5.5 Wave

MATH 294 SPRING 1983 FINAL # 5

5.5.1 **a**) Solve the wave equation with wave speed $c = 1$, boundary conditions: $u(0,t) = u(6,t) = 0$ and initial conditions $u(x,0) = 0, u_t(x,0) = 5\sin\left(\frac{\pi x}{3}\right)$.

b) Make a clearly labeled graph of $u(3,t)$ vs. t for your solution in part (a) above.

MATH 294 SPRING 1994 FINAL # 14

5.5.2 Verify that $u(t,x) = \frac{1}{2}[f(x+t) + f(x-t)]$ solves the initial value problem:

$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} \;\; t > 0, \;\; -\infty < x < \infty,$
$u(t=0,x) = f(x)$
$u_t(t=0,x) = 0.$

MATH 294 FALL 1986 FINAL # 9

5.5.3 **a**) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, $(0 < x < 1, 0 < y < 1)$ where $u = u(x,y)$ and $u(0,y) = 0, u(1,y) = 0, u(x,0) = 0$, and $u(x,1) = 2\sin(2\pi x)$.

b) Use your result from part (a) to solve

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, (0 < x < 1, 0 < y < 1)$$

where $u = u(x,y)$ and $u(0,y) = 0, u(1,y) = 2\sin(2\pi y), u(x,0) = 0, u(x,1) = 0$.

c) Use your result from part (a) and (b) to solve

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, (0 < x < 1, 0 < y < 1)$$

where $u = u(x,y)$ and $u(0,y) = 0, u(x,0) = 0, u(1,y) = 2\sin(2\pi y), u(x,1) = 2\sin(2\pi x)$.

MATH 294 FALL 1986 FINAL # 12

5.5.4 Find the solution to the initial/boundary value problem

$\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2}, 0 < x < L, t > 0$
$u(0,t) = u(L,t) = 0, t > 0$
$u(x,0) = 0, 0 < x < L$
$\frac{\partial u}{\partial t}(x,0) = \sin\left(36\pi \frac{x}{L}\right), 0 < x < L.$

MATH 294 SPRING 1987 PRELIM 2 # 2

5.5.5 Find the value of u at $x = t = 1$ if $u(x,t)$ satisfies:

$$\frac{\partial^2 u}{\partial t^2} = 2\frac{\partial^2 u}{\partial x^2}$$

$$0 = u(0,t) = u(3\pi,t)$$

with

$$u(x,0) = \sin(5x)$$

$$\frac{\partial u}{\partial t}(x,0) = \sin x$$

MATH 294 SPRING 1987 FINAL # 7

5.5.6 Find *any* non-zero solution $u(x,t)$ to

$$\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2} \text{with} 0 = \frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(1,t)$$

and the extra restriction that $u(0,0) \neq u(1,0)$.

MATH 294 FALL 1987 PRELIM 2 # 3

5.5.7 Find the solution of the initial-boundary-value problem

$$\begin{array}{l} \frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2} \quad 0 < x < 1 \quad t > 0 \\ u(0,t) = u(1,t) = 0 \quad t > 0 \\ \left.\begin{array}{r} u(x,0) = 0 \\ \frac{\partial u}{\partial t}(x,0) = x. \end{array}\right. \quad 0 < x < 1 \end{array}$$

MATH 294 SPRING 1988 PRELIM 2 # 5

5.5.8 Once released, the deflection u of a taught string satisfies the wave equation

$$\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$$

where x is position along the string and t is time. It is held fixed (no deflection) at its ends at $x = 0$ and $x = 2$. At time $t = 0$ it is released from rest with the deflected shape $u = 3\sin\left(\frac{\pi x}{2}\right)$. Make a plot of $u(1,t)$ versus t for $0 \leq t \leq 2$. Label the axes at points of intersection with the curve. (You may quote any results that you remember.)

MATH 294 FALL 1991 FINAL # 3

5.5.9 The displacement $u(x, y)$ of a vibrating string satisfies

$$\frac{\partial^2 x}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0$$

in $0 \leq x \leq 4,\ t \geq 0$ and the boundary and initial conditions

$$u(0,t) = 0, u(4,t) = 0, u(x,0) = 0, \frac{\partial u}{\partial t}(x,0) = f(x),$$

where

$$f(x) = \begin{cases} 1, & \text{when } 0 \leq x \leq 2 \\ 0, & \text{when } 2 \leq x \leq 4 \end{cases}$$

a) Find a series representation for the solution.
b) Write down the equation for the displacement of the string at $t = 4$.

MATH 294 FALL 1993 FINAL # 14

5.5.10 **a**) Solve the wave equation $(a^2 u_{xx} = u_{tt}$ to find the displacement $u(x, t)$ of an elastic string of length ℓ. Both ends of the string are always free $[u_x(0,t) = 0;\ u_x(\ell, t) = 0]$ and the string is set in motion from its equilibrium position, $u(x, 0) = 0$, with an initial velocity, $u_t(x, 0) = V_0 \cos \frac{3\pi x}{\ell}$. Assume for this problem that it is legitimate to differentiate any Fourier series term-by-term. If you use separation of variables, consider only the case with a negative separation constant.
b) Write the solution to the wave equation $\left(a^2 u_{xx} = u_{tt}\right)$ for the boundary conditions $u(0,t) = h_L$ and $u(\ell, t) = h_R$ with initial conditions $u(x, 0) = h_L + (h_R - h_L)\frac{x}{\ell}$ and $u_t(x, 0) = 0$.

MATH 294 SPRING 1994 FINAL # 8

5.5.11 Consider

$$u_{xx} = u_{tt} \quad -\infty < x < \infty,\ \ t > 0$$

$$u(x,0) = 0, u_t(x,0) = g(x),$$

where $g(x)$ is a given function.
a) Show that $u(x,t) = G(x+t) - G(x-t)$ satisfies the above wave equation and initial conditions for a suitable function $G(x)$. How are $G(x)$ and $g(x)$ related?
b) Find $u(x, t)$ if $u_t(x, 0) = g(x) = \frac{x}{1+x^2}$.

MATH 294 SPRING 1993 FINAL # 15

5.5.12 **a**) The solution to

$u_{tt} = u_{xx} \quad -\infty < x < \infty$
$u(x,0) = e^{-x^2}$
$u_t(x,0) = 0$ is of the form $u(x,t) = \varphi(x+t) + \varphi(x-t)$. Find the solution without using Fourier series.

b) Find the solution of

$u_{xx} = u_t \ \ 0 \leq x \leq 1$
$u(0,t) = 1$
$u(1,t) = 2$
$u(x,0) = 1 + x$

Hint: The solution may be time-independent.

MATH 294 FALL 1995 FINAL # 15

5.5.13 If $u(x,t) = F(x+t) + G(x-t)$ for some functions F and G,

a) Find expressions for $u(x,0)$ and $u_t(x,0)$ in terms of F and G.

b) If also $\left\{ \begin{array}{c} u_{tt} = u_{xx} \quad -\infty < x < \infty \\ u(x,0) = e^{-x^2} \\ u_t x, 0 = 0 \end{array} \right.$ find expressions for F and G, and sketch the graph of $u(x,t)$ when $t = 0, 1$, and 2.

MATH 294 SPRING 1998 PRELIM 1 # 4

5.5.14 Consider the following partial differential equation for $u(x,t)$,

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0, \ \ 0 \leq x \leq 1,$$

with boundary conditions $u(0,t) = u(1,t) = 0, \ t > 0$,
and initial conditions $u(x,0) = f(x)$ and $\frac{\partial u}{\partial t}(x,0) = 0, \ \ 0 \leq x \leq 1$.
which, if any, of the functions below is a solution to the initial/boundary-value problem? Justify your answer.

a) $u(x,t) = \sum\limits_{i=1}^{n} b_n e^{-\pi^2 n^2 t} \sin n\pi x, \ b_n = 2\int_0^1 f(x) \sin n\pi x dx$

b) $u(x,t) = \sum\limits_{i=1}^{n} b_n cos n\pi t \sin n\pi x, \ b_n = 2\int_0^1 f(x) \sin n\pi x dx$

MATH 294 SUMMER 1990 PRELIM 2 # 5

5.5.15 Consider the partial differential equation

$$(*)\ \frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2},\ \text{for } 0 \le x \le L,$$

with conditions

i) $u(0,t) = 0$,

ii) $u(\ell,t) = 0$

iii) $\frac{\partial u}{\partial t}(x,0) = 0$,

and

iv) $u(x,0) = f(x)$.

a) Verify that $u(x,t) = \sum_{i=1}^{n} C_n \sin\left(\frac{n\pi x}{L}\right)\cos\left(\frac{n\pi ct}{L}\right)$ is a solution to (*) and the conditions (i), (ii), and (iii).

b) Suppose $f(x) = \sin\left(\frac{\pi x}{L}\right)$. What values for the C_n's will satisfy condition (iv)?

c) For a general piecewise smooth function $f(x)$; Determine the formula for the C_n so that condition (iv) is satisfied.

MATH 294 FALL 1992 UNKNOWN # 4

5.5.16 Solve the initial-boundary-value problem

$$u_{tt} = u_{xx}, 0 < x < 1, t > 0,$$

$$u(0,t) = u(1,t) = 0, t > 0,$$

$$u(x,0) = 8\sin 13\pi x - 2\sin 31\pi x,$$

$$u_t(x,0) = -\sin 8\pi x + 12\sin 88\pi x.$$

MATH 294 SPRING 1996 FINAL # 5 MAKE-UP

5.5.17 Consider $u(x,y,z,t) = w(ax+by+cz+dt)$ where w is some differentiable function of one variable, and the expression $ax+by+cz+dt$ has been substituted for that variable.

a) Find restrictions on the constants $a, b, c,$ and d so that u will be a solution to the three dimensional wave equation $u_{xx} + u_{yy} + u_{zz} = u_{tt}$

b) Find a solution to the wave equation if (a) having $u(x,y,z,0) = 5\cos x$ and $u_t(x,y,z,0) = 0$.