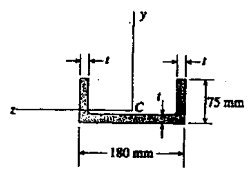
TAM 202 Spring 03 HW 15 Solution provided by Tian, Pankaj and Vijay.

5.6-16, 5.6-20, 9.3-11, 9.3-13, 9.3-6, synthesis problem (due 05/06)

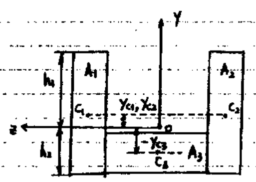
5.6-16 Page 1/15

5.6-16.

5.6-16 A beam having a cross section in the form of a channel (see figure) is subjected to a bending moment acting about the z axis. Calculate the thickness t of the channel in order that the bending stresses at the top and bottom of the beam will be in the ratio 7:3, respectively.



PROB. 5.6-16



σ_1: stress at the top

σ_2: stress at the bottom

① First, we can use the condition $\left|\frac{\sigma_1}{\sigma_2}\right| = \frac{7}{3}$ to find the position of neutral axis.

$\sigma = -\frac{My}{I} \Rightarrow \left|\frac{\sigma_1}{\sigma_2}\right| = \frac{h_1}{h_2} = \frac{7}{3}$ (1)

total height $h_1 + h_2 = 75\text{ mm}$ (2)

(1) & (2) $\Rightarrow$ $h_1 = 52.5\text{ mm}$ $h_2 = 22.5\text{mm}$

② Now, the areas A_i ($i = 1, 2, 3$) & the position of their centroid Y_{ci} depend on the thickness t, but the centroid of the whole section is point O, i.e.

$\sum_{i=1}^{3} Y_{ci} A_i = Y_o = 0$ (3)

where C_i is the centroid of the area A_i ($i = 1, 2, 3$)

(Continued)

5.6-16 (Cont'd) Page 2/15

So we can use (3) to find t. In (3)

$A_1 = A_2 = t\,(75\text{ mm})$

$A_3 = t\,(180\text{ mm} - 2t)$

$Y_{c1} = Y_{c2} = \frac{75}{2}\text{ mm} - h_2 = 15\text{ mm}$

$Y_{c3} = \frac{t}{2} - h_2 = \frac{t}{2} - 22.5\text{ mm}$

Sub into (3)

$\Rightarrow 2t\,(75\text{mm})(15\text{mm}) + t\,(180\text{mm} - 2t)\left(\frac{t}{2} - 22.5\text{mm}\right) = 0$

$\Rightarrow t^2 - (135\text{mm})\,t + 1800\text{ mm}^2 = 0$

$\Rightarrow t = \boxed{15\text{ mm}}$

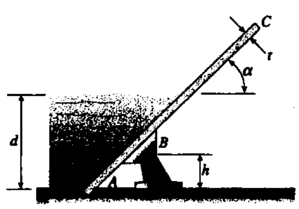
5.6-20

5.6-20 Water pressure acts against an inclined panel *ABC* that serves as a barrier (see figure). The panel is pivoted at point *B*, which is height *h* above the base, and presses against the base at *A* when the water level is not too high (note that the panel will rotate about the pin at *B* if the depth *d* of the water exceeds a certain maximum depth d_{max}). The panel has thickness *t* and is inclined at an angle α to the horizontal. The allowable bending stress in the panel is σ_{allow}.

Derive the following formula for the minimum allowable thickness of the panel:

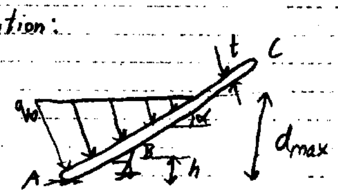
$$t_{min} = \sqrt{\frac{8\gamma h^3}{\sigma_{allow}(\sin^2\alpha)}}$$

(*Note:* To aid in deriving the formula, observe that the maximum stress in the panel occurs when the depth of the water reaches the maximum depth d_{max}. Also, consider only the effects of bending in the panel, disregard the weight of the panel itself, and let γ be the weight density of water.)



PROB. 5.6-20

Solution:



d_{max}: Maximum depth of water

(There is no reaction at end A when the water depth equals d_{max})

b = width of panel perpendicular to plane of figure

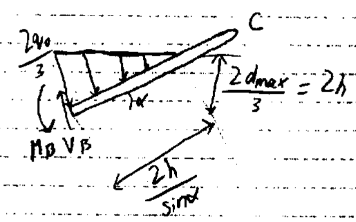
q_0 = Max. intensity of distributed load on panel

$= \gamma d_{max} b$

Resultant of the triangular load acts through the centroid of triangle. This must also act through B.

$\therefore \boxed{h = \frac{d_{max}}{3}}$

FBD of part BC of panel:



Maximum bending moment occurs at section B.

$$M_{max} = M_B = \frac{1}{2}\left(\frac{2q_0}{3}\right)\left(\frac{2h}{\sin\alpha}\right) \times \frac{1}{3}\left(\frac{2h}{\sin\alpha}\right)$$

$$= \frac{4q_0 h^2}{9\sin^2\alpha}$$

$$q_0 = \gamma d_{max} b = \gamma(3h)(b)$$

$$\therefore M_{max} = \frac{4\gamma b h^3}{3\sin^2\alpha} \longrightarrow (1)$$

Also, $M_{max} = (\sigma_{allow})\left(\frac{bt^2}{6}\right) \longrightarrow (2)$

From (1) & (2), we get (by equating M_{max})

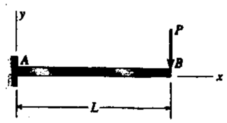
$$\sigma_{allow}\left(\frac{bt^2}{6}\right) = \frac{4\gamma b h^3}{3\sin^2\alpha}$$

$$\Rightarrow \boxed{t_{min} = \sqrt{\frac{8\gamma h^3}{\sigma_{allow}\sin^2\alpha}}}$$

Hence, the result.

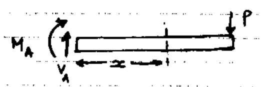
9.3/11

9.3-11 Derive the equation of the deflection curve for a cantilever beam AB supporting a load P at the free end (see figure). Also, determine the deflection δ_B and angle of rotation θ_B at the free end. (*Note:* Use the second-order differential equation of the deflection curve.)

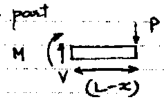


PROB. 9.3-11

Take a section at a distance x



take Right part



BCs at A:

$v(0) = 0$ (deflection)

$v'(0) = 0$ (angle)

$\Sigma M_x = 0 \Rightarrow M + P(L-x) = 0$

$$\boxed{M = -P(L-x)}$$

we know that

$$EI v'' = M = -P(L-x)$$

so $v'' = \frac{-P}{EI}(L-x)$

integration $v' = -\frac{P}{EI}\left(Lx - \frac{x^2}{2}\right) + C_1$

one more integration

$$v = -\frac{P}{EI}\left(\frac{Lx^2}{2} - \frac{x^3}{6}\right) + C_1 x + C_2$$

$v(0) = 0 \Rightarrow C_2 = 0$

$v'(0) = 0 \Rightarrow C_1 = 0$

so

$$v = -\frac{P}{EI}\left(\frac{Lx^2}{2} - \frac{x^3}{6}\right)$$

and $\theta = v' = -\frac{P}{EI}\left(Lx - \frac{x^2}{2}\right)$

so $\delta_B = v(L) = -\frac{P}{EI}\left(\frac{L^3}{2} - \frac{L^3}{6}\right)$

$$= \frac{-PL^3}{3EI}$$

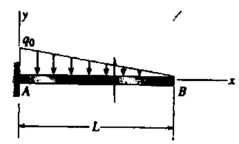
$$\theta_B = v'(L) = \frac{-P}{EI}\left(L^2 - \frac{L^2}{2}\right)$$

$$= \frac{-PL^2}{2EI}$$

θ_B is negative $\Rightarrow$ Clockwise rotation.

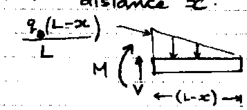
9.3/13

9.3-13 A cantilever beam AB supporting a triangularly distributed load of maximum intensity q_0 is shown in the figure. Derive the equation of the deflection curve and then obtain formulas for the deflection δ_B and angle of rotation θ_B at the free end. (*Note:* Use the second-order differential equation of the deflection curve.)



PROB. 9.3-13

Take right part of the FBD with section at a distance x.



$\Sigma M_x = 0 \Rightarrow M + \frac{1}{3}(L-x)\left[\frac{q_0}{2L}(L-x)(L-x)\right] = 0$

$$M = -\frac{q_0}{6L}(L-x)^3$$

$$EI v'' = M = -\frac{q_0}{6L}(L-x)^3$$

$$v'' = \frac{-q_0}{6EIL}(L-x)^3 \quad \text{—} ①$$

integrating ①

$$v' = \frac{q_0}{6(EIL)}\frac{(L-x)^4}{4} + C_1$$

at $x = 0$ $v'(0) = 0$

$$C_1 = \frac{-q_0}{24EI}L^3$$

$$v' = \frac{q_0}{24EIL}\left((L-x)^4 - L^4\right) \quad \text{—} ②$$

integrating ②

$$v = \frac{q_0}{24EIL}\left[-\frac{(L-x)^5}{5} - L^4 x\right] + C_2$$

at $x = 0$ $v(0) = 0$

$$C_2 = \frac{q_0}{24EIL}\frac{L^5}{5} = \frac{q_0}{120EI}L^4$$

$$v = \frac{q_0}{120EIL}\left[L^5 - (L-x)^5 - 5L^4 x\right]$$

at $x = L$

$$v_B = v(L) = \frac{q_0}{120EIL}\left(L^5 - 0 - 5L^5\right)$$

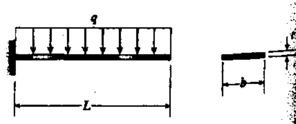
$$\boxed{v_B = -\frac{q_0 L^4}{30EI}}$$

$$\boxed{\theta_B = v'(L) = -\frac{q_0 L^3}{24EI}}$$

θ_B is negative $\Rightarrow$ Clockwise rotation.

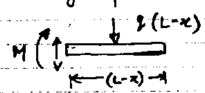
9.3-6 A gold-alloy microbeam attached to a silicon waf behaves like a cantilever beam subjected to a uniform lo (see figure). The beam has length $L = 25\ \mu m$ and rectang lar cross section of width $b = 15\ \mu m$ and thickn $t = 0.87\ \mu m$. The total load on the beam is $44\ \mu N$.

If the deflection at the end of the beam is $1.3\ \mu m$, wh is the modulus of elasticity E_g of the gold alloy? (*Hint:* U the formulas of Example 9-2.)



PROB. 9.3-6

consider a section at a distance x
consider right part as FBD



$$\Sigma M_x = 0 \Rightarrow M + \frac{q(L-x)^2}{2} = 0$$

$$\Rightarrow M = -\frac{q}{2}(L-x)^2$$

we know that

$$EI\,v'' = M = -\frac{q}{2}(L-x)^2$$

$$v'' = \frac{-q}{2EI}(L-x)^2 \quad \text{—①}$$

Integrating ①

$$v' = \frac{q}{2EI}\frac{(L-x)^3}{3} + C_1$$

at $x=0$ $v'(0)=0$

$$\Rightarrow 0 = \frac{q}{6EI}(L^3) + C_1 = 0$$

$$C_1 = -\frac{qL^3}{6EI}$$

so $$v' = \frac{q}{6EI}\left((L-x)^3 - L^3\right) \quad \text{—②}$$

integrating ②

$$v = \frac{q}{6EI}\left(-\frac{(L-x)^4}{4} - L^3x\right) + C_2$$

at $x=0$ $v(0)=0$

$$C_2 = \frac{q}{6EI}\frac{L^4}{4}$$

$$v = \frac{q}{24EI}\left(L^4 - (L-x)^4 - 4L^3x\right)$$

at $x = L$

$$v(L) = \frac{q}{24EI}\left(L^4 - 4L^4\right)$$

$$v(L) = \frac{-qL^4}{8EI} = -\delta_B$$

$$\Rightarrow E = \frac{qL^4}{8I\delta_B}$$

$$I = \frac{bt^3}{12} = \frac{(15\mu m)(0.87\mu m)^3}{12}$$

$$= .82\ (\mu m)^4$$

$$E = \frac{(qL)L^3}{8I\delta_B} = \frac{44\,\mu N\ (25\,\mu m)^3}{8 \times .82\ (\mu m)^4 \times 1.3\,\mu m}$$

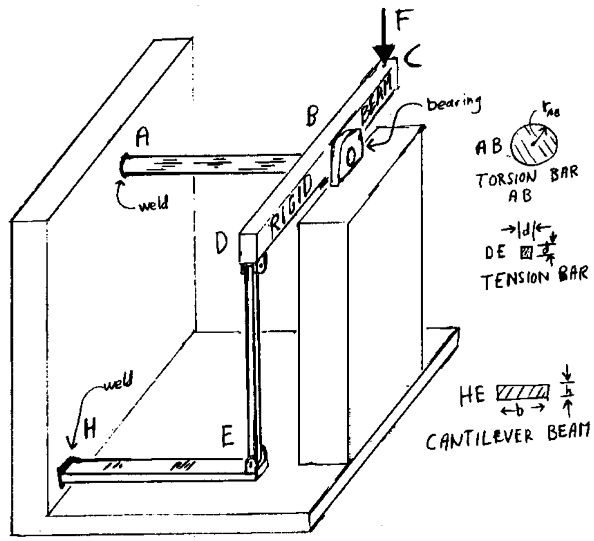
$$= 80.32\ GPa$$

T&AM 202 Synthesis HW question

Structure and geometry description:
Torsion bar AB is welded to a rigid support at its left end at A and is supported by a bearing at B. Rigid beam DBC is welded to, and rotates with, the right end of the bar AB. The load F is applied to point C on DBC. The width of this beam can be neglected when considering the length of AB. Tension rod DE hangs from a pin joint at D and pulls up on a pin joint at E. Cantilever beam HE is welded to the wall at H and has its end pulled up by the pin at E.

The load F tries to rotate the beam DBC clockwise (as viewed from the right). This motion is resisted by the torsional stiffness of rod AB and would also be resisted by the bending stiffness of HE but for the compliance of tension rod DE which diminishes this resistance.

Assume linear elastic behavior throughout. The structure is stress-free when there is no load ($F = 0$).



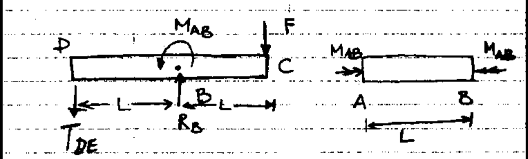
Given:
$L_{AB} = L_{HE} = L_{DB} = L_{BC} = L_{DE} = 0.5\,\mathrm{m}$
$r_{AB} = 2\,\mathrm{cm}$, $G_{AB} = 80$ GPa,
$d = 2\,\mathrm{mm}$, $E_{DE} = 200$ GPa,
$b = 2\,\mathrm{cm}$, $h = .5\,\mathrm{cm}$, $E_{HE} = 200$ GPa
$F = 1000\,\mathrm{N}$.

a) What is the deflection of point C?
b) What is the maximum shear stress on any surface in bar DE?
c) What is the maximum tension stress in bar HE?
d) What is the maximum tension stress on any surface in bar AB?

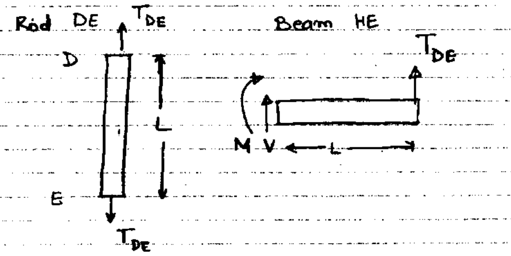
Sythesis HW Solution

FBD:

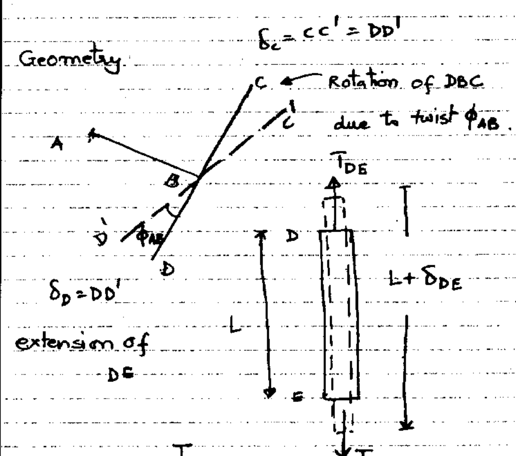
Rigid Member DBC   shaft AB

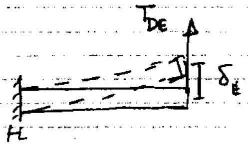


Rod DE   Beam HE



Geometry





deflection of point E due to bending

Note: 1. The member DBC is a rigid member so there will be no deformation of member DBC.

2. Deflection of point D in member CBD will be same as the sum of deflection of HE at E due to bending and extension of member DE due to tension.

So

$DD' = \phi_{AB} \cdot L$ (i) [Assuming small displacement. If displacement is large then we can consider

$$\frac{DD'}{L} = \sin(\phi_{AB})\Big]$$

also $DD' = \delta_E + \delta_{DE}$ —(ii)

from (i) and (ii)

$$\phi_{AB} L = \delta_E + \delta_{DE} \quad —①$$

strength of material result

A) Twist of AB

$$\phi_{AB} = \frac{M_{AB} L_{AB}}{G_{AB} J_{AB}} \quad —②$$

$$J_{AB} = \frac{\pi r_{AB}^4}{2} \; ; \; L_{AB} = L$$

B) Stretch of DE

$$\delta_{DE} = \frac{T_{DE} L_{DE}}{E_{DE} A_{DE}} \quad — (3)$$

$$A_{DE} = d^2 \; ; \; L_{DE} = L$$

C) Deflection of point E in beam HE

$$\delta_E = \frac{T_{DE} L_{HE}^3}{3 E_{HE} I_{HE}} \quad — (4)$$

$$I_{HE} = \frac{bh^3}{12} \; ; \; L_{HE} = L$$

(from problem 9.3/11)

FBDs and Mechanics

Consider FBD of DBC

$$\Sigma M_B = 0 \Rightarrow M_{AB} + T_{DE} L - F \cdot L = 0 \quad (5)$$

From eq (1)

$$\phi_{AB} L = \delta_E + \delta_{DE}$$

use eq^n (2) (3) and (4)

$$\frac{M_{AB} L}{G_{AB} J_{AB}} L = \frac{T_{DE} L}{E_{DE} d^2} + \frac{T_{DE} L^3}{3 E_{HE} \left(\frac{bh^3}{12}\right)}$$

and from eq (5)

$$T_{DE} = (FL - M_{AB}) \frac{1}{L}$$

$$\Rightarrow \frac{M_{AB} L^2}{G_{AB}\left(\frac{\pi r_{AB}^4}{2}\right)} = \left[\frac{L}{E_{DE} d^2} + \frac{L^3}{3 E_{HE}\left(\frac{bh^3}{12}\right)}\right] * (FL - M_{AB}) \frac{1}{L}$$

$$\Rightarrow M_{AB}\left[\frac{L^2}{G_{AB}\left(\frac{\pi r_{AB}^4}{2}\right)} + \frac{1}{L}\left\{\frac{L}{E_{DE} d^2} + \frac{L^3}{3 E_{HE}\left(\frac{bh^3}{12}\right)}\right\}\right] = F\left\{\frac{L}{E_{DE} d^2} + \frac{L^3}{3 E_{EH}\left(\frac{bh^3}{12}\right)}\right\}$$

$$\Rightarrow M_{AB} = \frac{(1000\,N)\left\{\frac{0.5\,m}{(200\,GPa)(2\,mm)^2} + \frac{(.5\,m)^3}{3\,(200\,GPa)\left(\frac{(2\,cm)(.5\,cm)^3}{12}\right)}\right\}}{\left[\frac{(.5\,m)^2}{(80\,GPa)\left(\frac{\pi}{2}(2\,cm)^4\right)} + \frac{1}{(.5\,m)}\left\{\frac{.5\,m}{(200\,GPa)(2\,mm)^2} + \frac{(.5\,m)^3}{3\,(200\,GPa)\frac{(2\,cm)(.5\,cm)^3}{12}}\right\}\right]}$$

$$\Rightarrow \boxed{M_{AB} \approx 496.91 \; N \cdot m}$$

use eq^n (5) $\quad 496.91\,Nm - 1000\,N * .5\,m + T_{DE} * .5\,m = 0$

$$\Rightarrow T_{DE} \approx \left(\frac{500 - 496.91}{.5}\right) N \approx 6.18\,N$$

a) deflection of pt C

as $L_{DB} = L_{BC}$

$\Rightarrow \delta_C = \delta_D$ (see fig on pg 6)

from eq^n (i)

$$\delta_C = \delta_D = DD' = \phi_{AB} L$$

$$= \frac{M_{AB} L^2}{G_{AB} J_{AB}}$$

$$= \frac{(496.91 \text{ N·m})(.5\text{ m})^2}{(80\text{ GPa})\left(\frac{\pi}{2}(2\text{cm})^4\right)}$$

$$= 6.18 \text{ mm}$$

b) Max shear stress on any surface in DE. DE is under pure tension and the max shear stress will be at an angle $45°$ from the longitudinal axis and its value will be

$$\tau_{max} = \frac{\sigma_x}{2}$$

$$\sigma_x = \frac{T_{DE}}{A_{DE}}$$

$$\tau_{max} = \frac{1}{2}\,\frac{6.18 \text{ N}}{(2\times10^{-3}\text{ m})^2}$$

$$= 772.25 \text{ KPa}$$

c) Max. Tension stress in beam HE (bending of beam solution)

$$\sigma_{max} = \frac{M\, y_{max}}{I}$$

$$M_{max} = T_{DE} * L$$

$$y_{max} = \frac{h}{2}$$

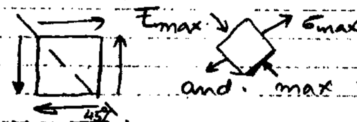
$$\sigma_{max} = \frac{T_{DE} * L \; h}{I * 2}$$

$$= \frac{(6.18\text{ N})(0.5\text{ m})(0.5\text{ cm})}{\frac{(2\text{cm})(.5\text{cm})^3}{12} \cdot 2}$$

$$= 37.068 \text{ MPa}$$

d) Max tension stress in bar AB.

From torsion theory we know that max shear stress in a shaft with a circular section occurs at the outer radius. Consider a stress element



and max tension stress will occur at a $45°$ angle.

$\sigma_{max} = \tau_{max}$ at a 45° section

$$\sigma_{max} = \tau_{max} = \frac{M_{AB}\, r_{AB}}{J} = \frac{2 M_{AB}}{\pi r_{AB}^3}$$

$$= \frac{2(496.91)}{\pi(2\times10^{-2}\text{ m})^3}$$

$$= 39.53 \text{ MPa}$$