MATH 293 PRELIM 2 SPRING 1994 # 6

13) a)

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 5 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\text{ROW 2 - 2 X ROW 3}} \left[\begin{array}{ccc|ccc} 1 & 3 & 5 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\text{ROW 1 - 5 X ROW 3}} \left[\begin{array}{ccc|ccc} 1 & 3 & 0 & 1 & 0 & -5 \\ 0 & 1 & 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{\text{ROW 1 - 3 X ROW 2}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -3 & 1 \\ 0 & 1 & 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$\Uparrow$ A^{-1}

b) TO CHECK THE RESULT USE THE FACT THAT $AA^{-1} = I$

$$\begin{bmatrix} 1 & 3 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -3 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I \checkmark$$

M294 PII SP95 #4

17) a)

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ -1 & 2 & 4 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow[\text{R03 - 2 ROW 1}]{\text{ROW 2 + ROW 1}} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 6 & 1 & 1 & 0 \\ 0 & 3 & -4 & -2 & 0 & 1 \end{array}\right] \xrightarrow{\text{ROW 3 - 3 ROW 2}}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 6 & 1 & 1 & 0 \\ 0 & 0 & -22 & -5 & -3 & 1 \end{array}\right] \xrightarrow{\text{ROW 3 / -22}} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 6 & 1 & 1 & 0 \\ 0 & 0 & 1 & 5/22 & 3/22 & -1/22 \end{array}\right] \xrightarrow{\text{ROW 2 - 6 ROW 3}}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & -8/22 & 4/22 & 6/22 \\ 0 & 0 & 1 & 5/22 & 3/22 & -1/22 \end{array}\right] \xrightarrow{\text{ROW 1 - 2 ROW 3}} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 6/11 & -3/11 & 1/11 \\ 0 & 1 & 0 & -4/11 & 2/11 & 3/11 \\ 0 & 0 & 1 & 5/22 & 3/22 & -1/22 \end{array}\right] \xrightarrow{\text{ROW 1 + ROW 2}}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2/11 & -1/11 & 4/11 \\ 0 & 1 & 0 & -4/11 & 2/11 & 3/11 \\ 0 & 0 & 1 & 5/22 & 3/22 & -1/22 \end{array}\right]$$

$\Uparrow$ A^{-1}

b)

$$A^{-1}A = AA^{-1} = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 2 & 4 \\ 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2/11 & -1/11 & 4/11 \\ -4/11 & 2/11 & 3/11 \\ 5/22 & 3/22 & -1/22 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I \checkmark$$

c)

$A\underline{x} = \underline{b}$

$$\underline{x} = A^{-1}\underline{b} = \begin{bmatrix} 2/11 & -1/11 & 4/11 \\ -4/11 & 2/11 & 3/11 \\ 5/22 & 3/22 & -1/22 \end{bmatrix} \begin{bmatrix} 5 \\ -2 \\ 7 \end{bmatrix} = \begin{bmatrix} 40/11 \\ -3/11 \\ 6/11 \end{bmatrix}$$

Section 1.8

MATH 293 FINAL SPRING 1996 #9

21)

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\text{Row3}-\text{Row1}} \left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & -2 & 0 & -1 & 0 & 1 \end{array}\right] \xrightarrow{\text{Row3}+2\text{Row2}}$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 2 & -1 & 2 & 1 \end{array}\right] \xrightarrow{\text{Row3}/2} \left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1/2 & 1 & 1/2 \end{array}\right] \xrightarrow{\text{Row2}-\text{Row3}}$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1/2 & 0 & -1/2 \\ 0 & 0 & 1 & -1/2 & 1 & 1/2 \end{array}\right] \xrightarrow{\text{Row1}+\text{Row3}} \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1/2 & 1 & 1/2 \\ 0 & 1 & 0 & 1/2 & 0 & -1/2 \\ 0 & 0 & 1 & -1/2 & 1 & 1/2 \end{array}\right] \xrightarrow{\text{Row1}-2\text{Row2}}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/2 & 1 & 3/2 \\ 0 & 1 & 0 & 1/2 & 0 & -1/2 \\ 0 & 0 & 1 & -1/2 & 1 & 1/2 \end{array}\right]$$

⇑

A^{-1}

$AA^{-1} = I$

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} -1/2 & 1 & 3/2 \\ 1/2 & 0 & -1/2 \\ -1/2 & \boxed{1} & 1/2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I \checkmark$$

$\therefore$ (c)

M293 F SP96 #10

22) The answer is e).

M293 PII FA96 #2

23) a) $[A \,\vdots\, b] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & -1 & 2 \\ 0 & 1 & 2 & -1 \end{bmatrix} \xrightarrow[\text{row 1}]{\text{row 2 -}} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & -2 & 1 \\ 0 & 1 & 2 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

2nd eqn: $x_2 = -1 - 2x_3$ 1st eqn: $x_1 = 1 - x_2 - x_3$
$= 2 + x_3$

$$\boxed{x = \begin{bmatrix} 2+x_3 \\ -1-2x_3 \\ x_3 \end{bmatrix}}$$

check: $Ax = \begin{bmatrix} (2+x_3) + (-1-2x_3) + (x_3) \\ (2+x_3) \quad - (x_3) \\ (-1-2x_3) + 2(x_3) \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ ✓

b) $[A \,\vdots\, c] = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 1 & 0 & -1 & 1 \\ 0 & 1 & 2 & 2 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & -1 & -2 & -1 \\ 0 & 1 & 2 & 2 \end{bmatrix}$ } rows 2 & 3 now contradict each other

$\therefore$ $\boxed{\text{no soln}}$

c) A^{-1} cannot exist because $Ax = c$ would have a soln if it did.

293 PII SP98 #4

$[A|I]$

31 a) $\det B = 0 \Rightarrow B^{-1}$ does not exist; for A: $\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & -1 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 2 & -1 & 1 & 1 \end{array}\right]$

$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 2 & -1 & 1 & 1 \end{array}\right] \rightarrow \left[I \;\middle|\; \frac{1}{2}\begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{pmatrix} \right]$

$\therefore A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{bmatrix}$

#