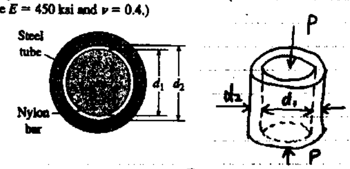
1.5.3   TAM 202 Spring 2003

HW 10 solution provided by Zhongping Bao.

1.5-3, 1.6-4, 1.6-11, 1.7-4, 1.7-14, 1.8-10 (due 04/01)

1.5-3 A nylon bar having diameter $d_1 = 2.75$ in. is placed inside a steel tube having inner diameter $d_2 = 2.76$ in. (see figure). The nylon cylinder is then compressed by an axial force P. At what value of the force P will the space between the nylon bar and the steel tube be closed? (For nylon, assume $E = 450$ ksi and $\nu = 0.4$.)



PROB. 1.5-3

i.e., find compression force P such that lateral strain

$$\varepsilon' = \frac{d_2 - d_1}{d_1} = -\nu\varepsilon$$

where ε is the axial strain. Thus

$$\varepsilon = -\frac{\varepsilon'}{\nu} = -\frac{(d_2 - d_1)}{\nu d_1}$$

$$\Rightarrow \quad P = \sigma A = E\varepsilon A = -EA\frac{(d_2 - d_1)}{\nu d_1}$$

(minus means compression).

Thus, in magnitude,

$$P = E \cdot \pi \cdot \left(\frac{d_1}{2}\right)^2 \cdot \frac{(d_2 - d_1)}{\nu d_1}$$

$$= 450 \text{ ksi} \cdot \pi \cdot \frac{2.75^2}{4} \text{ in}^2 \cdot \frac{0.01 \text{ in}}{0.4 \cdot 2.75 \text{ in}}$$

$$= 24.3 \text{ kip} \quad \text{i.e.} \quad 24.3 \times 10^3 \text{ lb}$$

Note: $\sigma = \frac{P}{A} = \frac{24.3 \times 10^3 \text{ lb}}{\pi \cdot \left(\frac{d}{2}\right)^2} = 4.1$ ksi. And σ_u for nylon is 6~12 ksi. σ_y of nylon is not available. Here, we have to assume it is still in elastic range for $\sigma = E\varepsilon$ to be valid.

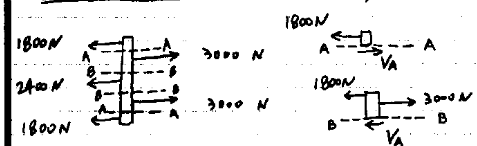
1.6.4

1.6-4 The connection shown in the figure consists of five steel plates, each 5 mm thick, joined by a single 6-mm diameter bolt. The total load transferred between the plates is 6000 N, distributed among the plates as shown.

(a) Calculate the largest shear stress in the bolt, disregarding friction between the plates. (b) Calculate the largest bearing stress acting against the bolt.

PROB. 1.6-4

FBD of bolt (this is the key part.)



Section A-A, shear force $|V| = 1800$ N

section B-B, shear force $|V| = 1200$ N

Thus, $|V|_{max} = 1800$ N

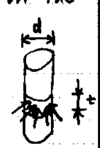
(a) maximum shear stress in bolt

$$\tau_{max} = \frac{V_{max}}{\pi\left(\frac{d}{2}\right)^2} = \frac{1800 \text{ N}}{\pi \cdot \left(\frac{6 \times 10^{-3} \text{ m}}{2}\right)^2}$$

$$= 63.7 \text{ MPa}$$

(b) maximum bearing stress

The maximum bearing force $F_{b\,max}$ on the bolt is 3000 N



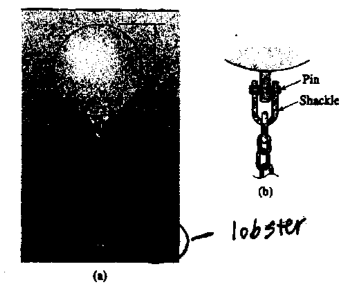
$$\sigma_{b\,max} = \frac{F_{b\,max}}{d \cdot t} = \frac{3000 \text{ N}}{6 \text{ mm} \cdot 5 \text{ mm}}$$

$$= 100 \text{ MPa}$$

1.6.11

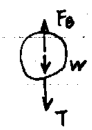
1.6-11 A spherical fiberglass buoy used in an underwater experiment is anchored in shallow water by a chain [see part (a) of the figure]. Because the buoy is positioned just below the surface of the water, it is not expected to collapse from the water pressure. The chain is attached to the buoy by a shackle and pin [see part (b) of the figure]. The diameter of the pin is 0.5 in. and the thickness of the shackle is 0.25 in. The buoy has a diameter of 60 in. and weighs 1800 lb on land (not including the weight of the chain).

(a) Determine the average shear stress τ_{aver} in the pin.
(b) Determine the average bearing stress σ_b between the pin and the shackle.



PROB. 1.6-11

FBD of the buoy



F_B = buoyant force of water pressure (sea water!)

$$= \rho g V$$

$$= 64 \text{ lb/ft}^3 \cdot \frac{\pi \cdot \left(\frac{60}{12} \text{ ft}\right)^3}{6}$$

$$= 4190 \text{ lb}$$

Equilibrium:

$$T = F_B - W = 4190 - 1800$$

$$= 2390 \text{ lb}$$

(a) Average shear stress in pin (under double shear)

$$\tau_{aver} = \frac{T/2}{A} = \frac{2390 \text{ lb}}{\pi \cdot \left(\frac{0.5 \text{ in}}{2}\right)^2} \cdot \frac{1}{2}$$

$$= 6086 \text{ psi}$$

(Continued)

ⓑ Bearing stress between the pin and shackle

The bearing area is

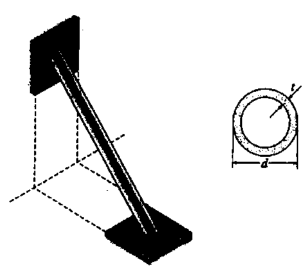
$$A = 2 \cdot 0.5\,in \cdot 0.25\,in = 0.25\ in^2$$

$$\sigma_b = \frac{T}{A_b} = \frac{2390\ lb}{0.25\ in^2}$$

$$= 9560\ psi$$

1.7-4 An aluminum tube serving as a brace in the fuselage of a small airplane (see figure) is designed to resist a compressive force. The outer diameter of the tube is $d = 25$ mm, and the wall thickness is $t = 2.5$ mm. The yield stress for the aluminum is $\sigma_Y = 270$ MPa and the ultimate stress is $\sigma_U = 310$ MPa.

Calculate the allowable compressive force P_{allow} if the factors of safety with respect to the yield stress and the ultimate stress are 4 and 5, respectively.



PROB. 1.7-4

$$A_{tube} = \pi\left(\frac{d}{2}\right)^2 - \pi\left(\frac{d-t}{2}\right)^2$$

$$= 176.7\ mm^2$$

$\sigma_y = 270$ Mpa with F.S. = 4

$\Rightarrow \sigma_{allow} = \sigma_y / F.S. = \frac{270\ Mpa}{4} = 67.5$ Mpa

$\sigma_u = 310$ Mpa with F.S. = 5

(Continued)

$\Rightarrow \sigma_{allow} = \sigma_u / F.S. = \frac{310\ Mpa}{5} = 62$ Mpa

Thus, the ultimate stress governs.

Allowable compressive force

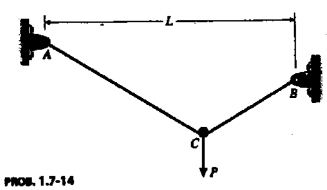
$$P_{allow} = \sigma_{allow} \cdot A_{tube}$$

$$= 62\ Mpa \cdot 176.7\ mm^2$$

$$= 11.0\ kN$$

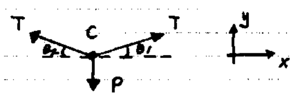
***1.7.14** A flexible string of length $L_S = 1.25$ m is fastened to supports at points A and B (see figure). Points A and B are at different elevations, with B being lower than A. The horizontal distance L between the supports equals 1.0 m. A load P hangs from a small pulley that rolls without friction along the string until it comes to rest in the equilibrium position at C.

If the string has a breaking strength $S = 200$ N, and if a factor of safety $n = 3.0$ is required, what is the allowable load P_{allow}?



PROB. 1.7-14

FBD of pt. C

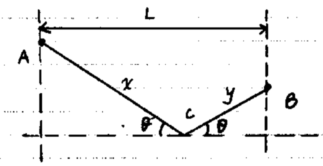


because of equilibrium of pt. C, and tension is the same everywhere in the string, to get $\Sigma F_x = 0$, $\theta_1 = \theta_2 = \theta$

$\Sigma F_y = 0 \Rightarrow P = 2T \sin\theta$

(Continued)

The next thing is to find $\sin\theta$.



suppose $|AC| = x$, $|BC| = y$

$$\begin{cases} x + y = L_s \\ x\cos\theta + y\cos\theta = L \end{cases}$$

$$\Rightarrow \cos\theta = \frac{L}{x+y} = \frac{L}{L_s}$$

$$\Rightarrow \sin\theta = \sqrt{L_s^2 - L^2} / L_s$$

Thus, $P = 2T\sin\theta = 2T \cdot \sqrt{1 - \left(\frac{L}{L_s}\right)^2}$

With breaking strength $S = 200\ N$ and

F.S. = 3.0, $T_{allow} = \frac{S}{F.S.} = \frac{200\ N}{3}$

$$\Rightarrow P = T_{allow} \cdot \sqrt{1 - \left(\frac{L}{L_s}\right)^2} \cdot 2$$

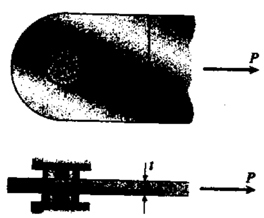
$$= \frac{200}{3} \cdot \sqrt{1 - \left(\frac{1}{1.25}\right)^2} \cdot 2$$

$$= 80\ N$$

#1.8.10

1.8-10 A bar of rectangular cross section is subjected to an axial load P (see figure). The bar has width $b = 60$ mm and thickness $t = 10$ mm. A hole of diameter d is drilled through the bar to provide for a pin support. The allowable tensile stress on the net cross section of the bar is 140 MPa, and the allowable shear stress in the pin is 80 MPa.

(a) Determine the pin diameter d_m for which the load P will be a maximum. (b) Determine the corresponding value P_{max} of the load.



PROB. 1.8-10

Allowable load based on tension in bar

$P_1 = \sigma_{allow} \cdot A_{net}$

$= 140\ Mpa \cdot (b-d) \cdot t$

$= 140\ Mpa \cdot (60\ mm - d) \cdot 10\ mm$

$= [1400 \cdot (60 - d)]\ N$ ①

(d in mm)

Allowable load based on shear in pin

$P_{allow} = 2\ \tau_{allow} \cdot \pi \cdot (\frac{d}{2})^2$

$= 2 \cdot 80\ Mpa \cdot \pi \cdot (\frac{d}{2})^2$

$= (40 \pi d^2)\ N$ ②

(d in mm)

Graph of eqn ① and ② (please see next page) and we can find where the two lines cross each other. And at that point, we will have max(P).

(Continued)

#1.8.10 (Cont'd)

ⓐ Maximum load occurs when $P_1 = P_2$

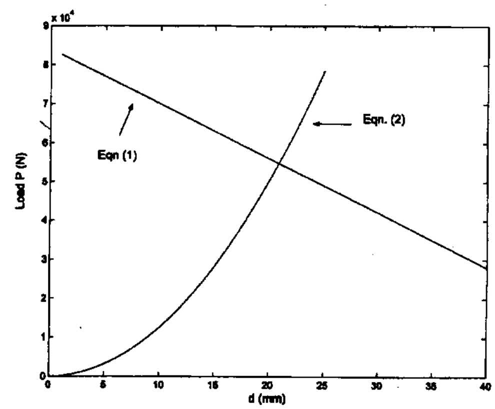
$8400 - 1400\,d = 40\,\pi\,d^2$

$\Rightarrow d_m = 20.88\ mm$

ⓑ Maximum Load

$P_{max} = 40\,\pi\,d_m^2 = 8400 - 1400\,d_m$

$= 54.8\ kN$



Matlab program:

```
d1 = [0:1:40];
P1 = 1400 * (60 - d1);

d2 = [0:1:25];
P2 = 40 * pi * (d2.^2);

plot (d1, P1);
hold on
plot (d2, P2);

Xlabel (' d (mm) ');
ylabel (' Load P (N) ');
```