M294 PII FA95 #1

25) (a) $\iint_{S_1} \text{curl}\,\vec{F}\cdot\vec{n}\,d\sigma = \int_C \vec{F}\cdot d\vec{r}$ by Stokes, where C

$= \int_C y\,dx + 8x\,dy = \iint_D (8-1)\,dx\,dy$ by Green, where D

$= 7(\text{area of } D) = \boxed{7\cdot\pi\cdot 9}$

(b) $\int_C d(xy^2z^2) = xy^2z^2\Big|_{(0,0,0)}^{(5,2,-1)} = \boxed{\cancel{9}}\ 20$

M294 SP96 F #2

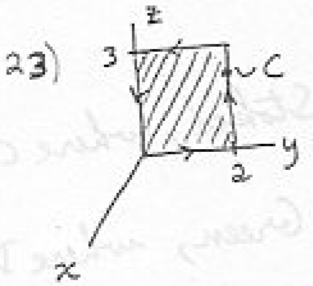
26)

(a) $x(t) = \cos t,\ y(t) = 2\sin t,\ 0 \le t \le 2\pi$ (b) $\dfrac{8x\vec{i} + 2y\vec{j} + \vec{k}}{\sqrt{64x^2+4y^2+1}}$

c) $\iint_S \vec{\nabla}\times\vec{v}\cdot\vec{n}\,d\sigma = \int_C \vec{v}\cdot d\vec{r} = \int_C x^3 dy = \int_0^{2\pi} \cos^3 t\ 2\cos t\,dt = 2\cdot\frac{3}{8}\cdot 2\pi$ (Stok

M294 PI FA93 #5

23)



$$\int_C (\underline{a}\times\underline{r})\cdot d\underline{r} = \iint_{\text{rectangle}} \text{curl}(\underline{a}\times\underline{r})\cdot\underline{i}\;dy\,dz \quad \text{by Stokes}$$

Calculate $\underline{a}\times\underline{r} = (a_2 z - a_3 y)\underline{i} + (a_3 x - a_1 z)\underline{j} + (a_1 y - a_2 x)\underline{k}$ if $\underline{a} = a_1\underline{i} + a_2\underline{j} +$

Then $\text{curl}(\underline{a}\times\underline{r}) = \left(\frac{\partial}{\partial y}(a_1 y - a_2 x) - \frac{\partial}{\partial z}(a_3 x - a_1 z)\right)\underline{i} + \cdots$

$= 2\underline{a}$

So $\int_C (\underline{a}\times\underline{r})\cdot d\underline{r} = \int_0^3\int_0^2 2\underline{a}\cdot\underline{i}\;dy\,dz = \boxed{12\,\underline{a}\cdot\underline{i}}$

M294 PI SP95 #2

24) Can evaluate directly or use Stoke's Thm., $\oint_C \vec{F}\cdot d\vec{r} = \iint_S \vec{\nabla}\times\vec{F}\cdot\vec{n}\,d\sigma$

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y+x & -x & zx^3y \end{vmatrix} = zx^3\hat{i} - 3zx^2\hat{j} - 2\hat{k}.$$

Choose S to be the surface $z=0$, $x^2+y^2\le 1$.

Note that a direction was not specified, but if you take the path C to be counterclockwise looking down on the x-y plane from $z>0$, then the corresponding $\vec{n}$ is $\vec{n}=\hat{k}$.

on S: $\vec{\nabla}\times\vec{F}\cdot\vec{n} = (0\hat{i} - 0\hat{j} - 2\hat{k})\cdot\hat{k} = -2$

$\therefore \iint_S \vec{\nabla}\times\vec{F}\cdot\vec{n}\,d\sigma = \iint_S -2\,dx\,dy = -2(\text{Area of } S) = \boxed{-2\pi}$

M294 PII FA90 #2

15) (a) $\underline{\nabla} \times \underline{F} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y\sin z & x\sin z & xy\cos z \end{vmatrix} = i(x\cos z - x\cos z) - \underline{j}(y\cos z - y\cos z) + \underline{k}(\sin z - \sin z) = \vec{0}$

(b). $\underset{\sim}{F} = \underline{\nabla} f \Rightarrow \frac{\partial f}{\partial x} = y\sin z \ (1)$; $\frac{\partial f}{\partial y} = x\sin z \ (2)$; $\frac{\partial f}{\partial z} = xy\cos z \ (3)$

$(1) \Rightarrow f = xy\sin z + g(y,z)$ $\quad (2) \Rightarrow f = xy\sin z + h(x,z)$ $\quad (3) \Rightarrow f = xy\sin z + \mathcal{J}(x,y)$

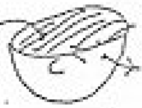
$1, 2, 3 \Rightarrow \boxed{f = xy\sin z + C}$

(c) $\int_{(0,0,0)}^{(1,1,\pi/2)} y\sin z\,dx + x\sin z\,dy + xy\cos z\,dz = \int_A^B \underline{F}\cdot d\underline{R} = \int_A^B \nabla f = f(1,1,\pi/2) - f(0,0,0) = 1.$

21) M294 F FA92 #5

(a) $\text{curl}\,\vec{F} = xe^{xy}\sin x\,\hat{i} - \hat{j}(ye^{xy}\sin x + e^{xy}\cos x - 2zx\ldots$
$+ (3)\hat{k}$
$\uparrow$ simple, try $\hat{n} = \pm\hat{k}$; flat surface
$S_1 : z = 1$

S_1



$$\iint_S (\vec{\nabla}\times\vec{F})\cdot\hat{n}\,d\sigma \overset{\text{stokes}}{=} \int_C \vec{F}\cdot d\vec{R}$$

$$\overset{\text{stokes}}{=} \iint_{S_1} 3\hat{k}\cdot(-\hat{k})\,dx\,dy = -3(\text{area inside}$$

$$= \boxed{-3\pi}$$

(b) $\iint_{\substack{\text{whole} \\ \text{sphere}}} (\vec{\nabla}\times\vec{F})\cdot\hat{n}\,d\sigma \overset{\text{div Thm}}{=} \iiint_{\text{inside}} \underbrace{\text{div}(\text{curl}\,\vec{F})}_{0}\,dx\,dy\,dz = \boxed{0}$

M294 PII SP87#6

5) $$\operatorname{curl}\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x-y & x+z & x^2 \end{vmatrix} = \vec{i}(0-1) - \vec{j}(0-0) + \vec{k}(1+1)$$

$$= -\vec{i} + 2\vec{k}$$

a) $\boxed{\operatorname{curl}\vec{F}(1,1,1) = -\vec{i} + 2\vec{k}}$

A small paddle wheel in the fluid rotates at a rate proportional to $|\operatorname{curl}\vec{F}|$ if $\vec{F}$ is velocity, provided the wheel axis is aligned with $\operatorname{curl}\vec{F}$. It rotates slower in other directions. So orient the wheel along the $\operatorname{curl}\vec{F}$ direction, which is

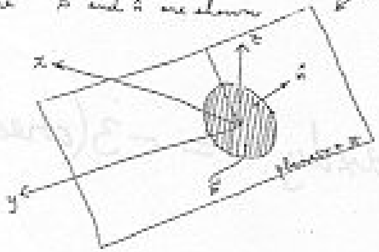
b) $\boxed{\dfrac{-\vec{i}+2\vec{k}}{\sqrt{5}}}$

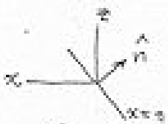
M294 PII SP88 #8

10) $$\int (x+z^2)\hat{j} \cdot d\vec{R} = \iint_{\Sigma} \operatorname{curl}(x+z^2)\hat{j} \cdot \hat{n}\, d\sigma$$

$\left.\begin{array}{l}\cos\theta\,\hat{i} + \sin\theta\,\hat{j} + \cos\theta\,\hat{k} \\ 0 \le \theta \le 2\pi\end{array}\right] \Rightarrow$ ellipse on plane shown

Where Σ and $\hat{n}$ are shown





$$\hat{n} = \frac{\hat{k}-\hat{i}}{\sqrt{2}}$$

$$\operatorname{curl}(x+z^2)\hat{j} = 0\hat{i} + 0\hat{j} + 1\hat{k} = \hat{k}$$

So the $\displaystyle\int = \iint_{\Sigma} \hat{k} \cdot \frac{\hat{k}-\hat{i}}{\sqrt{2}}\, d\sigma = \frac{1}{\sqrt{2}} \iint_{\Sigma} d\sigma$

$$= \iint_{\Sigma} \frac{d\sigma}{\sqrt{2}} = \iint_{\text{unit disk}} \frac{\sqrt{2}\,dx\,dy}{\sqrt{2}} = \pi 1^2 = \boxed{\pi}$$