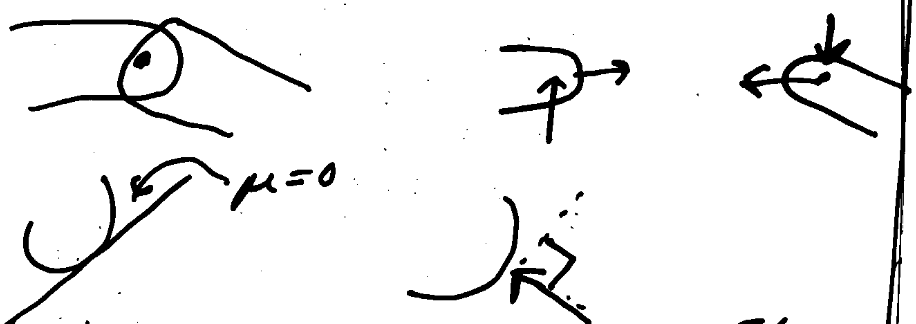
couple ≡ a representation of a force syst. equiv. to a couple (no net force) by a vector showing net moment.

3/14/03 ②
(cont'd)

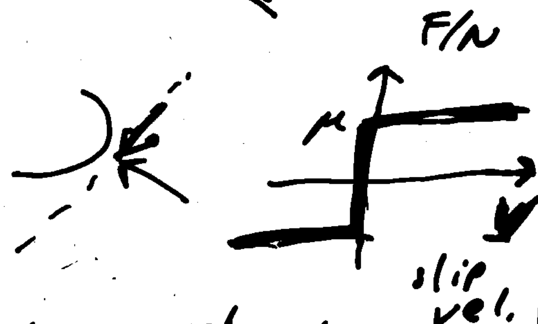
0. Principle of action & reaction.

if A causes $\underline{F}$ on B then
B " $-\underline{F}$ " A w/
same line of action.

Models of interactions and of things.



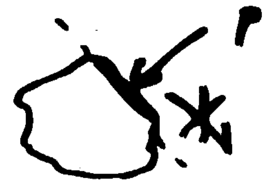
Friction

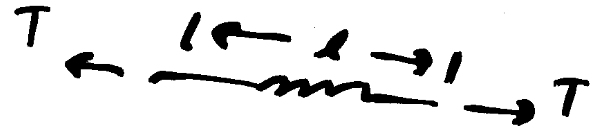


If a motion is caused or prevented a force or couple shows on FBD.

String

fluid



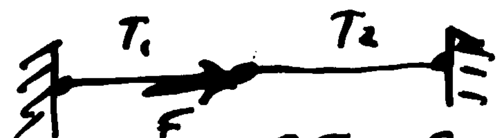


$$T = k(l - l_0)$$

STATICALLY DETERMINATE PROBLEMS.

Problems you can solve using laws of statics.

ex) Not statically determinate



T_1 & T_2 from
can't find statics

3/14 pg. 3

Need models of behavior (force/deformation relations) to solve statically indeterminate problems.

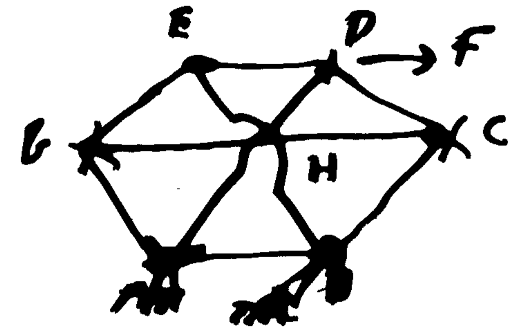
What problems are statically det.? No single precise answer.

$$\underbrace{\begin{bmatrix}\text{Known} \\ \text{coeff.}\end{bmatrix}\overset{\text{unknowns}}{\begin{bmatrix}\ \\ \ \end{bmatrix}} = \overset{\text{knowns}}{\begin{bmatrix}\ \\ \ \end{bmatrix}}}_{\text{eqs. are solvable}}$$

Rule of thumb:

eqs. = # unknowns

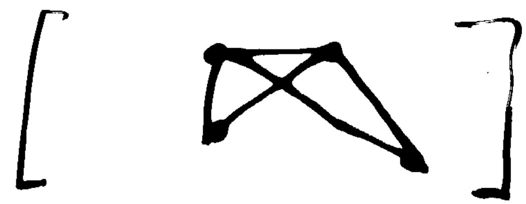
ex) difficult truss

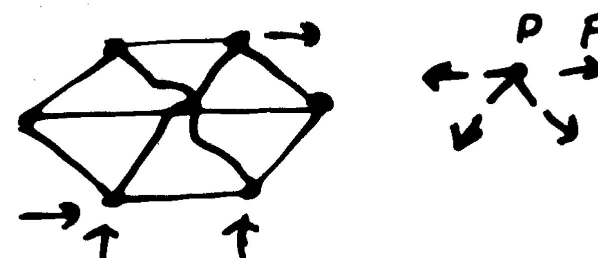


regular hexagon

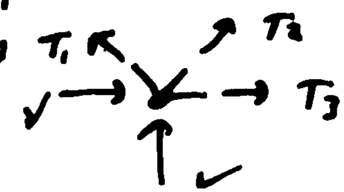
no connection at H

given F find all bar tensions





FBD of A



3/14 pt. 4

All joint eqs

2 × 6 eqs = 12 eqs.

12 unknowns

9 tensions
3 react. forces] 12 unknowns

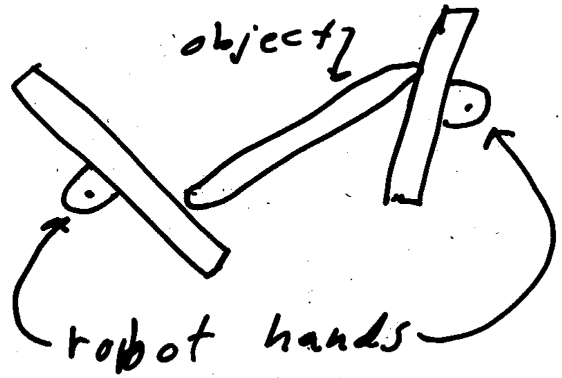
Need to set up matrix to solve.

Yet the matrix is singular (not solvable)!

3/24/03 ①

TODAY 1. Friction hints
2. Stress & strain

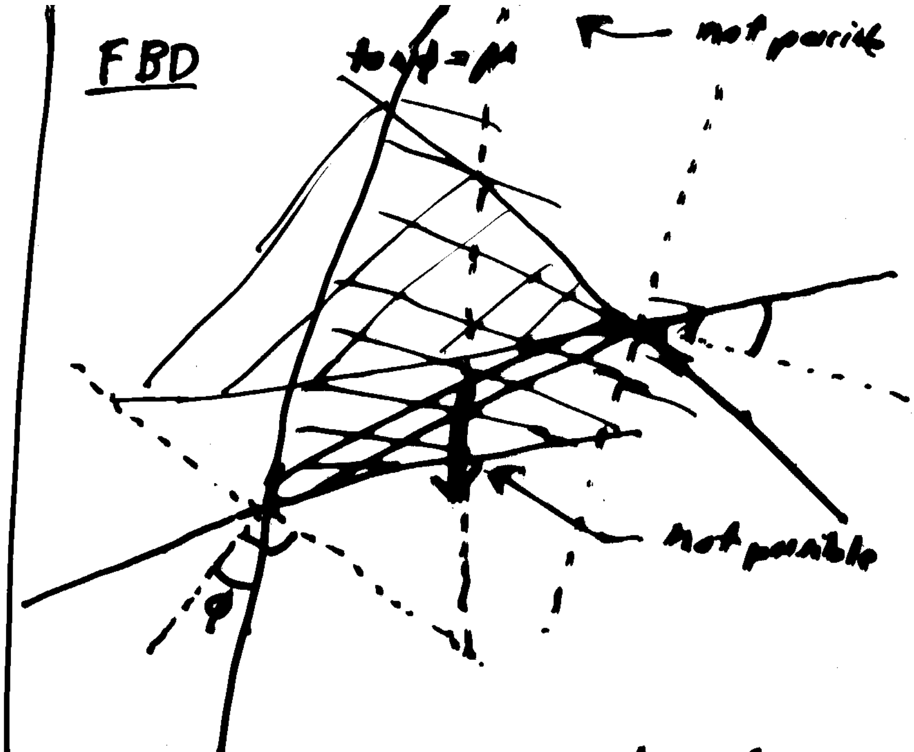
Friction problems.



Given a slow hand motion how does object move. (Assume statics.)

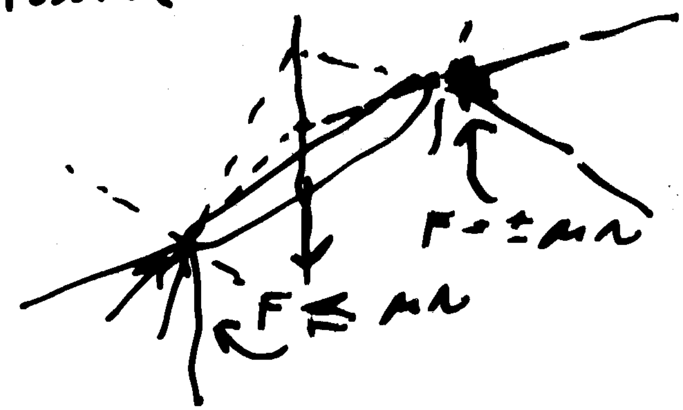
$$\boxed{\tan(\phi) \equiv \mu}$$

FBD



one of forces is on boundary of friction cone

Possible situations:

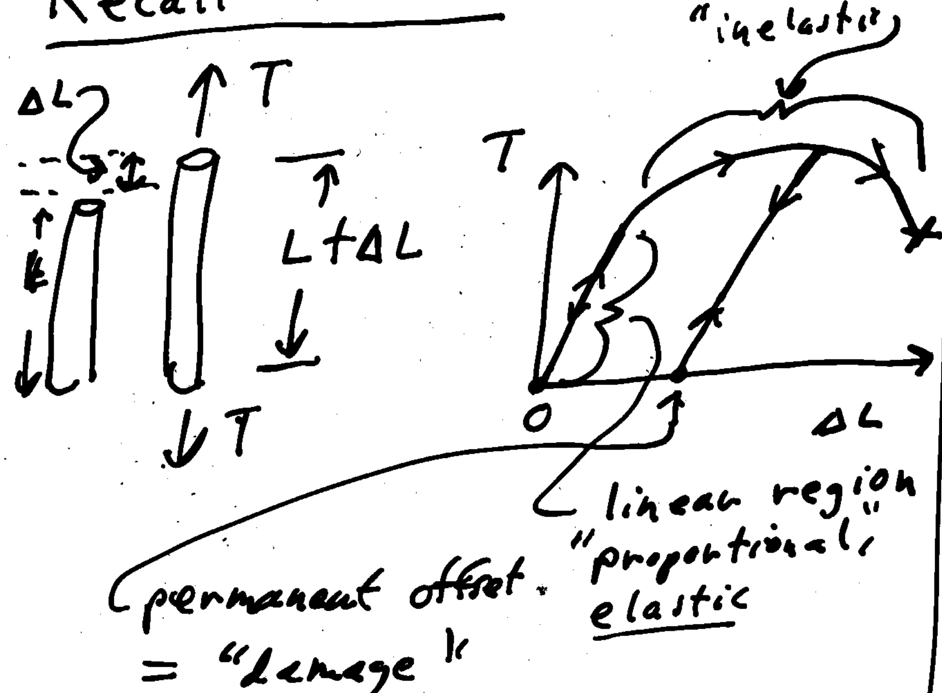


3/24 ②

STRESS & STRAIN (New changed world.)

[Steel is Jello (Agar Agar)]

Recall Lab 1 : tension test



Assume elastic (linear) for most structural analysis.

Linear regime

$$\boxed{\Delta L = \frac{TL}{AE}}$$

E = elastic modulus = Young's modulus

$$\frac{\Delta L}{L} = \frac{1}{E}\,\frac{T}{A}$$

$$\epsilon = \frac{1}{E}\,\sigma$$

ϵ = strain = measure of lengthening def. = "tension strain"

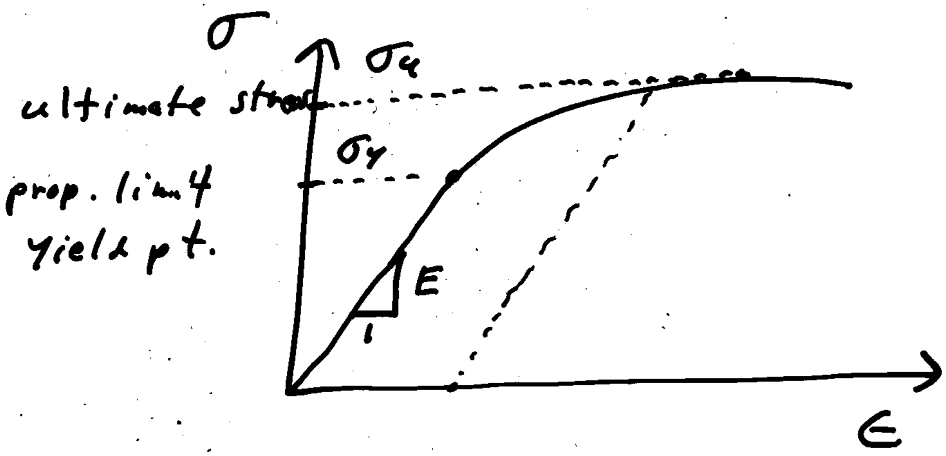
σ = stress = measure of force per unit area.

3/24/03

ϵ = "epsilon"
σ = "sigma"

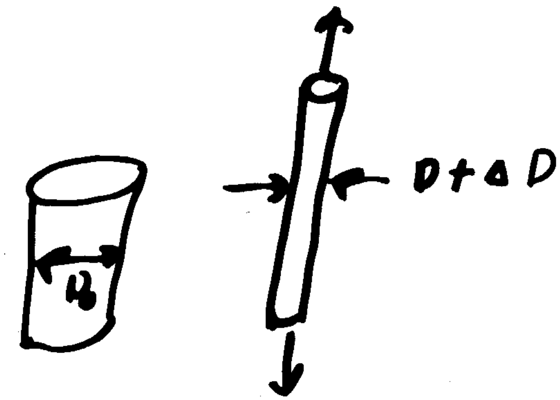
stress is <u>not</u> a synonym for strain in mechanics.

Redraw curve:



typically an engineer wants $\sigma < \sigma_y$ always.

<u>Poisson contraction</u>



ϵ_t = transverse strain

$\equiv \frac{\Delta D}{D_0}$

$\epsilon_t = -\nu \epsilon$

"Nu" = Poisson's ratio

$\nu = -\epsilon_t / \epsilon$

3/24/03 ①

Some numbers:

$E = \frac{\sigma}{\epsilon}$ in linear prop. regime

$= \left[\begin{array}{l} \left. \begin{array}{ll} 200 \times 10^9 & N/m^2 \\ 30 \times 10^6 & lbf/in^2 \end{array} \right] \text{steel} \\ \left. \begin{array}{ll} 70 \times 10^9 & N/m^2 \\ 10 \times 10^6 & lbf/in^2 \end{array} \right] \text{Al} \end{array} \right.$

$\sigma_y = \left[\begin{array}{l} \left. \begin{array}{l} 200 \times 10^6 \; N/m^2 \\ 30 \times 10^3 \; lbf/in^2 \end{array} \right] \text{weak steel} \\ \left. \begin{array}{l} 200 \times 10^7 \; N/m^2 \\ 30 \times 10^4 \; lbf/in^2 \end{array} \right] \text{strongest steel} \end{array} \right.$

Note: ϵ at yield is typically $\underbrace{.1\%}_{.001} \le \epsilon_y \le \underbrace{1\%}_{.01}$

σ_u usually not much bigger than σ_y (usually 10 - 20% but up to 3x)

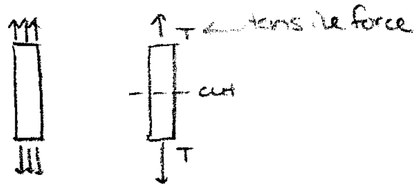
$0 \le \nu \le .5$
typically

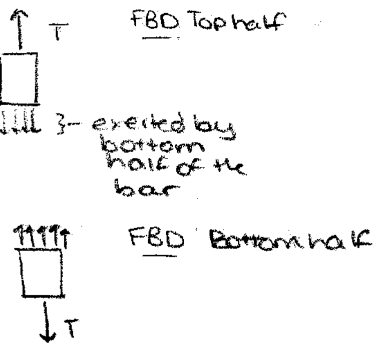
$-1 \le \nu \le .5$

Lecture given by: Phoebus Rosakis
Notes taken by: Bina Lokchander

3-26-03
202 lec

Stress - force per unit area
↳ tensile stress

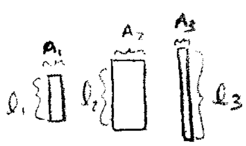




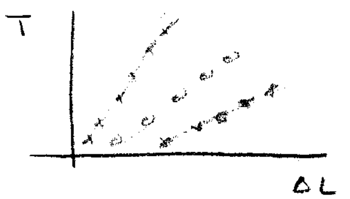
Stress - force (T) distributed over area (A)
$\sigma = T/A$ Normal stress
Force ÷ Area

units: N/m^2 = Pascal
psi

$\sigma > 0$ tension
$\sigma < 0$ compression



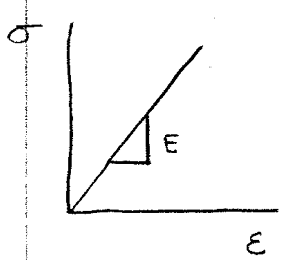
(same material)



how to get relationship that is independent of strain?

strain → $\varepsilon = \frac{\Delta \ell}{\ell}$
stress → $\sigma = \frac{T}{A}$

if plot these against eachother, the data points of will collapse together

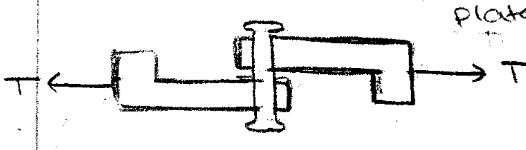


Hooke's Law - Linear relationship btw normal stress + normal strain
$\sigma = E\varepsilon$

* But tangential forces also exist.
 - when forces (V) are tangential over an area (A)

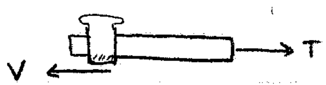
Shear stress
$\tau = \frac{V}{A}$

Ex.



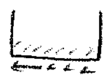
plates held together by rivet

Top half



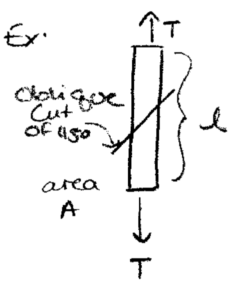
for equilibrium V must $= T$

distributed tangential force - which can vary along surface.



$\tau = \frac{V}{A}$

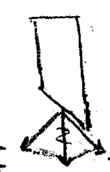
Ex.



— because of this cut + surface it has both tangential + normal stress.

∴ Resolve T into components of shear + normal to surface.

↓



turns out oblique Area (A') is greater $= A(\sqrt{2})$

$T_n = \frac{T}{\sqrt{2}}$ $\quad T \quad$ $T/\sqrt{2} = T_t$

On this cut

normal stress $\sigma = \frac{T_n}{A'} = \frac{T/\sqrt{2}}{A\sqrt{2}} = \frac{T}{2A}$

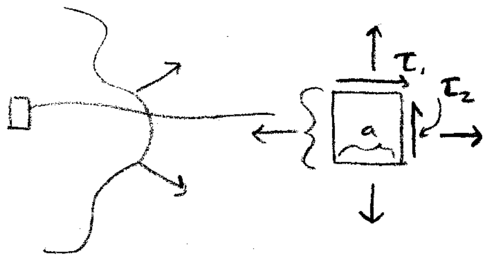
shear stress $\tau = \frac{T_t}{A'} = \frac{T}{2A}$

State of stress on a material surface depends on forces and orientation of surface

3-26-03

202 Lec

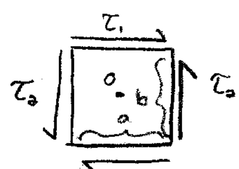
General State of Stress (2D)



shear stress

what is relation btw τ_1 + τ_2?

If No Normal Stess



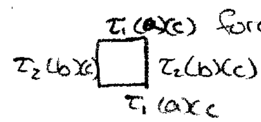
thickness is 'c'

$\Rightarrow$ relationship btw τ_1 + τ_2?

Moment Balance about o

FBD

forces = stresses * areas

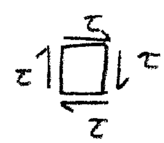


$$\Sigma M_{/o} = -2\tau_1(a)(c)\left(\frac{b}{2}\right) + \cancel{2}\left(\frac{a}{\cancel{2}}\right)(\tau_2)(b)(c) = 0$$

$$(-\tau_1 + \tau_2)abc = 0$$

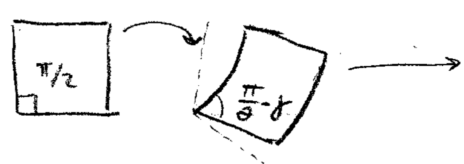
$$\tau_1 = \tau_2$$

☆ shear stress acting on ⊥ faces must be same



not a scalar or vector, stress turns out to be a matrix. [294 - shear transformation etc]

response to shear stress is a shape change



$\longrightarrow$ angle change is known as <u>shear strain</u> $\rightarrow \gamma$

Hooke's Law for Shear

$$\tau = G\gamma$$

G ↑ Shear Modulus

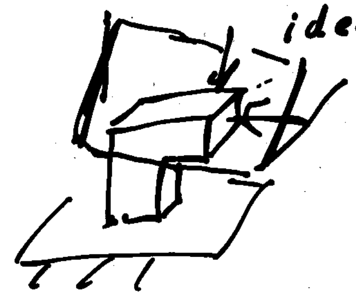
(μ) Poisson's Ratio, (E) Young's Mod, (G) Shear Mod] – not indep of each other. relation exists

3/28/01

TODAY: More stress & strain

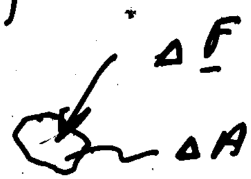
stress: Force per unit area on a surface exposed by a FBD cut. Usually inside some solid (same idea works for fluids)

cut plane



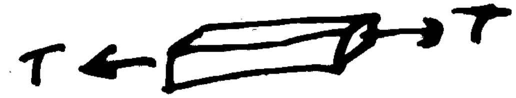
$\underline{n}$ = unit vector $\perp$ to surface pointing out

ΔA = a little bit of area $= (\Delta \ell)^2$



The word "stress" something used to describe:

<u>a scalar</u>: $\sigma = \frac{T}{A}$ = tension stress



in this context σ is a scalar like "tension" is a scalar.

<u>the stress vector</u>:

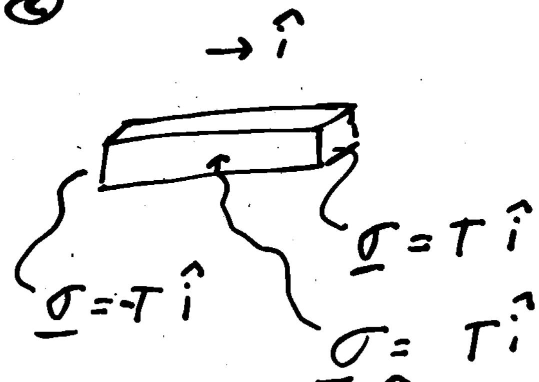
$$\underline{\sigma} = \frac{\Delta \underline{F}}{\Delta A}$$

for small ΔA "$\Delta A \to 0$"

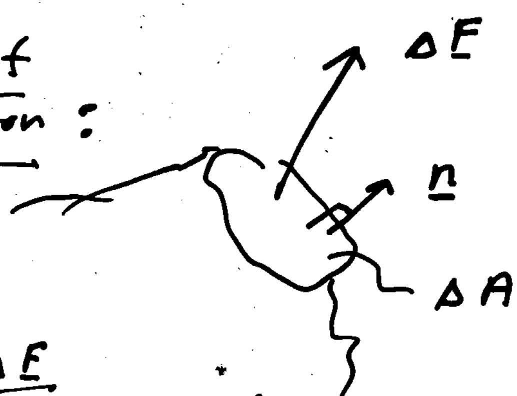
$\underline{\sigma}$ will depend on where in solid &

3/28/03 ⑥

ex)



<u>a component of stress vector</u>:



$$\underline{\sigma} = \frac{\Delta \underline{F}}{\Delta A}$$

σ = normal stress

= tension stress

= comp of $\underline{\sigma}$ in $\underline{n}$ dir.

= $\underline{\sigma} \cdot \underline{n}$

τ = shear stress

= comp of $\underline{\sigma}$ tangent to surface

$$= \begin{cases} |\underline{\sigma} - \sigma \underline{n}| & 3D \\ \underline{\sigma} \cdot \underline{e}_t & 2D \end{cases}$$

↑ unit tangent

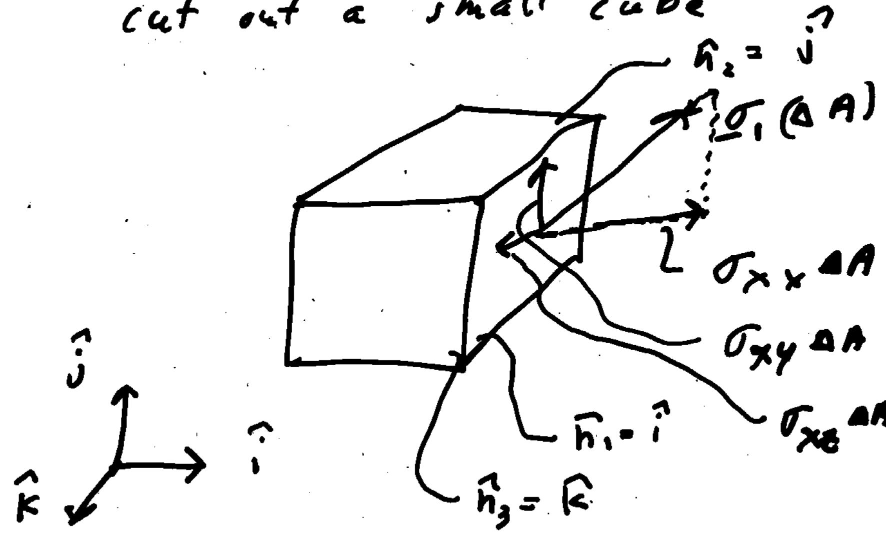
<u>The stress matrix</u>: 3×3

comps of $\underline{\sigma}$ for

$\underline{n} = \underline{e}_1, \underline{e}_2, \underline{e}_3$

3/28/03 (3)

cut out a small cube



$\underline{\sigma}$ on $\underline{n}_1 = \hat{i}$ face

$$\underline{\sigma} = \sigma_{xx}\,\hat{i} + \sigma_{xy}\,\hat{j} + \sigma_{xz}\,\hat{k}$$

Likewise for other 2 surfaces

$$[\underline{\sigma}] \text{ matrix} = \begin{bmatrix} \sigma_{xx} & \sigma_{yx} & \sigma_{zx} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{zy} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{bmatrix}$$

comps of $\underline{\sigma}$ vector for $\underline{n} = \hat{i}$

<u>stress tensor</u>:

<u>Explanation 1</u>: Huh?

<u>Expl. 2</u>: Forget about it you'll learn in courses like TAM 610, 663 etc.

<u>Expl. 3</u>: Analogy

$\underline{\underline{\sigma}}$ tensor is to $[\underline{\underline{\sigma}}]$ like

3/28/03 (4)

$\underline{V}$ is to its list of components.

<u>expl. 4</u>: $\underline{\underline{\sigma}}$ is a function

f : $f(\underline{n}) = \underline{\sigma}$

that has $\underline{n}$ as input and $\underline{\sigma}$ as output. [A linear function. "A linear operator"]

Aside on "continuum mechanics"

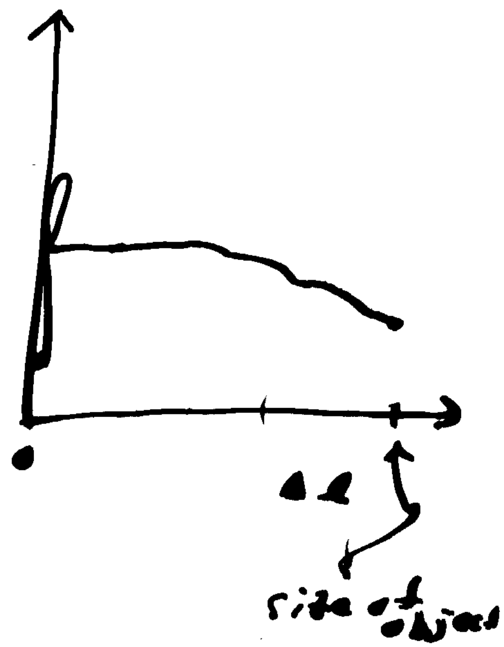
$$\underline{\sigma} = \frac{\Delta \underline{F}}{(\Delta l)^2}$$

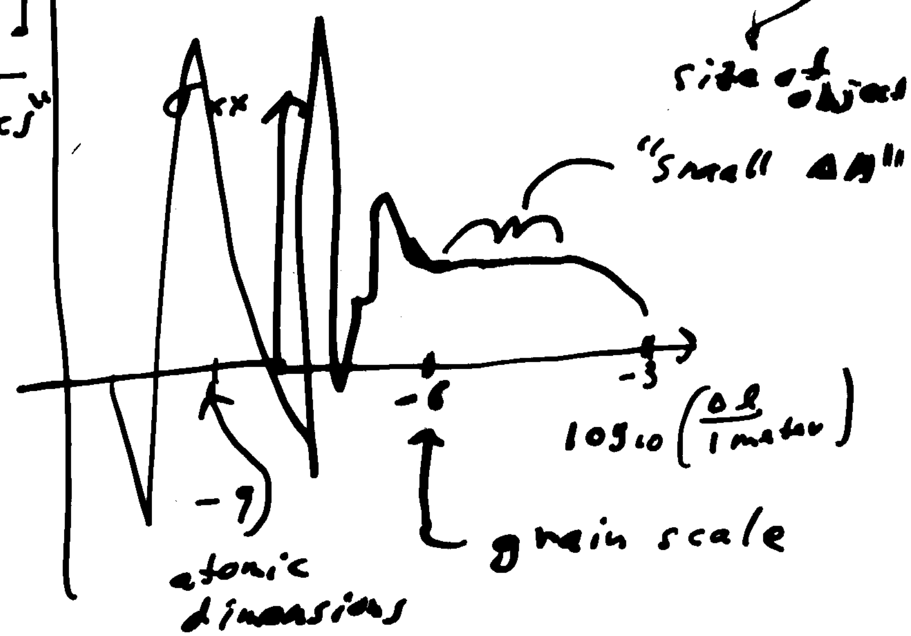
└ small Δl
small $\Delta A = \Delta l^2$

At some point in space inside a solid. (paperclip)

σ_{xx} $\frac{\Delta \underline{F}}{\Delta A} \cdot \hat{i}$

(fix $\underline{n}$) $= \hat{i}$





3/28/03 ⑤

Back to reality

Recall: Big & small numbers

March 31, 03 ① A. Poisson's ratio

TODAY: B. Collections of bars in tension

A. Poisson's ratio

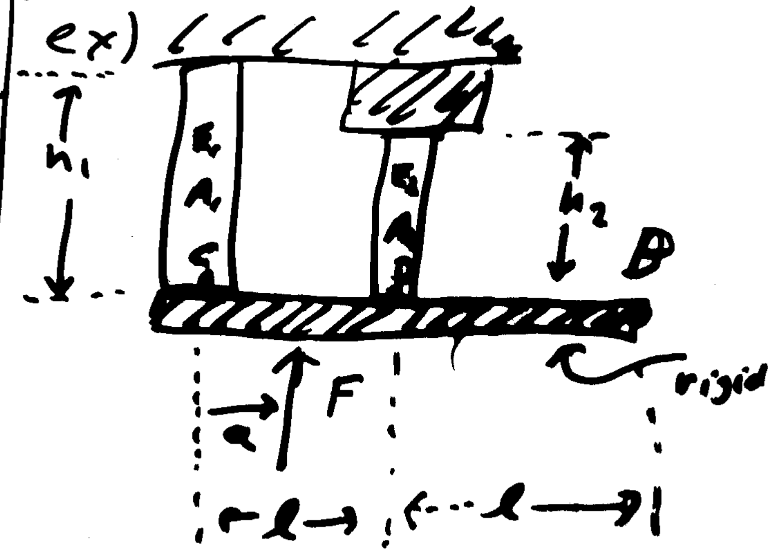
ex) Metals $\nu \approx .2$ or $.3$

Rubber, Jello $\nu \approx .5$ or $.49$

(.5 = ν corresponds to const. volume

Recall $\nu = -\dfrac{\epsilon_t}{\epsilon_\ell}$

ex)



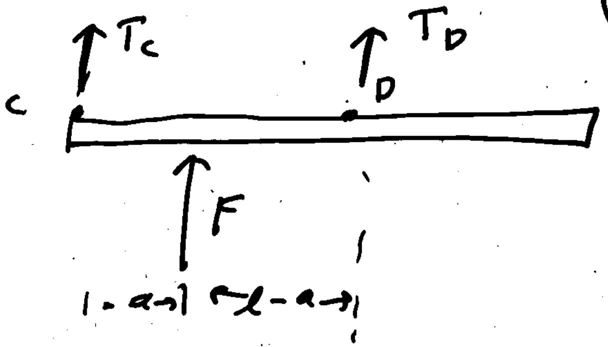
Given: $h_1, h_2, E_1, E_2, A_1, A_2, F, \ell, a$

Find deflection of pt. B.

Treat C & D as hinges.

3/31/03 ①

FBD of ''CDB (Σ ICDB Zzz)''



Sign Convention: Tension is positive

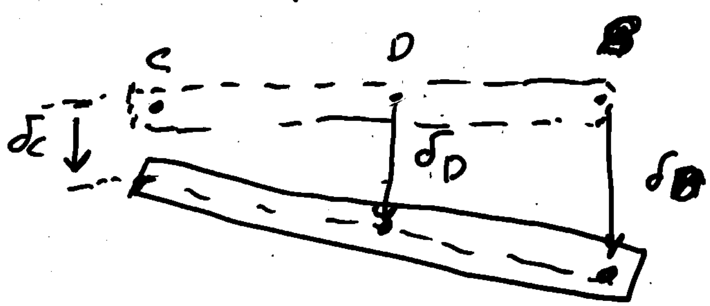
$\Sigma M = 0,\ \Sigma F_i = 0 \Rightarrow$

$$\left.\begin{aligned} T_D &= \frac{-a}{\ell} F \\ T_C &= \frac{-(\ell - a)}{\ell} F \end{aligned}\right] \quad (1)$$

Kinematics, Geometry

Assume small slopes $\Rightarrow$ neglect horiz. displacements

Increase in length is positive



slope = slope

$$\frac{\delta_D - \delta_C}{\cancel{\ell}} = \frac{\delta_B - \delta_D}{\cancel{\ell}}$$

$$\delta_A = 2\delta_D - \delta_C \quad (2)$$

Mech. Properties, Constit. Laws

bar C $\quad$ "$\delta = \dfrac{T\ell}{AE}$" $\quad \ell = h_1, h_2$

$$\epsilon_D = \frac{\sigma_D}{E}$$

$$\frac{\delta_D}{h_2} = \frac{\frac{-a}{\ell} F / A_2}{E_1}$$

$$\delta_D = \cancel{\frac{\quad}{E_1 A}} = \frac{-a\, h_2}{\ell\, A_2 E_2} F$$

$$\delta_D = \frac{-a\, h_2}{\ell\, A_2 E_2} F$$

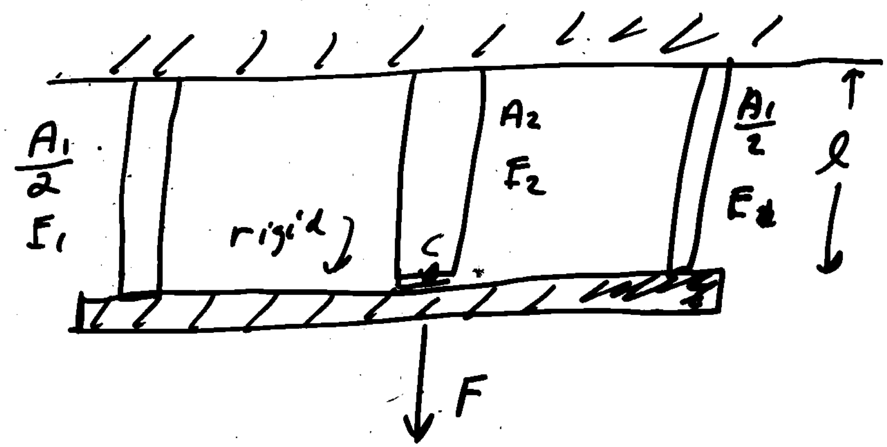
3/31/03 (3)

$$\delta_c = \frac{-(l-a)\,h_1}{l A_1 E_1} F$$

(3) apply to (2)

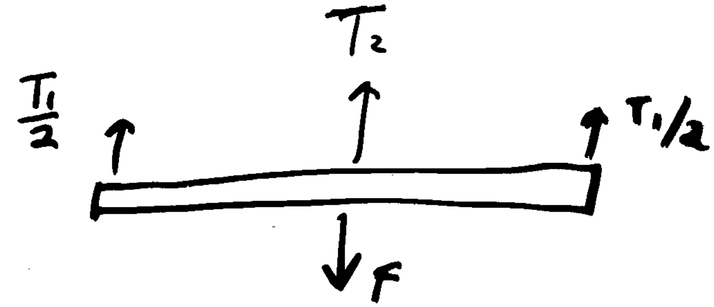
$\Rightarrow$

$\delta_A =$ mess - - - -
involving things we know



Given F, A_1, E_1, A_2, E, l
Find δ_c.
Symmetry $\Rightarrow$ no rotation

Mechanical



$\Sigma \underline{F}_i = \underline{0}$, $\Sigma M = 0 \Rightarrow$

$$T_1 + T_2 = F \qquad (1)$$

can't find T_1 & T_2 from statics alone $\Rightarrow$
statically indeterminate

3/31/03 (4)

Kinematics / Geometry

$$\delta_1 = \delta_2 \qquad (2)$$

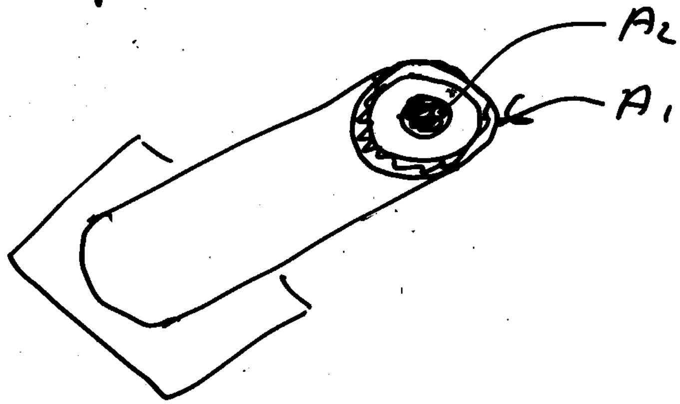
Material Properties

$$T_1 = \frac{E_1 A_1 \delta_1}{l} \qquad (3)$$

$$T_2 = \frac{E_2 A_2 \delta_2}{l}$$

(1), (2), (3) to get answer.

Graphical derivation



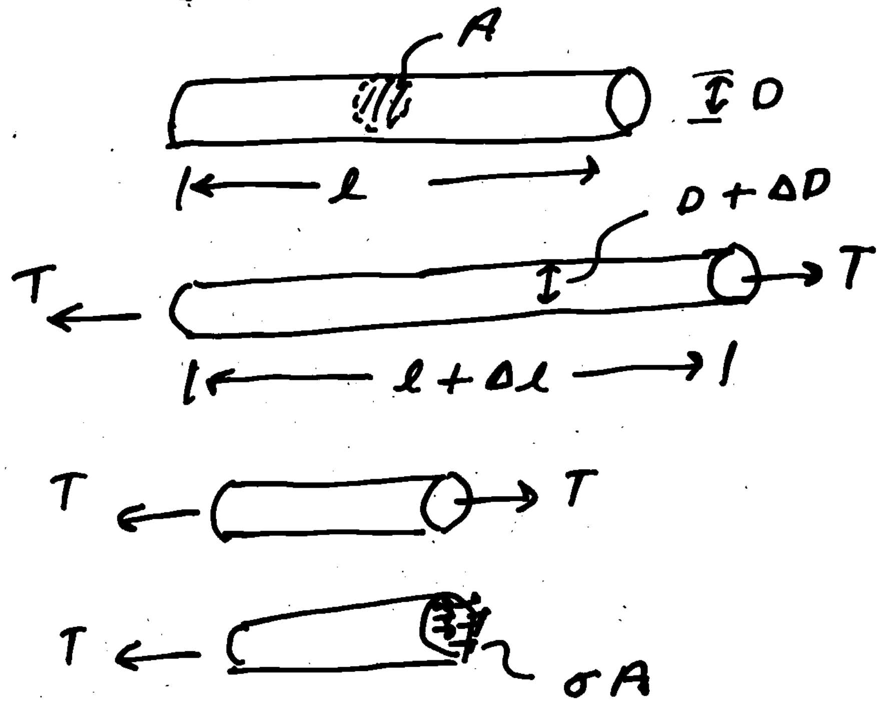
$$F = \underbrace{T_1}_{\frac{E_1 A_1 \delta}{l}} + \underbrace{T_2}_{\frac{E_2 A_2 \delta}{l}}$$

$$F = \left[\frac{E_1 A_1}{l} + \frac{E_2 A_2}{l}\right] \delta$$

$$\boxed{\delta = \frac{F}{[\cdot\,-\,-\,-\,-\,-]}}$$

$\boxed{4/02/03}$ ①

TODAY: More tension



T = tension in bar (pos. if pulling on ends)

A = cross section area

σ = tension stress
$\equiv T/A$ (same sign convention as for T)

$\epsilon = \epsilon_\ell$ = elongation strain
$\equiv \frac{\Delta\ell}{\ell}$

$\Delta\ell$ = change in length
increase

$\boxed{\sigma = E\,\epsilon}$ Linear Elastic constitutive law

"Young"

$\boxed{T = \frac{EA}{\ell}\,\Delta\ell}$ "$T = k\,\delta$"

$\left(\epsilon = \frac{1}{E}\sigma,\ \Delta\ell = \frac{\ell}{EA}T\right)$

Think of rod as a spring w/ spring constant

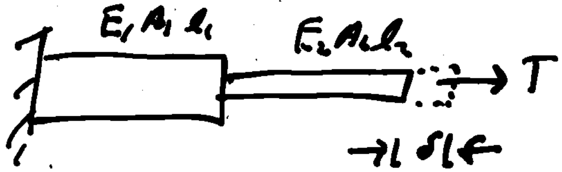
$\boxed{k = \frac{EA}{\ell}}$

$\frac{\Delta D}{D} = -\nu \frac{\Delta\ell}{\ell}$ $\boxed{\nu \equiv \frac{-\epsilon_t}{\epsilon_\ell}}$

Poisson's ratio

4/2/03 (2)

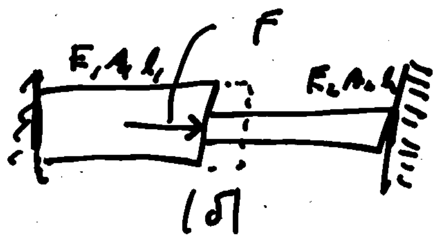
ex 1)



compare

given δ find T

ex 2)



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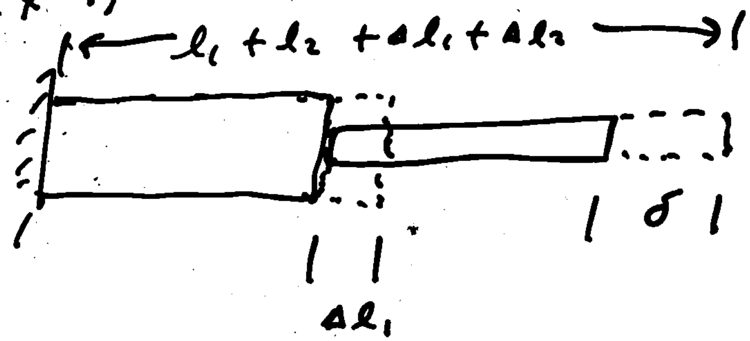
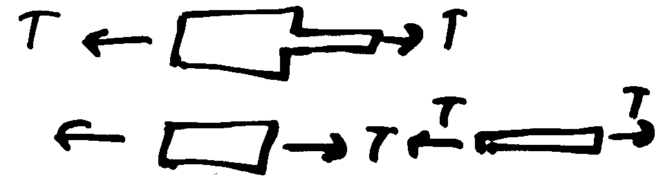
ex 1)



FBDs



Geometry

$$\delta = \overbrace{\delta_1}^{\Delta l_1} + \overbrace{\delta_2}^{\Delta l_2}$$

δ_2: $\dfrac{T l_2}{A_2 E_2}$

δ_1: $T l_1 / A_1 E_1$

$$\delta = \Big[\underbrace{\frac{l_1}{A_1 E_1}}_{1/k_1} + \underbrace{\frac{l_2}{A_2 E_2}}_{1/k_2}\Big] T$$

$$T = \frac{\delta}{[l_1/E_1 A_1 + l_2/E_2 A_2]}$$

$$\boxed{K = \frac{1}{1/k_1 + 1/k_2}}$$

4/2/03 ③

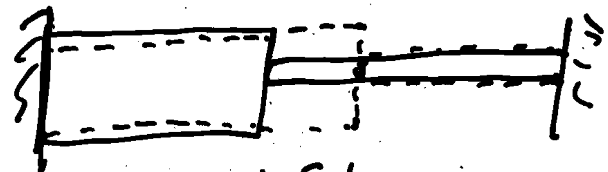
$$\Rightarrow \delta_1 = \frac{T l_1}{A_1 E_1}$$

$$= \frac{l_1 / A_1 E_1}{[\;\cdots\cdots\;]}\,\delta$$

$$\delta_2 = \ldots$$

---

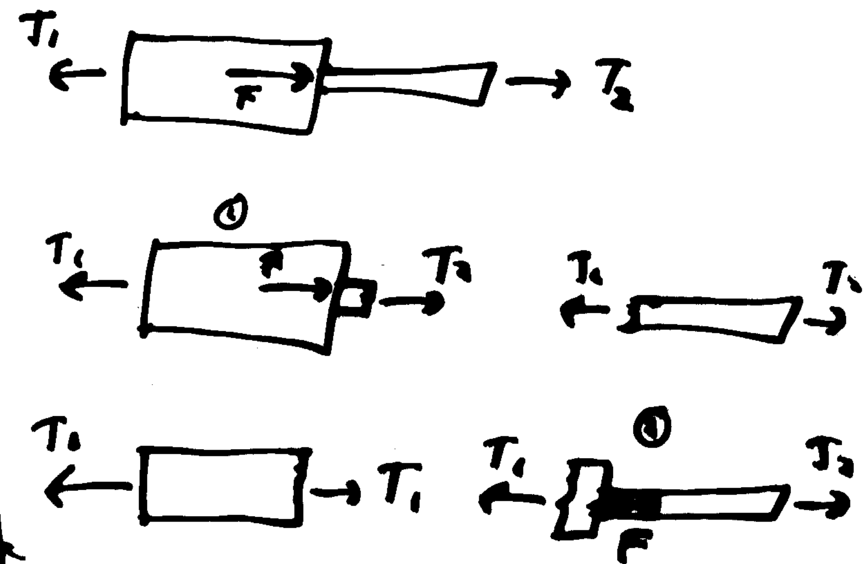
ex 2)



Geometry : $\boxed{\delta_1 = -\delta_2}$

Assume : $T_1 = T_2 = 0$ when $F = 0$

("No prestress"



FBDs ① or ②

$$\Rightarrow F + T_2 - T_1 = 0$$

$$T_1 = \delta_1 \frac{E_1 A_1}{l_1}, \quad T_2 = \delta_2 \frac{E_2 A_2}{l_2}$$

$$F + \delta_2 \frac{E_2 A_2}{l_2} + \delta_2 \frac{E_1 A_1}{l_1} = 0$$

$$\boxed{\delta_2 = -\frac{F}{\left[\frac{E_1 A_1}{l_1} + \frac{E_2 A_2}{l_2}\right]}}$$

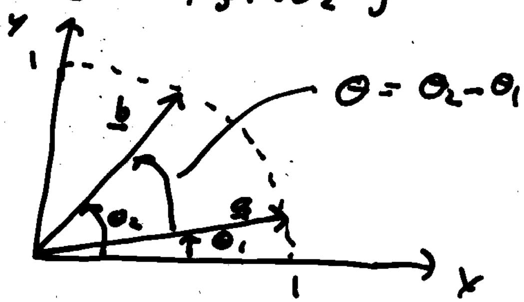
4/7/03 © (Ruina not Hui)

<u>TODAY</u> 1) Trig. add. formulas
2) Stress on inclined sections

TRIG. ASIDE

Consider 2 unit vectors $\underline{a}$ & $\underline{b}$

$$\underline{a} = \cos\theta_1 \hat{i} + \sin\theta_1 \hat{j}$$
$$\underline{b} = \cos\theta_2 \hat{i} + \sin\theta_2 \hat{j}$$



1)
$$\underline{a} \cdot \underline{b} = \underline{a} \cdot \underline{b}$$
$$\underbrace{|\underline{a}|}_{1}\underbrace{|\underline{b}|}_{1} \cos\underbrace{\theta}_{\theta_2-\theta_1} = \cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2$$

$$\boxed{\begin{aligned}\cos(\theta_2-\theta_1) &= \cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2 \\ \cos(\theta_2+\theta_1) &= \cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2\end{aligned}}$$

"addition formula for cosines"

2)
$$\underline{a} \times \underline{b} = \underline{a} \times \underline{b}$$
$$\left\{\underbrace{|\underline{a}|}_{1}\underbrace{|\underline{b}|}_{1} \sin\underbrace{\theta}_{\theta_2-\theta_1} \hat{k} = (\cos\theta_1 \sin\theta_2 - \cos\theta_2 \sin\theta_1)\hat{k}\right\}$$
$$\{\ \} \cdot \hat{k} =$$

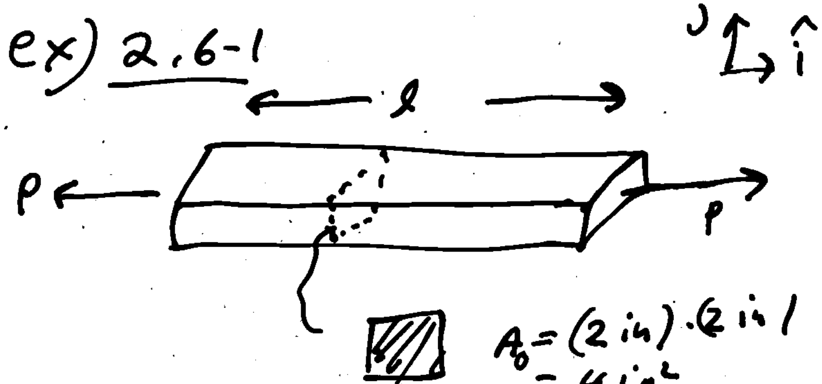
$$\boxed{\begin{aligned}\sin(\theta_2-\theta_1) &= \cos\theta_1 \sin\theta_2 - \cos\theta_2 \sin\theta_1 \\ \sin(\theta_2+\theta_1) &= \cos\theta_2 \sin\theta_1 + \cos\theta_1 \sin\theta_2\end{aligned}}$$

"addition formula for sines"

4/7/01 ②

Stresses on inclined sections

ex) 2.6-1



$A_0 = (2\text{ in})\cdot(2\text{ in}) = 4\text{ in}^2$

Material strength

$\sigma_{ult} = 16000$ psi (lbf/in²)

$\tau_{ult} = 9000$ psi

MAX value of P to prevent failure?

Look at tension stress first:

$\sigma = \frac{P}{A} \le \sigma_{ult}$

$P \le \sigma_{ult} A$

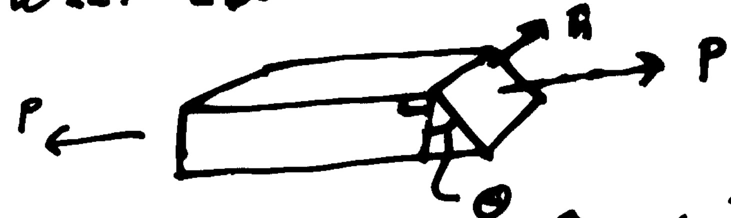
⇒ To avoid failure from σ_{ult} ⇒ $P \le \sigma_{ult} A$

⇒ $P \le 16000\left(\frac{\text{lbf}}{\text{in}^2}\right)(4\text{ in}^2)$

$\boxed{P \le 64 \times 10^3 \text{ lbf.}}$

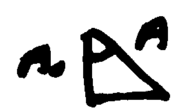
Hey! no shear on surfaces w/ normal in î direction.

What about crooked surfaces?



$\hat{n} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j} \ne \hat{i}$

Area of surface:



$\frac{A_0}{A} = \cos\theta$

$A = \frac{A_0}{\cos\theta}$

$\left[\begin{array}{ll} A_0 & \theta = 0 \\ \infty & \theta = \pi/2 \end{array}\right.$

4/7/03 (3)

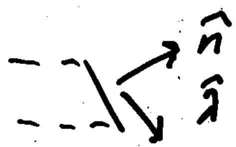
stress vector $\bar{\sigma} = \dfrac{P\,\hat{i}}{A}$

$\dfrac{\text{force}}{\text{area}}$ on cut surface $= \dfrac{P\cos\theta}{A_0}\,\hat{i}$

Want to look at normal & shear comps.

$$\underline{\sigma} = \sigma_n\,\hat{n} + \tau\,\hat{\lambda}$$

$\hat{\lambda}$ ← a unit vector tangent to surface



(See text for FBD approach)

$$\underline{\sigma} = \underline{\sigma}$$

$$\underline{\sigma}\cdot\hat{n} = \underline{\sigma}\cdot\hat{n}$$

$$(\sigma_n\,\hat{n} + \tau\,\hat{\lambda})\cdot\hat{n} = \left(\frac{P\cos\theta}{A_0}\,\hat{i}\right)\cdot\hat{n}$$

$$\sigma_n = \frac{P\cos\theta}{A_0}\underbrace{\cos\theta}_{\hat{i}\cdot\hat{n}}$$

$$\boxed{\sigma_n = \frac{P}{A_0}\cos^2\theta}$$

note σ_n is max for $\theta = 0 \Rightarrow$ answer used before for tension stress

$$\underline{\sigma}\cdot\hat{\lambda} = \underline{\sigma}\cdot\hat{\lambda}$$

$$(\sigma_n\,\hat{n} + \tau\,\hat{\lambda})\cdot\hat{\lambda} = \left(\frac{P}{A_0}\cos\theta\,\hat{i}\right)\cdot\hat{\lambda}$$

$$\tau = \frac{P}{A_0}\cos\theta\,\underbrace{\hat{i}\cdot\hat{\lambda}}_{\sin\theta}$$

$$\boxed{\tau = \frac{P}{A_0}\cos\theta\sin\theta}$$

4/7/03 ①

Can use our trig. formulas

$$\sin\theta\cos\theta = \frac{1}{2}\Big[\underbrace{\sin\theta\cos\theta + \sin\theta\cos\theta}_{\sin(\theta+\theta)}\Big]$$

$$= \frac{\sin 2\theta}{2}$$

$$\boxed{\tau = \frac{P}{A_0}\cos\theta\sin\theta = \frac{P}{A_0}\,\frac{\sin 2\theta}{2}}$$

$\sigma = \ldots\ldots = \left(\frac{1}{2} + \frac{\cos 2\theta}{2}\right)\frac{P}{A_0}$

↑ see text

$$\cos^2\theta = \frac{\cos^2\theta}{2} + \frac{\cos^2\theta}{2} + \frac{\sin^2\theta}{2} - \frac{\sin^2\theta}{2}$$

$$= \frac{(\cos^2 + \sin^2)}{2} + \frac{\cos^2 - \sin^2}{2}$$

Back to problem at hand.

Shear stress is biggest when $\frac{\sin(2\theta)}{2}$ biggest

$(\sin(2\theta) = 1)$

$\Rightarrow \theta = \pi/4$

Shear biggest on surfaces at 45° from tension dir.

$$\tau_{max} = \frac{P}{2A_0}$$

$$\tau_{max} \le \tau_{ult}$$

$$\frac{P}{2A_0} \le 9000 \text{ psi}$$

$$P \le \left(9000 \frac{\text{lbf}}{\text{in}^2}\right) 8 \text{ in}^2$$

$$\le 72000 \text{ lbf}$$

2 answers

1) $P \le 64000$ lbf

2) $P \le 72000$ lbf

take smaller

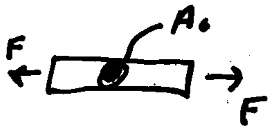
$\Rightarrow \boxed{P \le 64000 \text{ lbf}}$

4/9/03 ①

TODAY: a) Tension b) Torsion

TENSION (CONT'D)

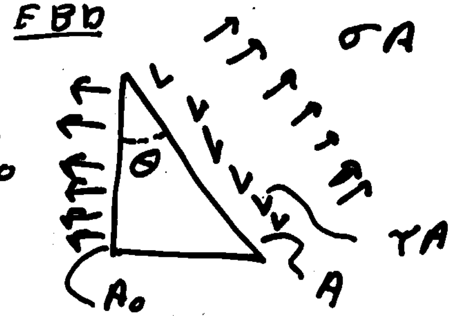
Recall

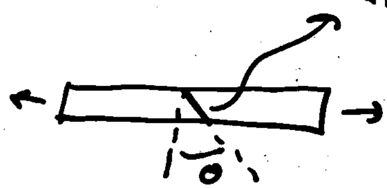


$\sigma_0 A_0$

$\sigma_0 = \frac{F}{A_0}$

EBD





$\Sigma F_{\sigma\,dir} = 0 \Rightarrow -\sigma_0 A_0 \cos\theta + \sigma \underbrace{A}_{A\cos\theta = A_0} = 0$

$\Rightarrow \boxed{\sigma = \sigma_0 \cos^2\theta}$

$\sigma = \frac{1}{2}(1+\cos 2\theta)\sigma_0$

$\Sigma F_{\tau\,dir} = 0 \Rightarrow -\sigma_0 A_0 \sin\theta + \tau A = 0$

$\Rightarrow \boxed{\tau = \sigma_0 \sin\theta\cos\theta}$

$= \sigma_0 \sin 2\theta$

②

Why interesting?

A. Might want to know surface w/ biggest σ $(\theta = 0)$ or biggest τ $(\theta = \pi/4)$ ⟵▭⟶

B. For a weld or glue joint σ_{max} &/or τ_{max} may be specified for a given θ.

4/09/03 (2)

TORSION

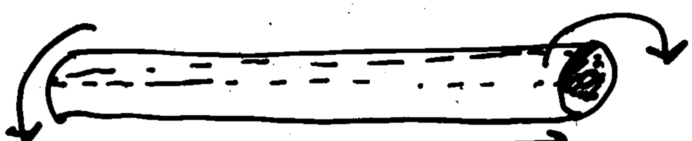
WARNING!! For round bars only!! If you care about not-round bars it is incorrect to use the formulas below by just recalculating J!!!

TORSION OF ROUND BARS

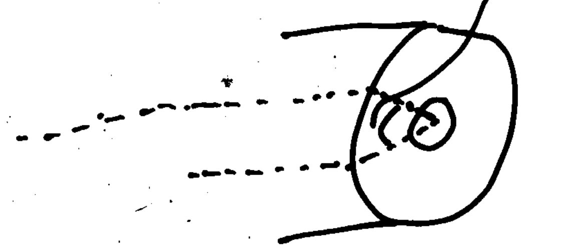
(possibly hollow)

T = torque

T



ℓ



ϕ

ϕ = net rotation of one end rel. to other

(3)

$$\phi = \frac{T\ell}{G(\text{some measure of fatness})}$$

σ

ϵ

material properties

G = the shear modulus

- $T = K_{torsion}\,\phi$

$$K_{torsion} = \frac{G \cdot (\text{fatness})}{\ell}$$

What about stress?

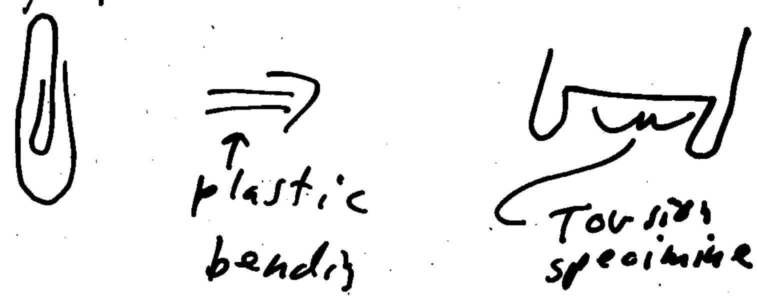
$$\tau = \frac{T \cdot (\text{where in bar})}{(\text{fatness})}$$

4/16/03 ①

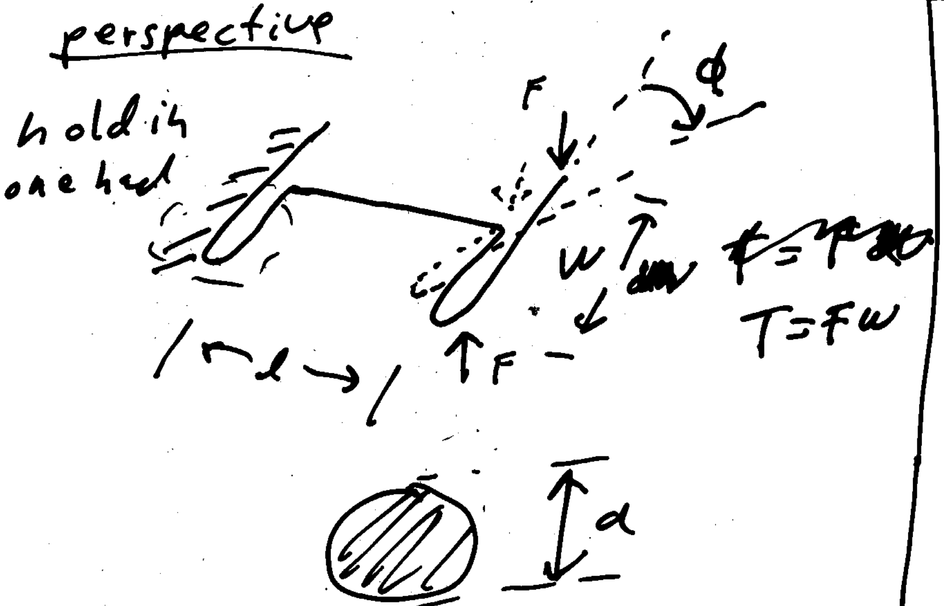
TODAY: TORSION

TORSION

ex/ paperclip



perspective



question:

Given l, d, material, w Find F to twist a given ϕ.

Recall from last lecture

$$T = \frac{\cancel{(\text{proportionality factor})}}{l}\,\phi$$

↑ common sense reasoning

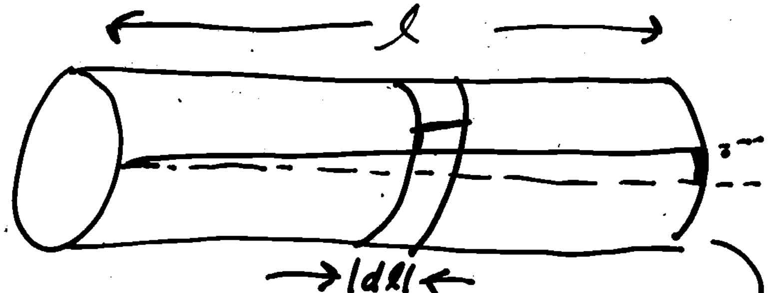
Torsion Theory

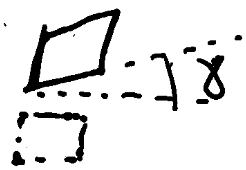
(Geometry, Mat. props, Mechanics)

Geometry

Assume deformation is like a bunch of stacked pennies each rotated slightly compared to neighbor.

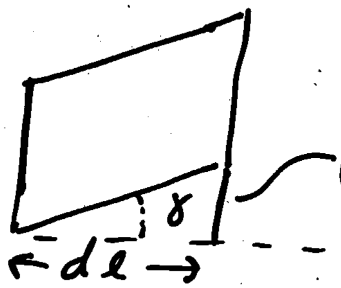
4/11/83 5





rectangle drawn on outside

"rho"



$(d\phi)\rho$

$d\phi = \frac{\phi}{l}\, dl$

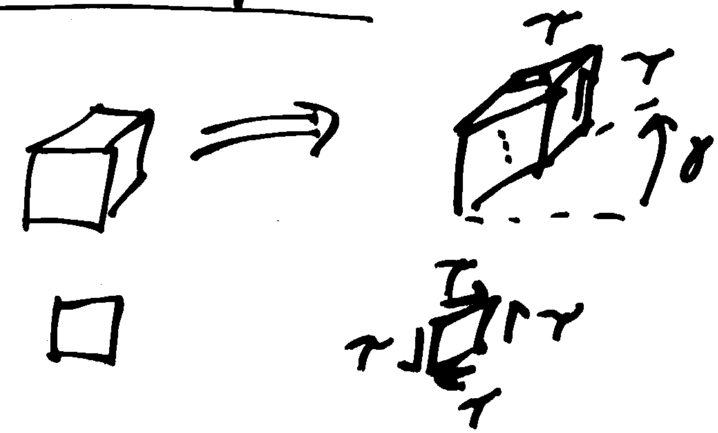
ρ = radius from centerline

$$\gamma = \frac{(d\phi)\rho}{dl} = \frac{(\phi/l)\rho\, dl}{dl}$$

↑ "gamma"

$$\boxed{\gamma = \frac{\rho\phi}{l}} \quad (1)$$

Material Properties



Linear elastic:

$$\tau = G\,\gamma$$

↑ shear modulus

"τ"

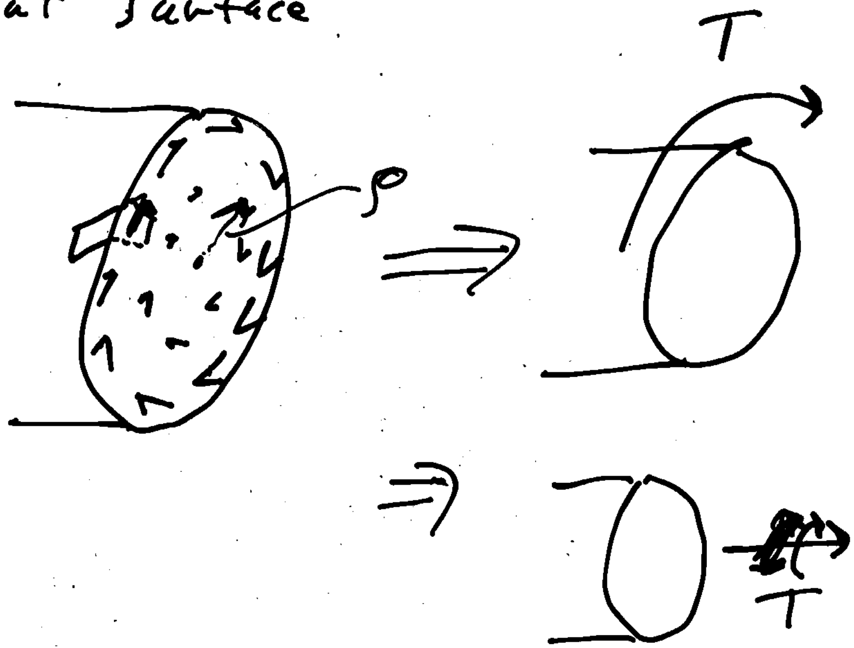
"G"

Mechanics

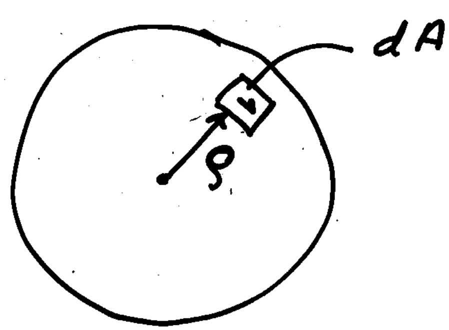
recall "Tension = σA"
we want something like this for torsion.

4/1/03 (3)

Look at an end cap (internal cut surface)



T for torque (not tension)



$$\boxed{T = \int_{\substack{\text{cross}\\ \text{section}}} \underbrace{\rho \, \underbrace{\tau \, dA}_{dF}}_{dT}}$$

(3)

All linear elastic torsion formulas come from (1), (2), (3).

Algebra & Calculus:

$$T = \int \rho \, \tau \, dA$$

$$\tau = G\gamma, \quad \gamma = \frac{\phi \rho}{\ell}$$

$$= \int G \rho^2 \frac{\phi}{\ell} dA$$

$$= \frac{G\phi}{\ell} \int \rho^2 dA$$

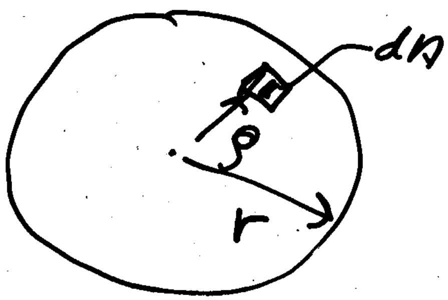
$$\boxed{T = \frac{G\phi}{\ell} J}$$

(A)

$J \equiv \int \rho^2 dA$ = polar area moment of inertia

4

Theory only holds for round bars (penny stacking geometry is otherwise wrong.)



J for round bar

$$\int \rho^2 dA = \int_0^r \int_0^{2\pi} \rho^2 \underbrace{\rho \, d\rho \, d\theta}_{dA}$$

$$= 2\pi \int_0^r \rho^3 d\rho = \frac{\pi r^4}{2}$$

$$\boxed{J = \frac{\pi r^4}{2}}$$ 4!!

twice the radius ⇒
16 times the torsion stiffness

What about failure

$$\gamma = \frac{\rho \phi}{L} = \frac{\rho (T l / G J)}{l}$$

$$= \frac{\rho T}{G J}$$

$$\tau = G \gamma \Rightarrow \boxed{\tau = \frac{T \rho}{J}}$$ B

Back to paperclip

$$T = \frac{\phi G J}{l}$$

$$w F = \frac{\phi G J}{l}$$

$w \approx .5''$, $l \approx 1''$

$G \approx 12 \times 10^6$ lbf/in^2

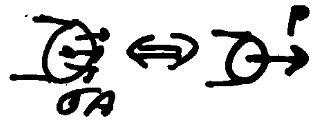
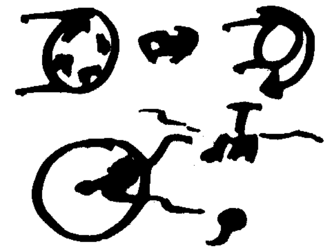
$J = \frac{\pi}{2} \left(\frac{1}{64}\right)^4$ in^4

$\phi = \pi/18$

$\Rightarrow F$

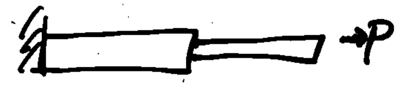
4/14/03 ①

Torsion (w/ Tension Review)

	Tension	Torsion
Mechanics	$P = \sigma A$  $[P = \int \sigma \, dA]$	$T = \int \rho \tau \, dA$ 
Constit. Law	$\sigma = E \epsilon$ $[\epsilon = \frac{\sigma}{E} + \alpha \Delta T]$	$\tau = G \gamma$ 
Geometry of Def.	$\epsilon = \frac{\delta}{l}$	$\gamma = \frac{\rho \phi}{l}$ (straight remains)
Summary of commonly used formulas	$\sigma = \frac{P}{A}$, $\delta = \frac{Pl}{AE}$	$\tau = \frac{\rho T}{J}$, $\phi = \frac{Tl}{JG}$ $J = \int \rho^2 dA = \frac{1}{2}\pi r^4$ circle

4/14/03

bars in series



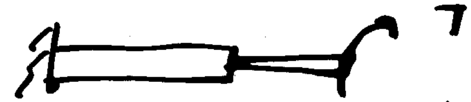
$P_1 = P_2, \quad \delta = \delta_1 + \delta_2$

$$\exists\, K_{eff} = \frac{1}{\frac{1}{K_1} + 1/K_2}$$

$K_1 = A_1 E_1 / \ell_1$

$K_2 =$

$C = C_1 + C_2$



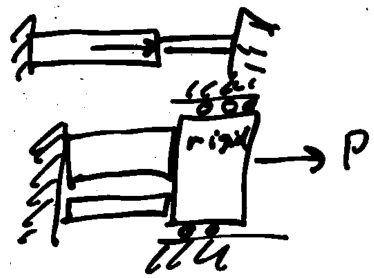
$T_1 = T_2, \quad \phi = \phi_1 + \phi_2$

$$K_{eff} = \frac{1}{\frac{1}{K_1} + 1/K_2}$$

$K_1 = \frac{J_1 G_1}{\ell_1}, \quad K_2 =$

$C = C_1 + C_2 \qquad C_1 = 1/K_1$

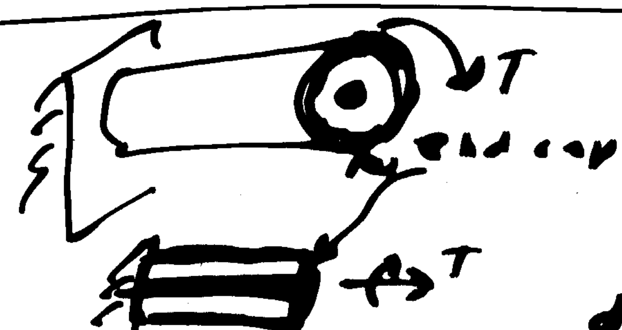
bars in parallel



$P = P_1 + P_2, \quad \delta_1 = \delta_2$

$K_{eff} = K_1 + K_2$

$$C_{eff} = \frac{1}{1/C_1 + 1/C_2}$$



$\phi_1 = \phi_2$

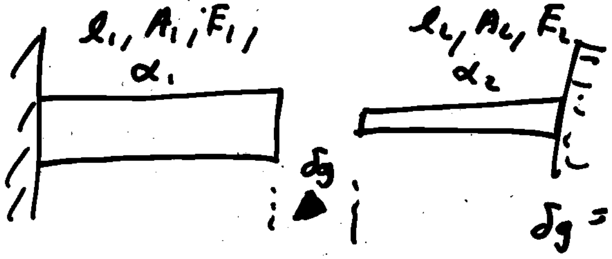
$T = T_1 + T_2,$

$K_{eff} = K_1 + K_2$

$$C_{eff} = \frac{1}{1/C_1 + 1/C_2}$$

4/14/03 (3)

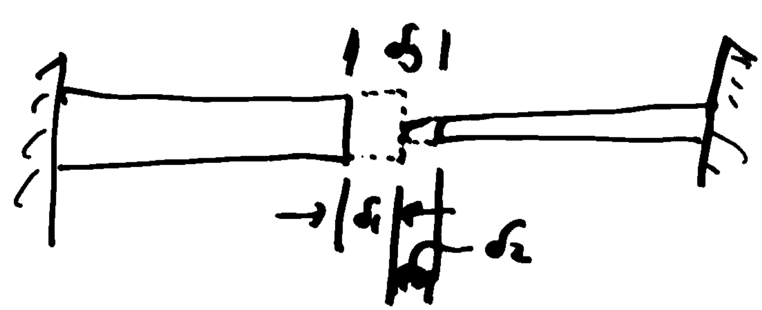
statically ind. problems require mixing up diff. parts of the theory (hint: be especially careful w/ signs).



δ_g = initial gap

Given all geometry & properties
Given ΔT, find σ_1.
temp. rise

Geometry



δ_1, δ_2 are increase in lengths

[we expect that if A_1, E_1, α_1 big & ΔT big then $\delta_2 < 0$]

(1) $\delta_1 + \delta_2 = \delta_g$ [if ΔT big enough]

(2) $\delta_2 = \left[\frac{\sigma_2}{E_2} + (\Delta T)\,\alpha_2\right] l_2$

$\delta_1 = \left[\frac{\sigma_1}{E_1} + (\Delta T)\,\alpha_1\right] l_1$

(3)

$p = P$

$\sigma_1 A_1 = \sigma_2 A_2$

4/18/03

Apply ③ to ②, ② to ①

$\Rightarrow$ 1 eqn. for 1 unknown σ_1

① $\delta_1 + \delta_2 = \delta_g$

$$\Rightarrow \left[\frac{\sigma_1}{E_1} + \Delta T \alpha_1\right] l_1 + \left[\frac{\sigma_1 A_1}{A_2 E_2} + \Delta T \alpha_2\right] l_2 = \delta_g$$

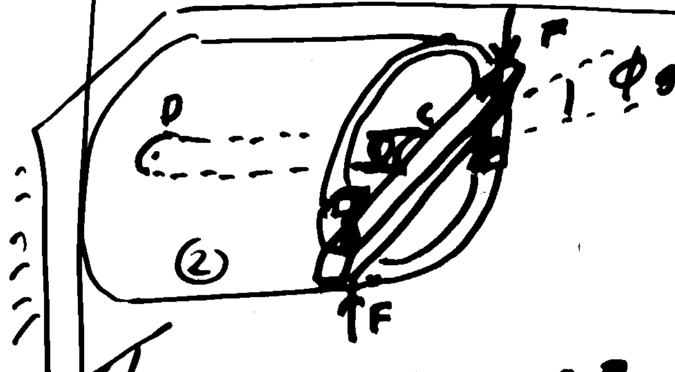
1 eqn. in 1 unknown σ_1

$\sigma_1 =$ ------

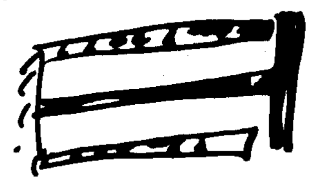
if answer is $\sigma_1 > 0$
replace w/ $\sigma_1 = 0$

if answer is $\sigma_1 < 0$
leave alone $\boxed{\sigma_1 =}$

TORSION indet. prob.



rigid bar AB is welded to end of shaft DC. When gap ϕ_g is covered ($\phi_{DC} = \phi_g$) then contact made w/ outer tube



Given F & all geom & props. Find τ in tube ② outer wall.

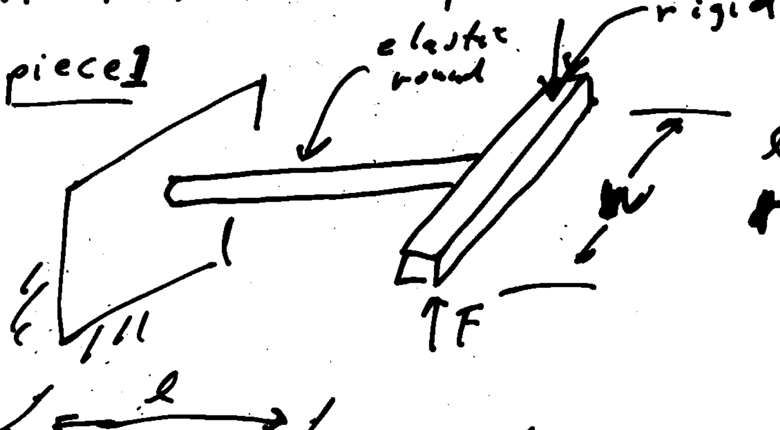
4/16/03 ①

TODAY

1) Torsion ex cont'd
2) More about tension & torsion
3) long narrow things
4) intro to bending.

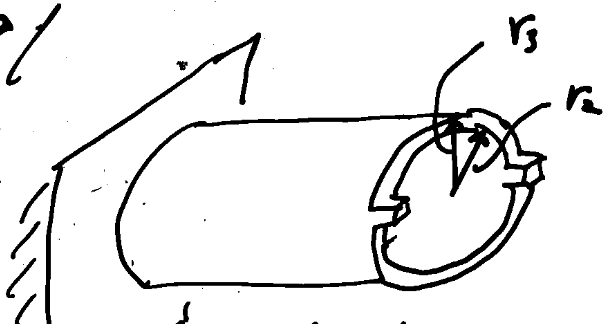
ex/ Torsion: 2 pieces.

piece 1



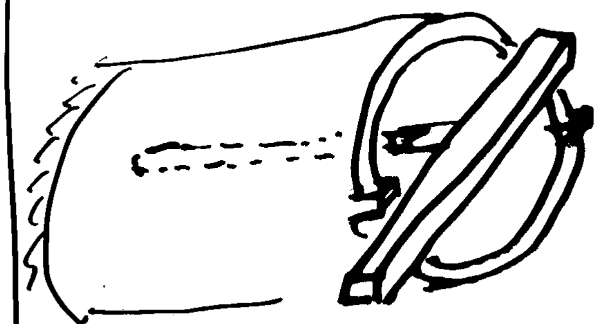
ℓ_1, r_1, G_1

piece 2

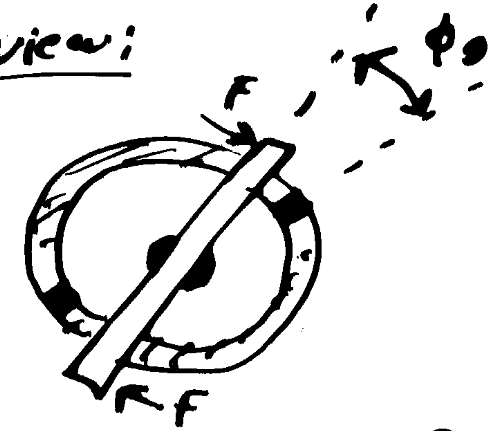


ℓ_2, r_2, r_3, G_2

put together



end view:



Given all geometry & properties & F find max stress in bar 2.

Assume F big enough to close gap:

$$\frac{(Fw)\ell}{J_1 G_1} > \phi_g$$

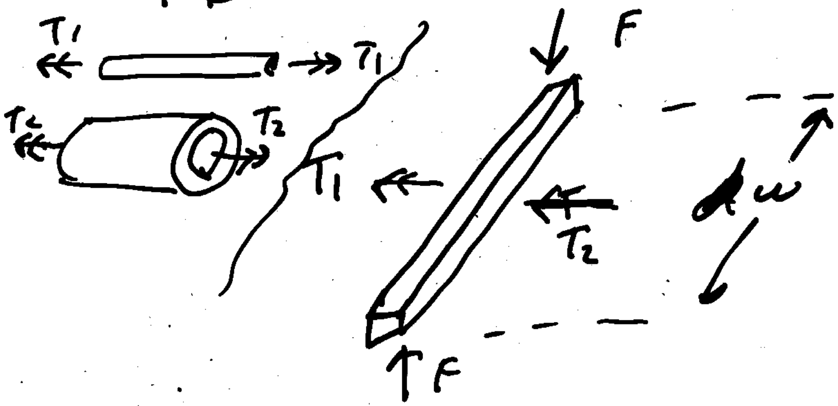
4/16/03 ②

Geometry

$$\boxed{\phi_2 = \phi_1 - \phi_g} \quad ①$$

Mechanics

FBD of rigid bar & shafts



T_2 comes from 2 nubs

$$\boxed{-T_1 + -T_2 = Fw} \quad ②$$

Note! We neglect funny effects at ends

Mat prop

$$\boxed{T_1 = \frac{J_1 \phi_1}{l_1 G_1}, \quad T_2 = \frac{J_2 \phi_2}{l_2 G_2}} \quad ③ ④$$

$l_1 = l_2$

$J_1 = \pi r_1^4/2$

$J_2 = \frac{\pi}{2}(r_3^4 - r_2^4)$

$\approx ((r_3 - r_2)\, 2\pi r_3)\, r_3^2$

$\approx t\, 2\pi r^3$

$t = r_3 - r_2$

$r = r_2$ or r_3

① → ④ can solve for T_2

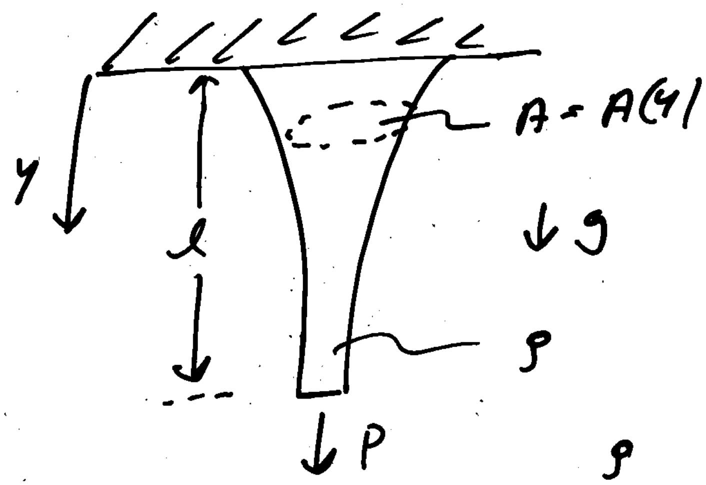
$$\boxed{\tau_{max\,2} = \left| \frac{T_2\, r_{max}}{J_2} \right|}$$

answer

4/16/03 ③

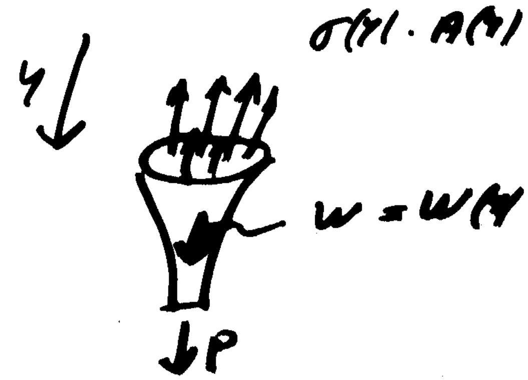
Tension P, Torsion T can vary along length, as can properties & geometry

ex)



given E, A(y), l, P, what is change in length

FBD



$$W(y) = \left[\rho \int_y^l A(y')\, dy'\right] g$$

$$\sigma(y) = \frac{P + W(y)}{A(y)}$$

$$\epsilon(y) = \sigma(y)/E$$

$$\delta = \int d\delta = \int_0^l \overbrace{\epsilon(y)\, dy}^{d\delta}$$

4/16/03 ④

Likewise for torsion

$$\left[\phi_{tot} = \int d\phi = \int_0^{\ell} \frac{T}{JG}\, d\ell' \right]$$

HW hint:

1) "rate of twist" meant is

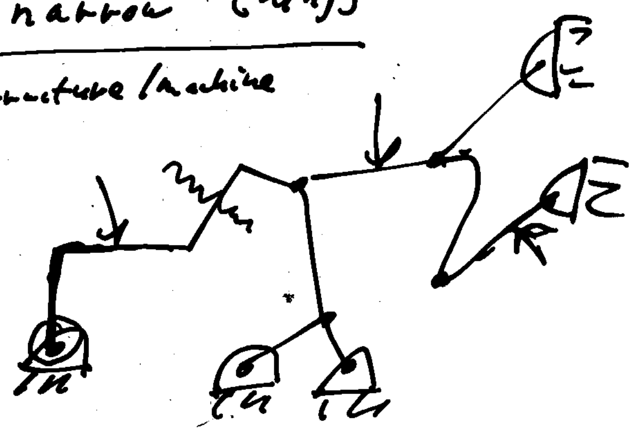
ave. rate of twist = ϕ_{tot}/ℓ

2) exercise your integration skills.

Long narrow things

some structure/machine

3D



force & moment at cut can be resolved into

P tension

$\underline{V}$ shear

T torque

$\underline{M}$ bending moment

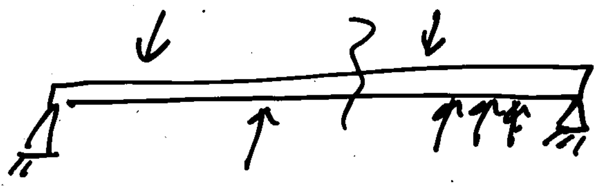
We are dealing w/ these 1 at a time.

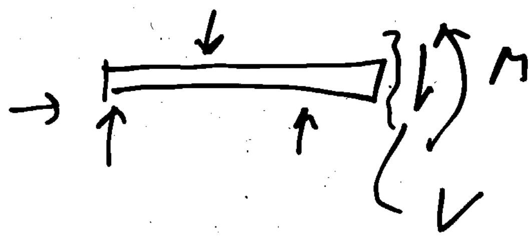
4/16/03

5

NOW, new topic, bending
Most important of all

ex)



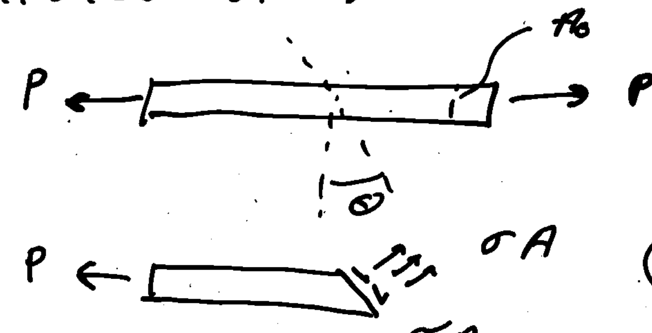


Use statics to find V &
M at cut.
↑ important.

4/21/03

TODAY: 1) Q&A, 2) V, M diagrams

Q. Shear stress



σA

τA

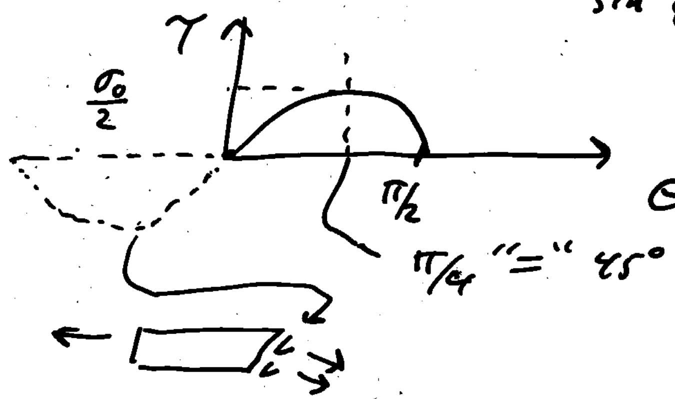
$\llcorner A = A_0 / \cos\theta$

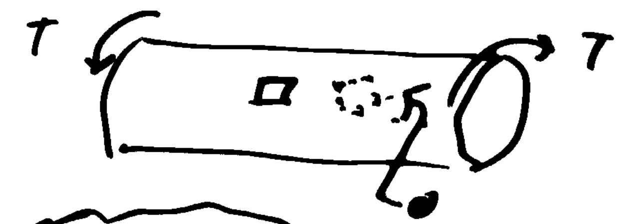
$\sigma_0 = \frac{P}{A_0}$

$\theta = 0$

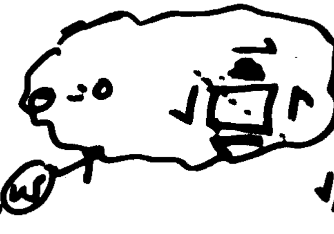
$$\tau = \sigma_0 \cos\theta \sin\theta$$

$\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$





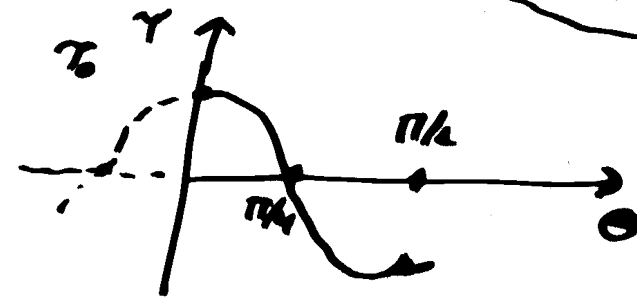
$\theta = 0$



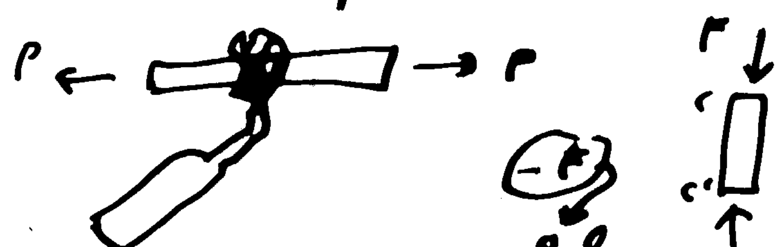
$\tau_0 = \frac{T\rho}{J}$

$\theta = 45°$

$$\tau = \tau_0 \cos 2\theta$$



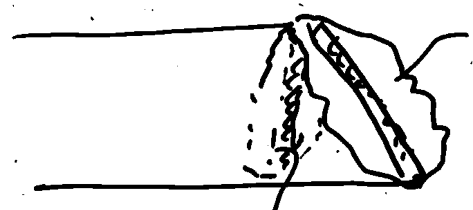
Q. Thermal Expansion

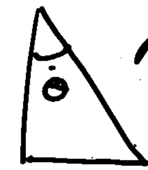


$$\delta = \alpha(\Delta T) l \pm \frac{Pl}{AE}$$

4) Mat Props.

4/23/07 ©



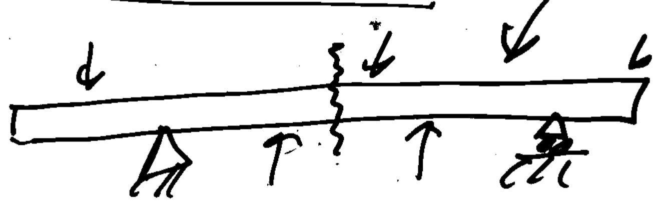


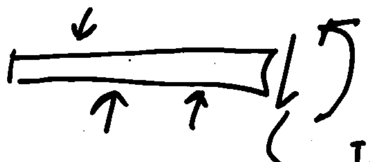
$A_0/A = \cos\theta$

$A_{02}/A = \sin\theta$

V, M diagrams

beams

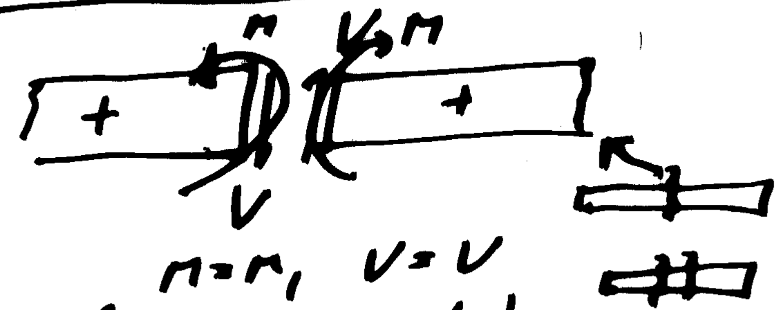




"Sign Convention" issues:

Need to have pre-defined defs of up ↑ & clockwise ↻.
(A drawing defines this.)

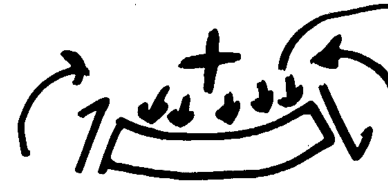
Sign Convention



M = M, V = V
(but vectors ≠)

OR

smiling beam



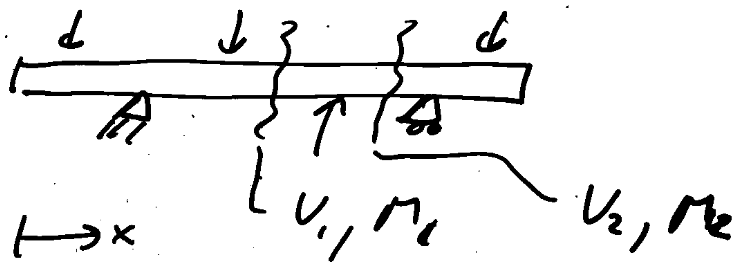
distributed load & + is down

Moment is + if causing smiling
Shear is + if causing clockwise rotation: (down on right, up on left.)

4/23/03

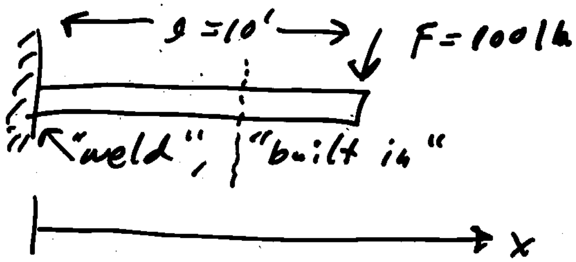
③

A common issue: How do V & M depend on pos. in beam?



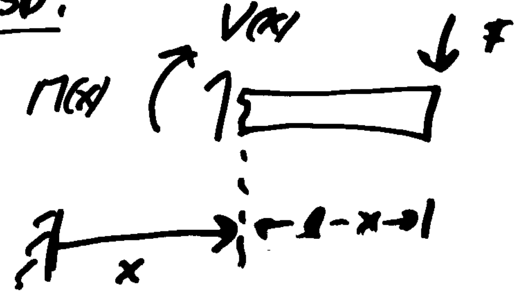
$V(x) = ?$, $M(x) = ?$

ex)



called "clamped-free" beam or "cantilever" beam

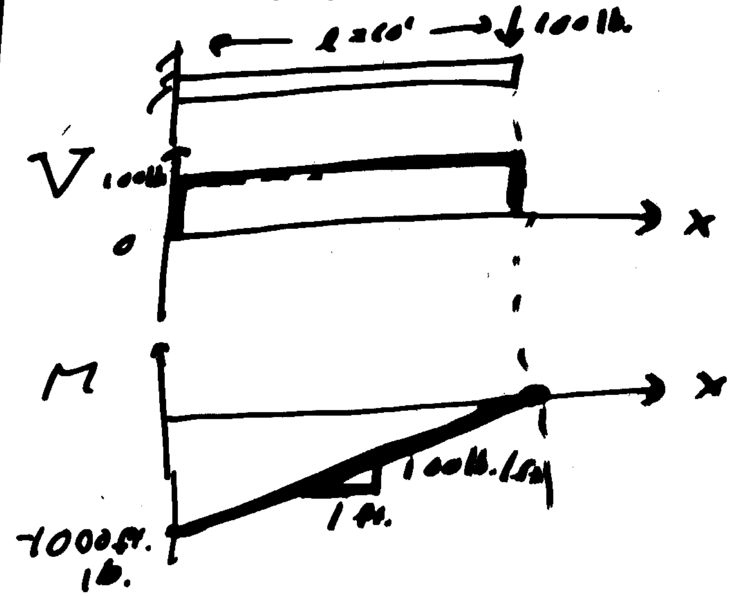
FBD:



$\sum F_y = 0 \Rightarrow V(x) = F = 100\text{lb}$
const.

$\circlearrowleft + \sum M_{(x} = 0 \Rightarrow -M(x) - F(\ell - x) = 0$

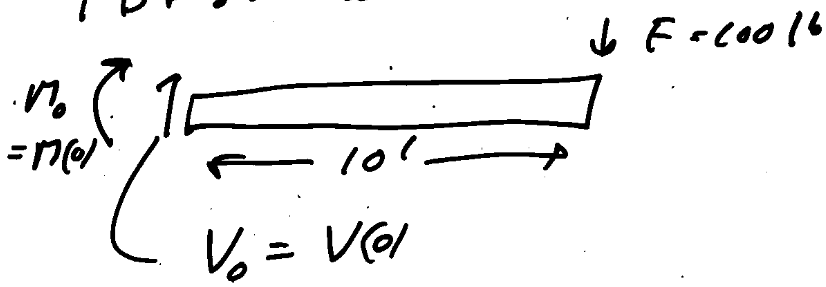
$M(x) = -100\text{lb}(10\text{ft.} - x)$



4/21/03

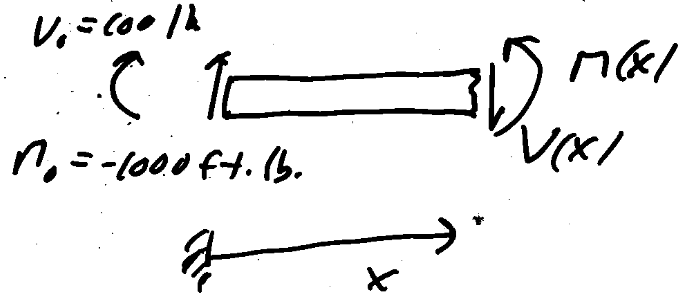
Alternative Soln.

FBD of whole beam



$\Sigma M_0 = 0$, $\Sigma F_y = 0 \Rightarrow$ $V_0 = 100$ lb
$M_0 = -1000$ ft.lb

Now make cut at x, but look at left piece



$\Sigma F_y = 0 \Rightarrow V(x) = 100$ lb const.

$\circlearrowleft \Sigma M_0 = 0 \Rightarrow$

$-(-1000 \text{ ft.lb}) - V(x)\cdot x + M(x) = 0$

$M(x) = -1000 \text{ ft.lb} + (100 \text{ lb.})\cdot x$

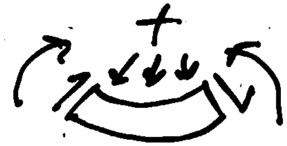
same as before

Moral: same answer for left FBD as right FBD.

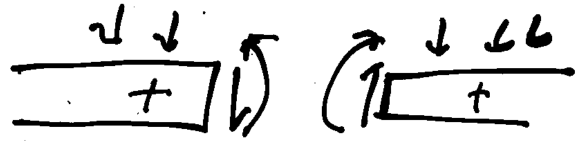
4/23/03 ① RP section 4.4

TODAY: V, M diagram cont'd

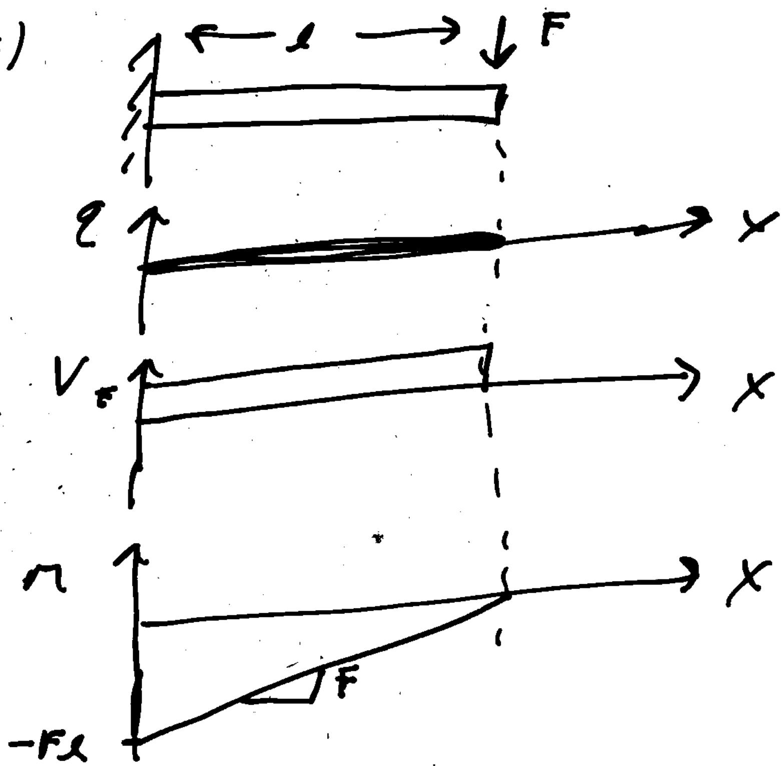
Recall



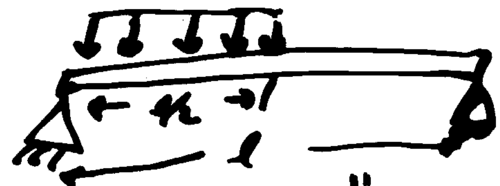
smiling beam

⇒ 

ex)

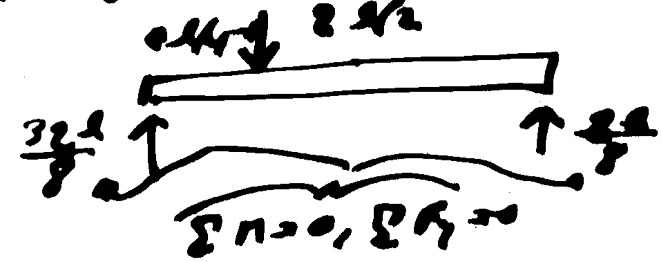


ex) 2



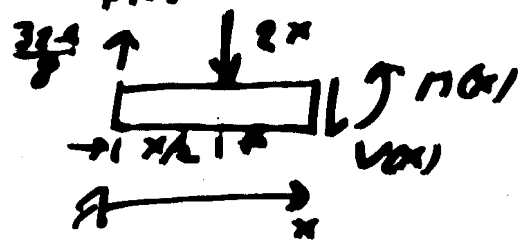
"simply supported" beam

FBD of whole beam



cut at x

first $x < \ell/2$



[4/23/09] ②

$\Sigma F_x = 0 \Rightarrow \quad 0 = 0$

$\Sigma F_y = 0 \Rightarrow \frac{3ql}{8} - q\cdot x - V(x) = 0$

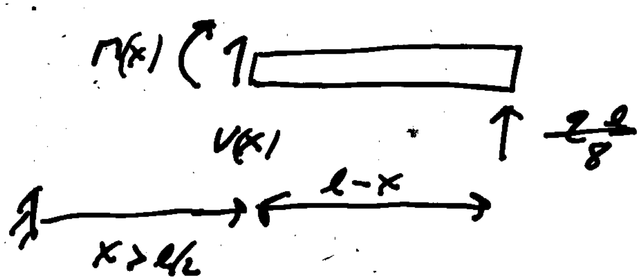
$$\boxed{V(x) = \frac{3ql}{8} - q\cdot x} \quad x \le l/2$$

$+\circlearrowleft \Sigma M_{/x} = 0 \Rightarrow M(x) - \left(\frac{3ql}{8}\right)\cdot x + (q\cdot x)\frac{x}{2} = 0$

$$\boxed{M(x) = \frac{3ql}{8}x - q\,x^2/2}$$

$x \le l/2$

Next look at $x > l/2$

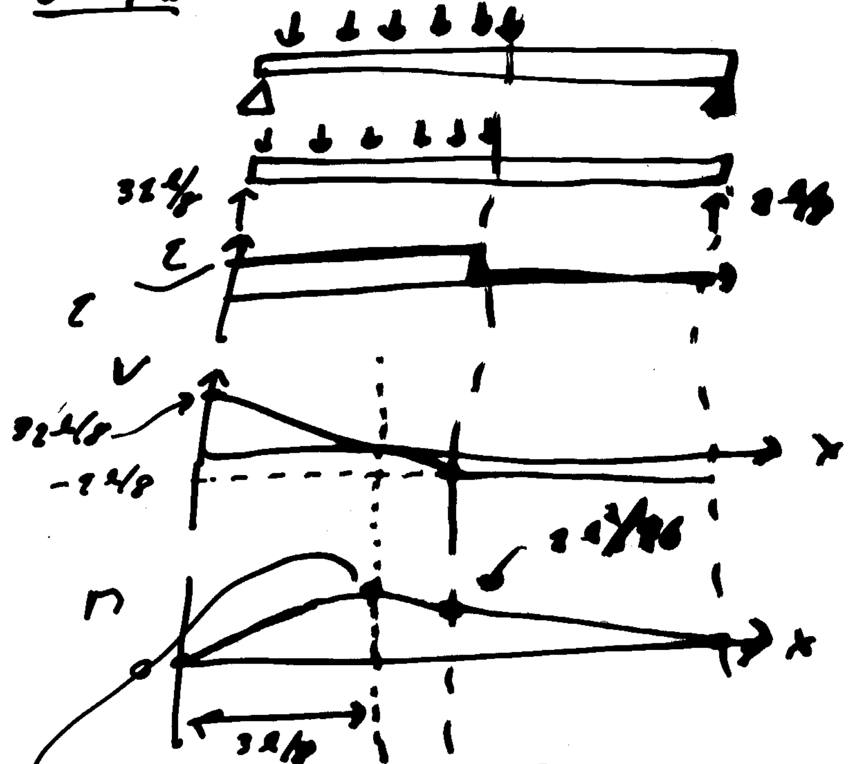


$\Sigma F_y = 0 \Rightarrow \boxed{V(x) = -ql/8} \quad x \ge l/2$

$+\circlearrowleft \Sigma M_{/x} = 0 \Rightarrow -M(x) + \frac{ql}{8}(l-x) = 0$

$$\boxed{M(x) = \frac{ql}{8}(l-x)} \quad x \ge l/2$$

Graphs



$M_{max} = \frac{9}{128} q l^2$

4/23/03 ③

Fact: **Always**

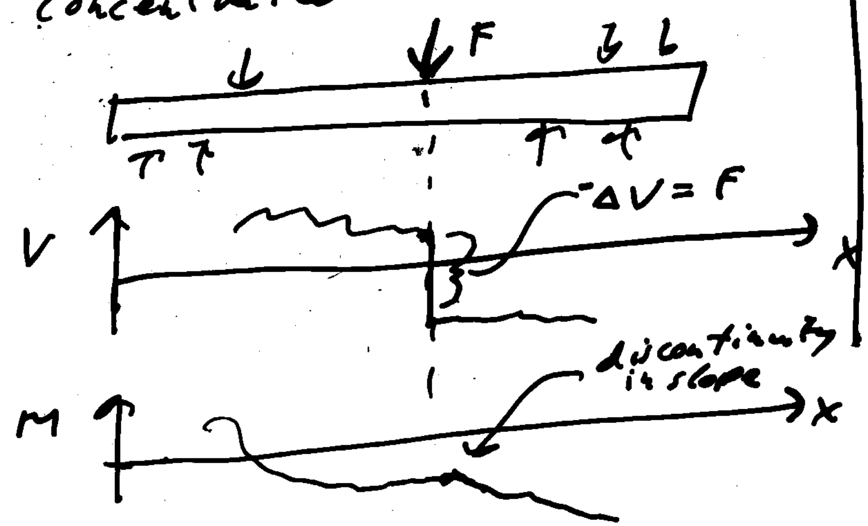
$$\frac{dM}{dx} = V(x)$$

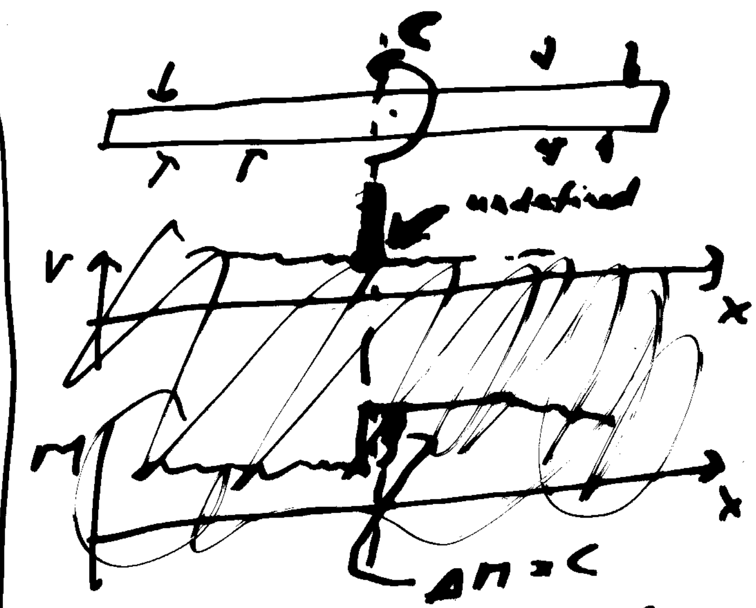
$$\frac{dV(x)}{dx} = -q(x)$$

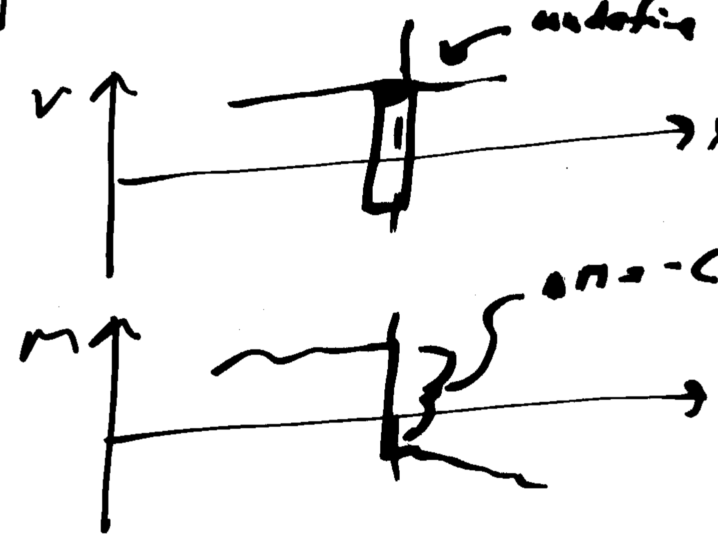
$$\Rightarrow \quad V(x) = V_0 + \int_0^x -q(x')\,dx'$$

$$M(x) = M_0 + \int_0^x V(x')\,dx'$$

Need to worry about concentrated loads & couples

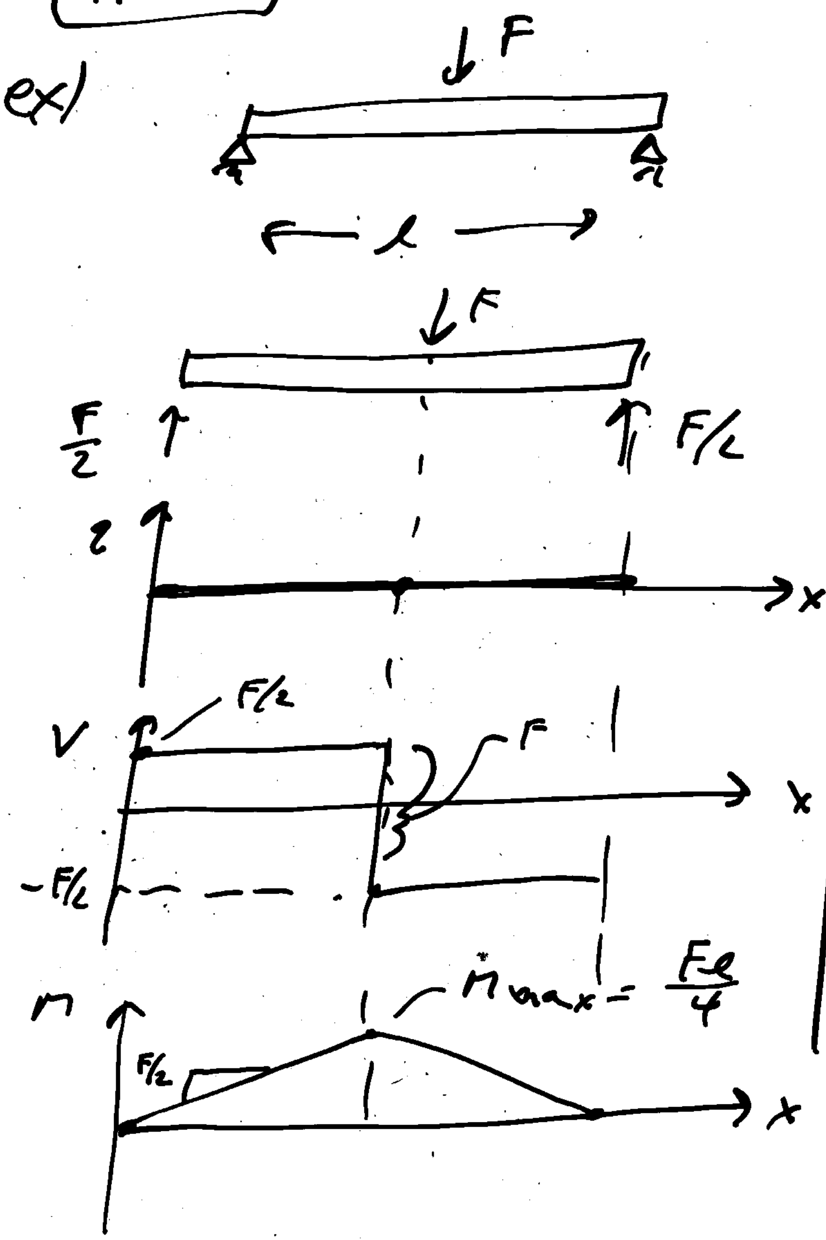






4/23/23 4

ex)



Note: $\frac{dV}{dx} = -q$

$\frac{dM}{dx} = V$

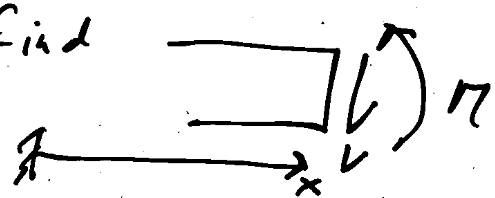
is a shortcut.

Can always use FBDs instead.

4/25/03

TODAY: σ, τ in beams

Procedure: Use statics to find



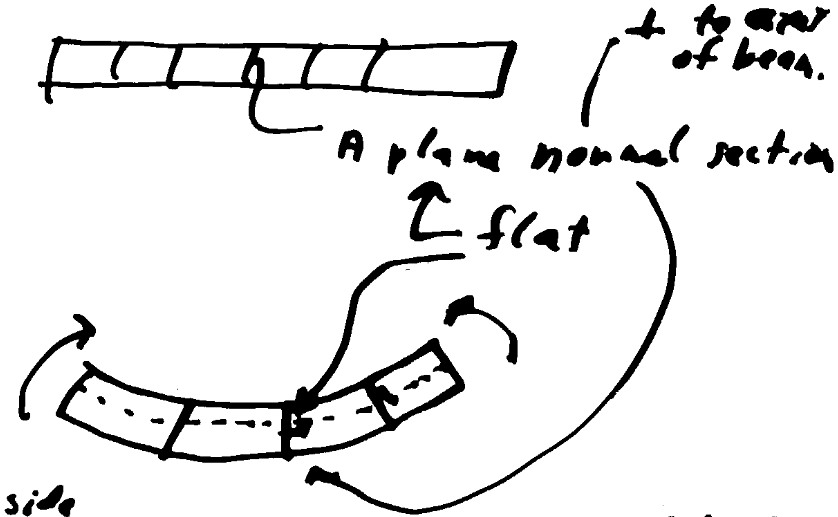
V, M at pt. x of interest

Goal: to find stresses.

Theory of stresses in beams

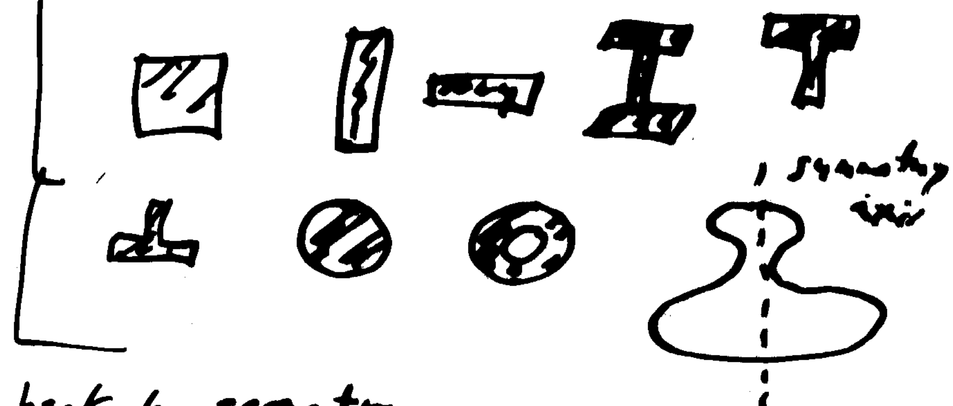
Geometric assumption

plane-normal sections remain plane & normal



aside

Cross sections of interest/relevance



back to geometry

4/25/03

Similar Δs!

$$\frac{\Delta l}{\rho - y} = \frac{\Delta l_0}{\rho} \Rightarrow \epsilon = \frac{\Delta l - \Delta l_0}{\Delta l_0}$$

$$\Delta l = \frac{\rho - y}{\rho} \Delta l_0$$

$$\epsilon = \frac{\Delta l - \Delta l_0}{\Delta l_0} = \frac{\frac{\rho - y}{\rho} \Delta l_0 - \Delta l_0}{\Delta l_0}$$

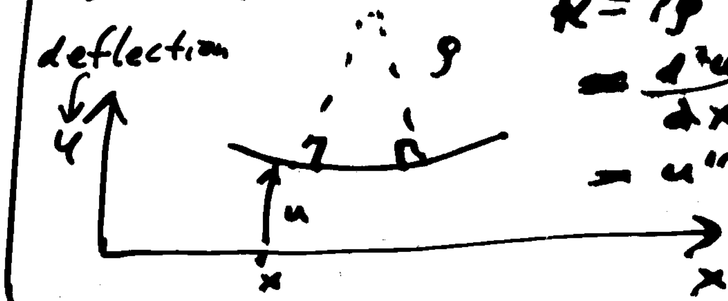
$$= \frac{\frac{\rho - y}{\rho} - 1}{1}$$

$$= 1 - \frac{y}{\rho} - 1 = \frac{-y}{\rho}$$

$$\boxed{\epsilon = -y/\rho}$$

$$\boxed{\begin{aligned} \epsilon &= -yK \\ &= -y u'' \end{aligned}}$$

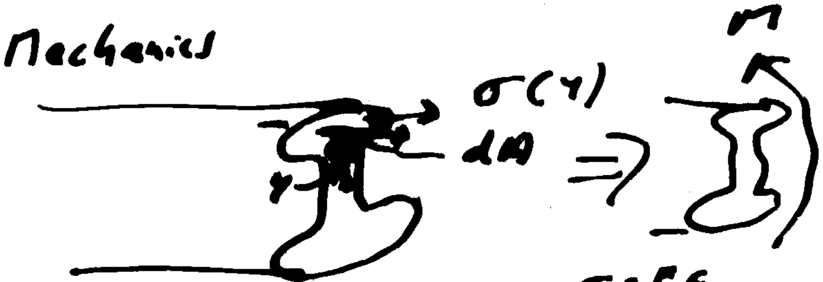
geometry con'td



$$K = 1/\rho = \frac{d^2 u}{dx^2} = u''$$

Mat. Prop. $\boxed{\sigma = E \epsilon}$ ← linear elastic

Mechanics



$$M = -\int_{\text{cross section}} y \, \sigma \, dA \qquad \sigma = E\epsilon, \quad \epsilon = -yK$$

$$= -\int y E(-yK) \, dA$$

$$= KE \int y^2 dA = EI u''$$

$K = 1/\rho = u''$

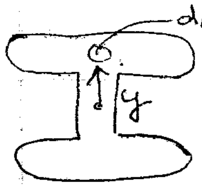
$$\boxed{M = EI/\rho}$$

courtesy of:
Julia Ferullo

Geometry: $\varepsilon = -y/\rho \longrightarrow \frac{1}{\rho} = \kappa = u'' = \frac{d^2 u}{dx^2}$

Mat. Props: $\sigma = E\sigma$; Mechanics: $M = EI/\rho = EI\kappa = EIu''$

I = area moment of inertia = $\int_{\text{cross section}} y^2 \, dA$

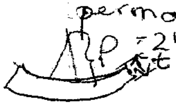


Location of neutral axis: Net tension = 0 $\Rightarrow \int \sigma \, dA =$
$\Rightarrow \int y \, dA = 0 \Rightarrow$ neutral axis at centroid

of interest is stress: $\sigma = E\varepsilon = E(-y\kappa)$. $\sigma = \frac{-My}{I}$

$\sigma_{max} = \left| \frac{M|y|_{max}}{I} \right| = \frac{M}{S}$ S = geometric quantity = section modulus = $\frac{I}{|y|_{max}}$

Example: What is the yield stress of Andy's ruler?
bend ruler to radius of 2 in ($1/50$ in thick) to get permanent Δ.



$t = 1/50$ in ε_y = yield strain = $\left| \frac{y_{max}}{\rho} \right| = \frac{|1/50|/2}{2 \text{ in}}$ $\varepsilon_y = \frac{1}{200}$

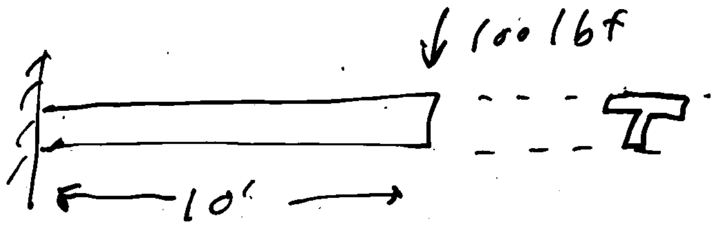
σ_y: yield stress = $E \varepsilon_y$
$\uparrow$ 30×10^6 lbf/in^2 } for all steel = 200×10^9 Pa

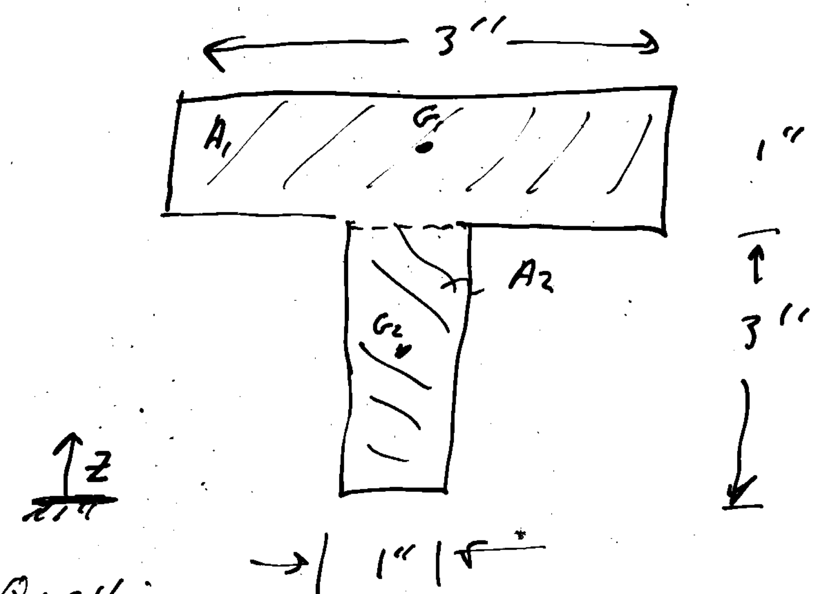
$\sigma_y \approx 10^9$ Pa or $.15 \times 10^6$ lb/in^2
= 150,000 lb/in^2 } hard steel

Tedious Skill: Calculating I for various cross sections

4/28/03

TODAY: I, deflection





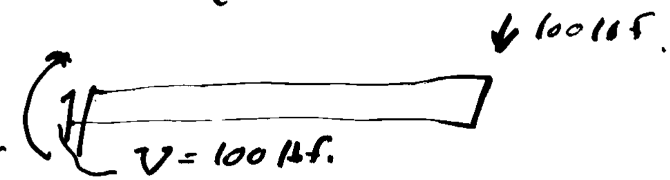
Question

$\sigma_{tension\ max} = ?$

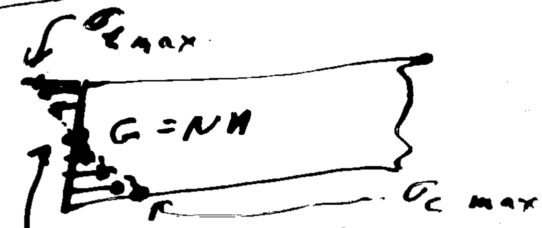
$\sigma_{comp\ max} = ?$

First find M

Max M at left end



$\boxed{M = -1000\ \text{lbf}\cdot\text{ft.}}$



$\sigma = -My/I$

Need to find I, y values

(find G = N.A.).

which z_i

	Area	z_i	I_i
1	3 in²	3.5 in	$\frac{1}{4}$ in⁴
2	3 in²	1.5 in	$\frac{9}{4}$ in⁴

$= \frac{1}{12} b h^3$

Area = area of region

z_i = vert. loc. of centroid of area

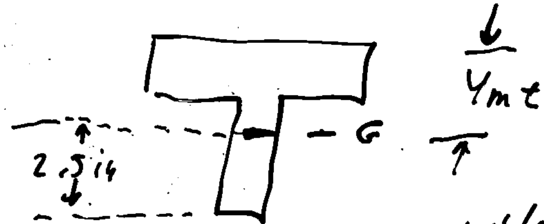
I_i = area moment of inertia about centroid

4/29/09 ②

Where is G? z value of centroid of composite.

$$z_G = \frac{\sum A_i z_i}{\sum A_i} = \frac{3\,in^2\,[3.5\,in + 1.5\,in]}{(3+3)\,in^2}$$

$$= \frac{15}{6}\,in = 2.5\,in.$$



parallel axis thm.

$$I = \sum \left(I_i + d_i^2 A_i\right)$$

$$d_i = |z_i - z_G|$$

dist. from total G to the individual G_i.

$$= \left[\left(\tfrac{1}{4} + 1^2 \cdot 3\right) + \left(\tfrac{9}{4} + 1^2 \cdot 3\right)\right] in^4$$

$$\boxed{I = \tfrac{17}{2}\,in^4}$$

For tension $|y_{max}| = 1.5\,in$

for comp. $|y_{max}| = 2.5\,in.$

$$\sigma_{t\,max} = \left|\frac{M_{max}\, y_{max\,t}}{I}\right|$$

$=$ arithmetic

$$|\sigma_{c\,max}| = \left|\frac{M_{max}\, y_{max\,c}}{I}\right|$$

$=$ arith. $\geq |\sigma_{t\,max}|$

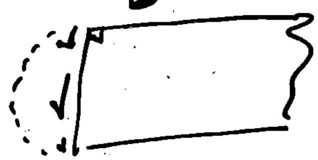
because bottom surface is further from G than top surface.

4/28/03 ③

What is shear stress in beam?

$\bar{\tau}$ = ave. shear stress

$$= \frac{V}{A} = \int \tau \, dA$$



not const. in cross section

varies according to

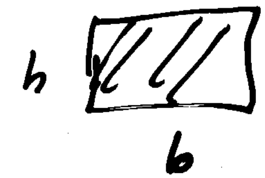
$$\approx \frac{VQ}{It}$$

which is covered in lab but not HW or lecture

This formula not on final exam.

④

ex)



$$I = \frac{1}{12} b h^3$$

ex)

$$I = \frac{\pi}{4} r^4 = \frac{1}{2} J$$

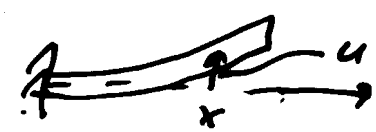
Deflection in beams

$$M = EI/\rho$$

$$\Rightarrow \kappa = \frac{M}{EI}$$

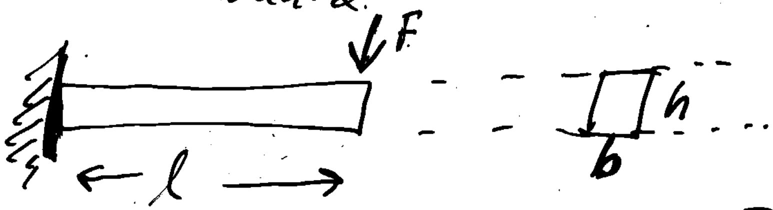
$$\frac{d^2u}{dx^2} = \frac{M}{EI}$$

$$u'' = M/EI$$

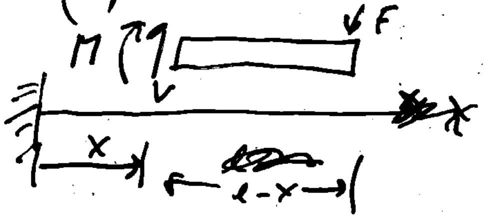


5/02/03 ①

TODAY 1) Deflection of beams
2) Something you've always wanted.



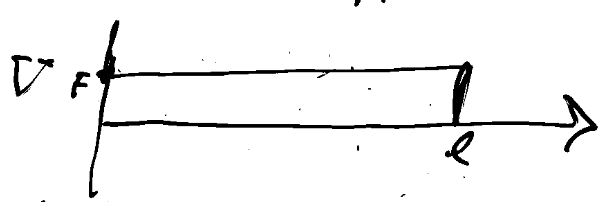
deflection at end = ?
(Know all moduli!)

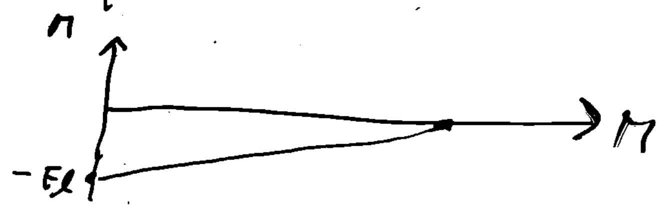


$\Sigma F_y = 0 \Rightarrow \quad V = F$

$+\circlearrowleft \Sigma M_{/x} = 0 \Rightarrow \quad -F(l-x) - M = 0$

$M = -F(l-x)$

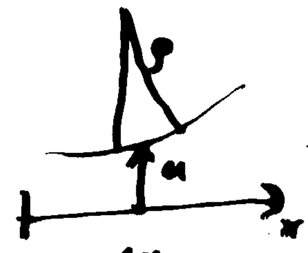




$M = EI/\rho$

$\frac{1}{\rho} = \frac{M}{EI}$

$$\boxed{u'' = \frac{M}{EI}}$$



$u' = \frac{du}{dx}$

$u'' = \frac{1}{EI}(Fx - Fl)$

$u' = \int_0^x u''(x')\,dx' + u'(0)$

$= \frac{1}{EI}\left(\frac{Fx^2}{2} - Flx\right) + 0$

B.C.: slope at left end is zero

$u(x) = \int_0^x u'(x')\,dx' + u(0)$

$= \frac{1}{EI}\left(\frac{Fx^3}{6} - \frac{Flx^2}{2}\right) + 0$

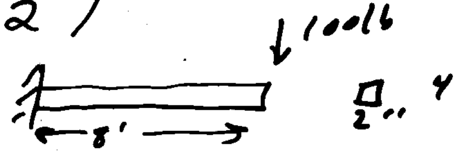
B.C.: value of u at left edge = 0

5/2/03 ②

Deflection at end = $u(l)$.

$$u(l) = \frac{F}{EI}\left(\frac{l^3}{6} - \frac{l^3}{2}\right)$$

$$\boxed{u(l) = -\frac{F\, l^3}{3EI}}$$



ex) $l = 8'$, $b = 2''$, $h = 4''$

$F = 100\text{lb}$ Real, as opposed to "Nominal," 2×4.

$E = 1{,}000{,}000\ \text{lb/in}^2$

$I = \frac{1}{12} b h^3 = \frac{1}{12} 2 \cdot 4^3 \text{ in}^4$

$= \frac{32}{3} \text{ in}^4$

$$u(l) = \frac{-(100\text{ lb})\,(8\text{ ft})^3}{3 \cdot \left(10^6 \frac{\text{lb}}{\text{in}^2}\right) \cdot \left(\frac{32}{3} \text{ in}^4\right)}$$

$$= \frac{-100 \cdot 8^3}{3 \cdot 10^6 \cdot 32/3} \frac{\text{ft}^3}{\text{in}^2} \left(\frac{12 \text{ in}}{1 \text{ ft}}\right)^3$$

$$= \frac{-100 \cdot 2^9}{10^6 \cdot 2^5} \cdot 3^3 \cdot 2^6 \text{ in}$$

$$= -10^{-4} \cdot \cancel{2^{10}}\, 2^{10} \cdot 3^3 \text{ in}$$

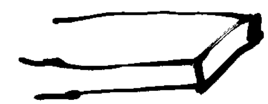
$2^{10} \approx 10^3$

$3^2 \approx 10$

$u = -10^{-4} \cdot 10^3 \cdot 10^1 \cdot 3$ in

$= -3$ in

ex) same 2×4 sideways



Key variable is I

more floppy by factor of I_{new} / I_{old}

$$= \frac{4 \cdot 2^3/12}{2 \cdot 4^3/12}$$

$$= 2^5/2^7$$

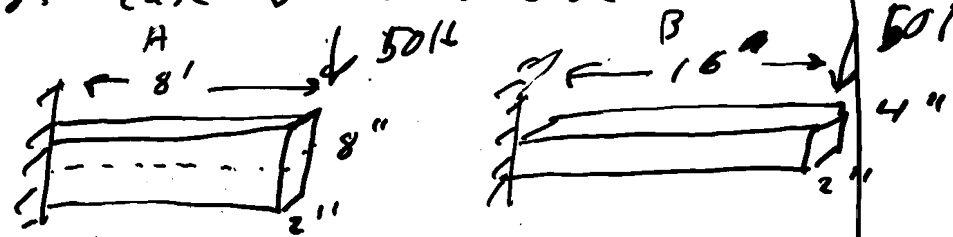
$$= \frac{1}{4}$$

⇒ 4 times floppier.

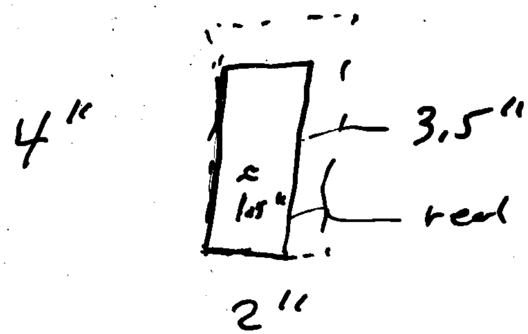
5/02/03

③

How much bigger is deflection of case B than case A.

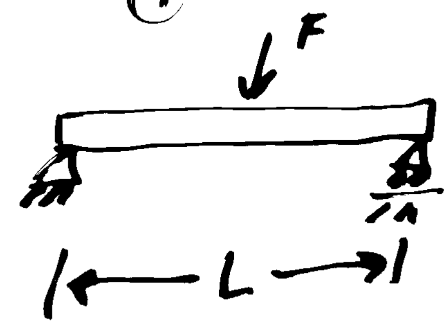


Aside: nominal 2×4



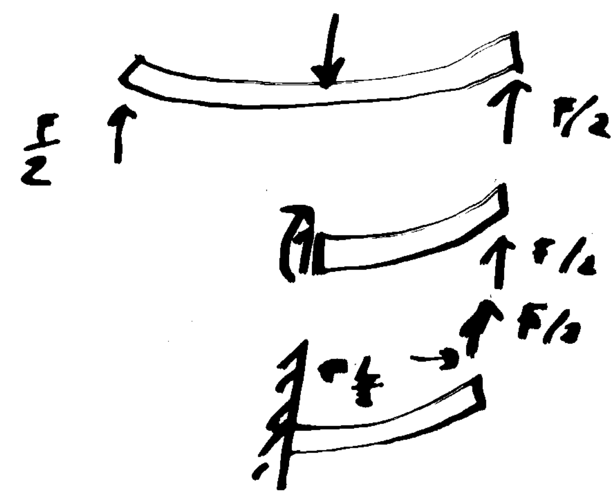
④

ex)



deflection in middle

Trick: symmetric



$\boxed{5/02/07}$ (4)

cantilever: $u(\ell) = \dfrac{\text{"}F\text{"}\,\ell^3}{3EI}$

simply supported load in middle: $u\left(\frac{L}{2}\right) = \dfrac{\left(\frac{F}{2}\right)\left(\frac{L}{2}\right)^3}{3EI}$

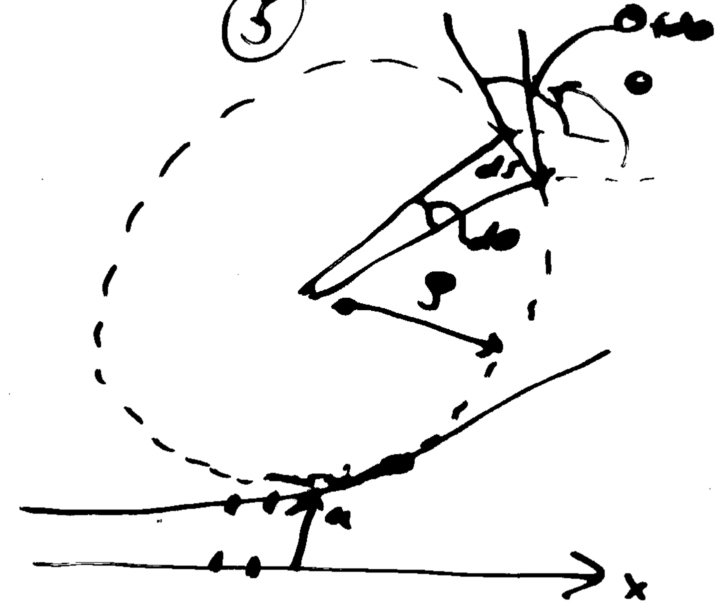
$= \dfrac{FL^3}{48EI}$

Q & A

big formula

$$\kappa = \frac{d^2u/dx^2}{\sqrt{1+\left(\frac{du}{dx}\right)^2}^{\,3}}$$

(5)



small slope

$ds \approx dx$

$\theta \approx \tan\theta = u'$

circle: $\rho\, d\theta = ds$

$\frac{1}{\rho} = \frac{d\theta}{ds}$

$\frac{1}{\rho} \approx \frac{d}{dx}\left(\frac{du}{dx}\right) = u''$