3.1 1^{st} Order ODEs

MATH 294 ???? FINAL # 4 294UFQ4.tex

3.1.1 Find the general solution $y(x)$ of each of the following ODE's

a) $y' = cosecy$

MATH 294 FALL 1990 PRELIM 2 # 4 294FA90P2Q4.tex

3.1.2 Determine the general solutions of

a) $\dfrac{dy}{dx} = x^2(e^{-x^3} - 3y)$

b) $\dfrac{dy}{dx} - y = 3 + 2e^x$

MATH 294 FALL 1991 PRELIM 1 # 4 294FA91P1Q4.tex

3.1.3 **a**) Determine the general solution of

$$\frac{dy}{dx} = \frac{(x-y)y}{x^2}$$

b) Determine the solution to the initial value problem

$$\frac{dy}{dx} - ycosx = -4cosx, \ with \qquad y(\pi) = 2$$

MATH 294 FALL 1991 FINAL # 5 294FA91FQ5.tex

3.1.4 Determine the general solutions of the following first order equations:

a) $2x^2 + 2xcosy\dfrac{dy}{dx} + 1 = 0$

b) $(x^2 - y^2dy - xydx = 0$

c) $\dfrac{dy}{dx} = \dfrac{x[y + (x+1)e^{-x}]}{(x+1)}$

MATH 294 SPRING 1991 PRELIM 1 # 3 294SP91P1Q3.tex

3.1.5 Determine the solutions to the initial value problems

a) $y' - 3y = e^{-3x} - 2sinx, \qquad y(0) = 0$

b) $y' = \dfrac{y^3}{2x^2}, \qquad y(1) = 1$

c) $y' + \dfrac{2y}{x} = 5x^2, \qquad y(1) = 1$

MATH 294 SPRING 1992 FINAL # 1 294SP92FQ1.tex

3.1.6 Solve the following differential equations. You may leave your solution in implicit form.

a) $e^{x^2}dy + [\frac{ye^{x2}}{x} - 1]dx = 0$

b) $xe^y dy + [\frac{x^2+1}{y}]dx = 0, \qquad y(1) = 0$

c) $x^2\frac{dy}{dx} = y(x+2y)$

MATH 294 SPRING 1992 FINAL # 6 294SP92FQ6.tex

3.1.7 Evaluate the line integral $\int_C \mathbf{F}\cdot d\mathbf{R}$ where $\mathbf{F}(x,y) = (x+y^2)\hat{\imath} + (2xy+1)\hat{\jmath}$ and C is the curve given by $\mathbf{R}(t) = \sin(t^2\pi)\hat{\imath} + t^3\hat{\jmath},\ 0 \le t \le 1$.

MATH 293 FALL 1992 PRELIM 1 # 1 293FA92P1Q1.tex

3.1.8 Find all solutions of the following differential equations and where indicated, find the solution of the initial value problem

a) $\frac{dy}{dx} + \frac{2}{x}y = 4x, \qquad y(1) = 2$

b) $3y^2\frac{dy}{dx} - x = xy^3$

MATH 294 FALL 1992 PRELIM 1 # 1 294FA92P1Q1.tex

3.1.9 **a)** Find the general solution of

$$y' + 2xy = e^{-x^2}$$

b) Find the solution satisfying the initial condition y(0) = 2

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3.1.10 **a)** Find the general solutions of the following differential equations:

$$e^{2y}dx + 2(xe^{2y} - y)dy = 0$$

Hint for a): Write this equation as a linear differential equation of the form

$$\frac{dx}{dy} + P(y)x = Q(y)$$

and solve for x as a function of y.

MATH 294 SPRING 1993 PRELIM 1 # 1 294SP93P1Q1.tex

3.1.11 Solve the following initial value problems:

a) $y' + 3y = e^{-2x}, \qquad y(0) = 5$

b) $y'(x) = xy^3, \qquad y(2) = -1$

MATH 293 SPRING 1993 PRELIM 1 # 2 293SP93P1Q2.tex

3.1.12 **a**) Solve the initial value problem

$$\frac{dy}{dx} + \frac{y}{x} = x^3$$

if $y = 0$ when $x = 1$

MATH 294 SPRING 1993 FINAL # 1 294SP93FQ1.tex

3.1.13 Solve the initial value problem for y(x)

$$y' + xy = x \qquad y(0) = 2$$

MATH 294 FALL 1993 PRELIM 2 # 1 294FA93P2Q1.tex

3.1.14 Find y(x) if $\frac{dy}{dx} = x^3$ and $y(0) = -4$. For what values of x is y defined? (x and y are real)

MATH 294 FALL 1993 MAKE UP PRELIM 2 # 1 294FA93MUP2Q1.tex

3.1.15 Solve the following initial value problem in explicit form.

$$y' = \frac{2x}{(y + x^2 y)}, \qquad y(0) = -2$$

Determine the interval in which the solution is defined.

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3.1.16 Consider the differential equation

$$\frac{dx}{dt} + cot(t)x = sec(t), for 0 < t < \frac{\pi}{2}$$

a) What is the order of the differential equation?
b) What type of equation is it?
c) If possible, find an integrating factor for this equation.
d) Solve the equation for its general solution, x(t).
e) Find the unique solution which satisfies the condition that $x = \frac{\sqrt{2}}{2} ln(2)$ when $t = \frac{\pi}{4}$

MATH 294 SPRING 1994 PRELIM 2 # 1 294SP94P2Q1.tex

3.1.17 Solve the initial-value problems

a) $xy' + 3y = 3e^{x^3}, \qquad y(1) = 0$

b) $y' = e^{x-y}, \qquad y(0) = 0$

MATH 294 SPRING 1994 PRELIM 2 # 2 294SP94P2Q2.tex

3.1.18 Use the change of variables $y(x) = u(x)x$ to find the general solution $y(x)$ of

$$y' = \frac{y^2 + xy}{x^2}$$

MATH 293 FALL 1994 PRELIM 1 # 4 293FA94P1Q4.tex

3.1.19 Solve $\dfrac{dy}{dx} + \dfrac{y}{x} = x^3, \qquad y(1) = 0$

MATH 293 FALL 1994 FINAL # 1 293FA94FQ1.tex

3.1.20 Find the general solution of $\dfrac{dy}{dx} + 2y = 2xe^{-2x}$

MATH 294 SPRING 1995 PRELIM 2 # 1 294SP95P2Q1.tex

3.1.21 Find $y(x)$ for $xy' + 2y = sinx;\ y(\frac{\pi}{2}) = 1$

Do not solve for undetermined coefficients.

MATH 293 FALL 1995 PRELIM 1 # 5 293FA95P1Q5.tex

3.1.22 **a**) Find the unique solution of the problem, in the form $y = f(x)$, of

$$\frac{dy}{dx} = y^3, \qquad y(0) = \frac{1}{2}$$

b) Check your solution for part (a).

c) Solve, in the form $y = f(x)$,

$$\frac{dy}{dx} - e^{-x} = -4y, \qquad y(0) = 1$$

d) Check your solution for part (c).

MATH 294 FALL 1995 PRELIM 2 # 2 294FA95P2Q2.tex

3.1.23 **a**) Solve $y' - 3y = e^{-x}, \qquad y(0) = 5$

MATH 293 SPRING 1995 PRELIM 1 # 4 293SP95P1Q4.tex

3.1.24 Find the general solution of the differential equation

$$\frac{dy}{dx} + y = \frac{1}{e^x}$$

MATH 294 SUMMER 1995 PRELIM 2 # 3 294SU95P2Q3.tex

3.1.25 Solve the following

a) $y' + 2y = xe^{-x} + 1, \qquad y(0) = 2$

b) $\dfrac{dr}{d\theta} = \dfrac{r^2}{\theta}, \qquad r(1) = 2$

MATH 293 SPRING 1996 FINAL # 26 293SP96FQ26.tex

3.1.26 The solution to the initial-value problem

$$x\frac{dy}{dx} = x^3 + 3y,\ x > 0,\ y(1) = 4\ is$$

a) $-x^3(ln(x) - 4)$ b) $x^3(ln(x) + 4)$ c) $-x^3 ln(x) + 4$

d) $-x^3(ln(-x) - 4)$ e) none of the above

MATH 293 FALL 1996 PRELIM 1 # 4 293FA96P1Q4.tex

3.1.27 Any solutions? How many solutions $y(x)$ to the differential equation

$$\frac{dy}{dx} = \sqrt{y}$$

have both $y(0) = 0$ *and* $y(3) = 1$? (0?, 1?, 2?, 3?, ...17? ...∞?). Why?

MATH 293 FALL 1996 FINAL # 1 293FA96FQ1.tex

3.1.28 Simple ODEs. For each of the problems below find the most general solution to the given problem. If no solution exists answer 'no solutions'. You may use any method *except* computer commands.

a) $\dfrac{dx}{dt} = -7x \qquad with \qquad x(0) = 3$

MATH 293 FALL 1996 FINAL # 2 293FA96FQ2.tex

3.1.29 More ODEs. For each of the problems below find the most general solution to the given problem. If no solution exists answer 'no solutions'. You may use any method *except* computer commands.

c) $\dfrac{dx}{dt} = x^{\frac{1}{7}} \qquad with \qquad x(3) = 0$

MATH 294 FALL 1992 PRELIM 1 # 2 294FA92P1Q2.tex

3.1.30 Consider the equation $y' = y^2$: (a) Sketch the slope field in the region $-2 < x < 2, -2 < y < 2$. (b) Sketch the solution curve that satisfies $y(0) = \frac{1}{2}$. (c) Find an exact expression for the solution in part (b).

MATH 294 FALL 1994 FINAL # 1 294FA94FQ1.tex

3.1.31 b) Solve $y' - y = 2xe^{2x}$, $y(0) = 1$

Also sketch slope marks along the x axis and along the y axis, for problem (b).

MATH 294 FALL 1994 PRELIM 2 # 1 294FA94P2Q1.tex

3.1.32 **a**) Sketch the direction field for the equation

$$\frac{dy}{dx} = x(y-1).$$

b) Find and plot two solutions having initial conditions $y(0) = -2$, and $y(3) = 1$, respectively, showing the relation between the direction field and the solution curves.

MATH 294 SPRING 1995 PRELIM 2 # 2 294SP95P2Q2.tex

3.1.33 Consider the following two differential equations:

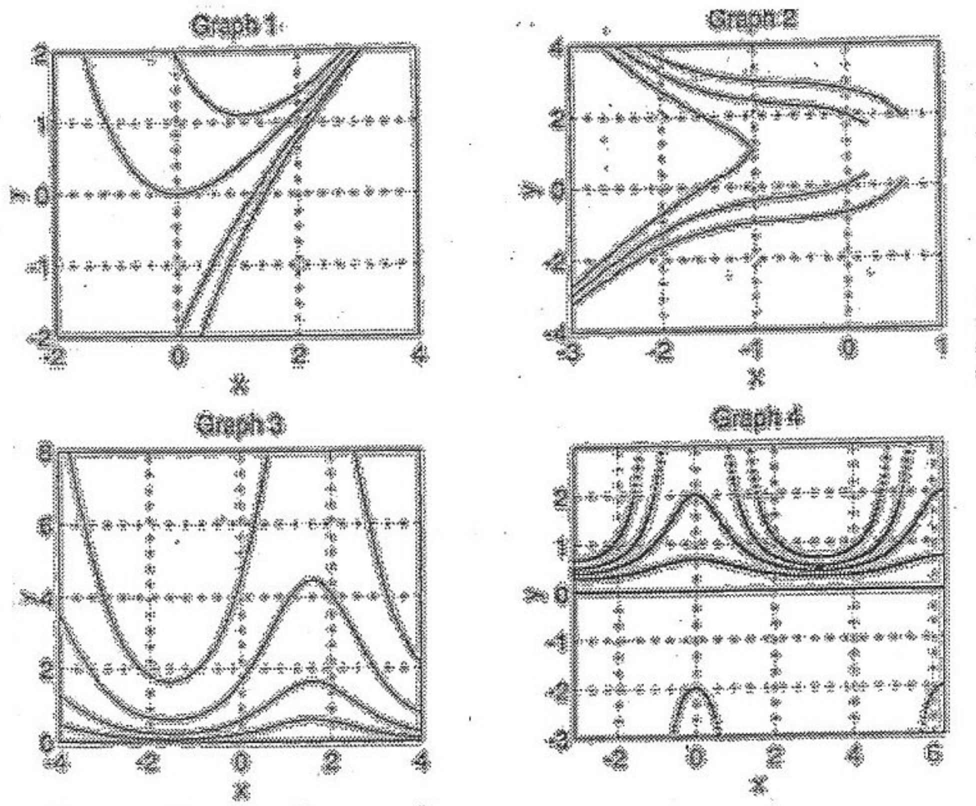
a) $y' = x - y$
b) $y' + y^2 sinx = 0$

Shown below are four sets of curves and four expressions, which are solutions to first-order equations; two of these are the integral curves and the solutions to the above equations. For both a) and b), identify the associated integral curve and solution. For each choice, give reason(s) why you believe your selections of a particular integral curve and the corresponding solution are correct. It should not be necessary to solve the equation to answer this, but you are welcome to solve them, if you wish.

Once you have selected the integral curves and solutions, sketch (on the appropriate diagram) the integral curve that passes through $y(0) = 1$. Do the same thing for the individual solutions: that is, choose the appropriate C so that the solution satisfies $y(0) = 1$.

FOUR POSSIBLE SOLUTIONS:

i) $ln|y| = sinx + C$
ii) $y^2 - 2y = x^3 + 2x^2 + 2x + C$, with condition $x^3 + 2x^2 + C < 1$
iii) $y^{-1} + cosx = C$ if y not $= 0$, or $y = 0$
iv) $(x - y - 1)e^x = C$


Graph 1
Graph 2
Graph 3
Graph 4
x
y

MATH 293 FALL 1996 PRELIM 1 # 1 293FA96P1Q1.tex

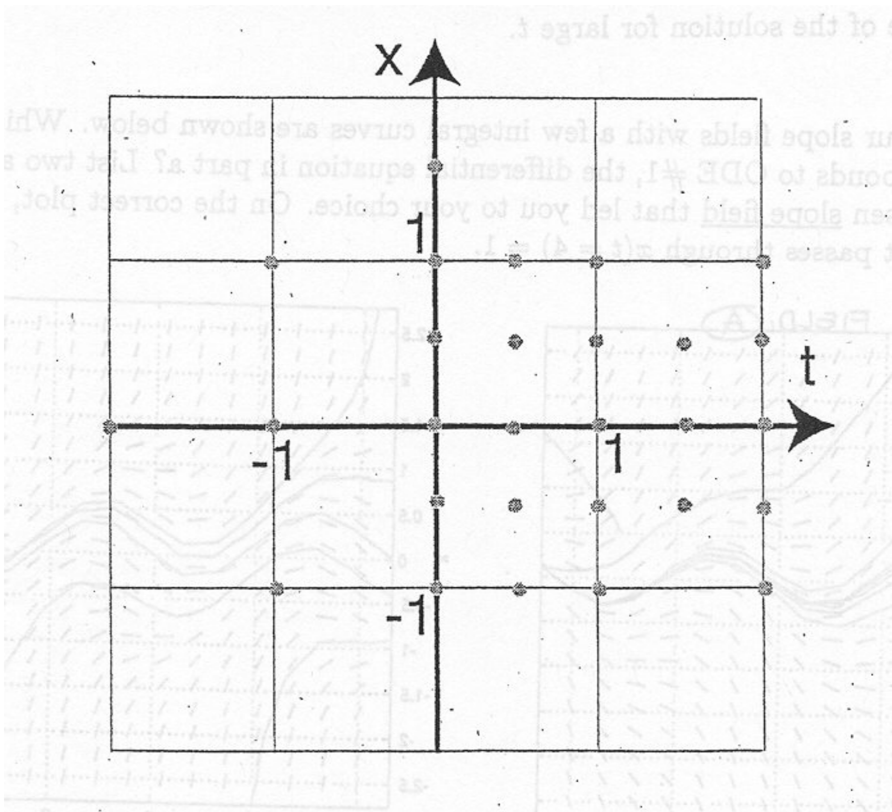
3.1.34 An initial value problem. You are given that $\frac{dx}{dt} = -xt$ **and** $x(0) = -1$

a) Find $x(2)$ (a tidy analytic expression is desired), and

b) (i) Sketch the slope field by putting appropriate firm dark marks at places where there are gray dots on the graphs below. You may put marks other places as well if you like.

(ii) Sketch the solution curve to part (a) on this graph for $-2 \leq t \leq 2$.

[Hint: If you had a calculator you would find a numerical value of $x(1) = -0.606..$]



MATH 294 FALL 1996 PRELIM 2 # 1 294FA96P2Q1.tex

3.1.35 **a**) Find the solution to

$$y' = y^3 \qquad (' = d/dx)$$

that passes through $y(0) = 1$

b) Sketch the slope field in the region $-2 < x < 2,\ -2 < y, 2$. Sketch the solution curve that satisfies $y(0) = 1$

c) Does this initial value problem have a unique solution? Why or why not? Does the solution exist everywhere in the interval $-2 < x < 2$? Why or why not?

d) What is the solution passing through $y(0) = 0$

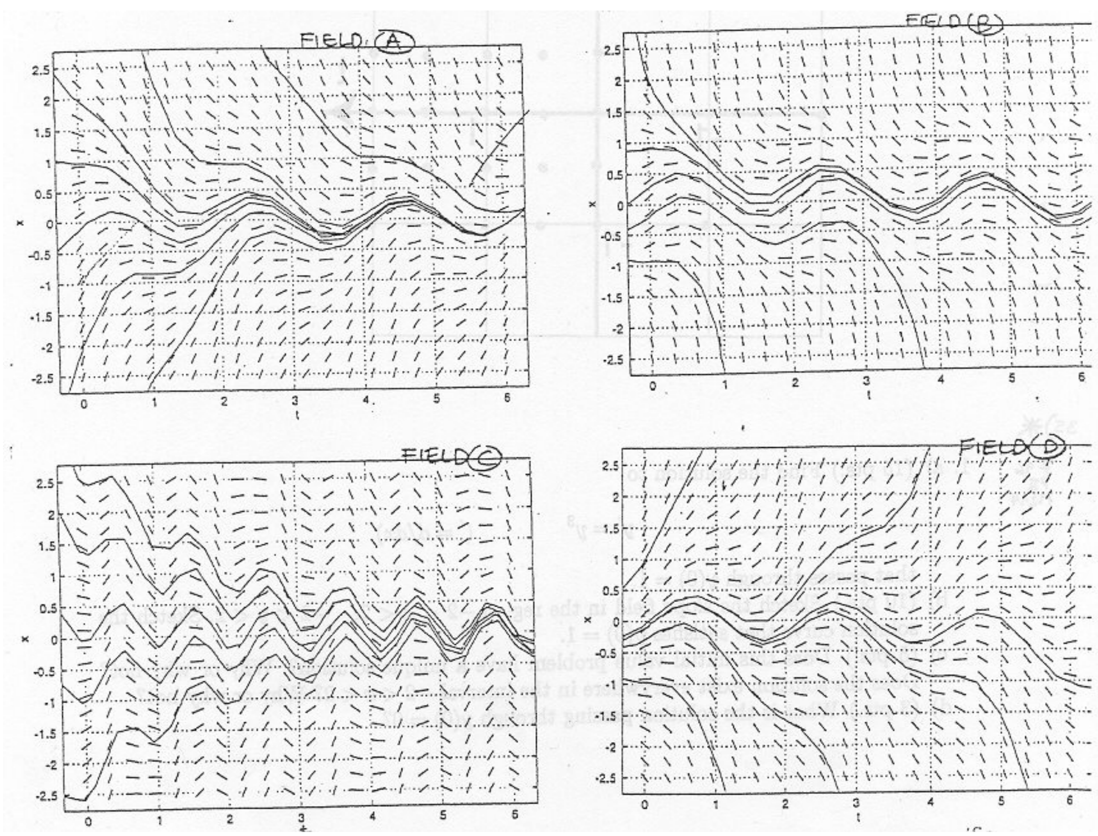
MATH 294 SPRING 1996 PRELIM 2 # 1 294SP96P2Q1.tex

3.1.36 **a**) Solve

$$\frac{dx}{dt} + x = cos3t \qquad ; x = x_o \;\; at \;\; t = 0. \qquad ODE\,1$$

Describe the nature of the solution for large t.

b) Four slope fields with a few integral curves are shown below. Which one of them correspond to ODE 1, the differential equation in part a? List two attributes* of the chosen slope field that led you to your choice. On the correct plot, show the solution that passes through $x(t = 4) = 1$



* By "attributes" we mean either a specific description of over-all behavior or a specific description of properties holding at a particular point of the picture.

c) The other plots correspond to the equations

$$x' + \frac{1}{2}x = 2sin6t \qquad ODE\,2$$

$$x' - x = cos3t \qquad ODE\,3$$

$$x' + x^2 = cos3t \qquad ODE\,4$$

Identify the slope fields that correspond to the first two equations. List two attributes* of each slope field or integral curve that led to your choice.

MATH 294 FALL 1996 PRELIM 2 # 2 294FA96P2Q2.tex

3.1.37 a) The equation

$$\frac{dy}{dx} = y^3 - y$$

has three equilibrium solutions. What are they? State whether they are stable or unstable, giving specific arguments for your choices. A graphical method will suffice; you do not need to solve the equation.

b) Use Euler's method with step size h = 0.5 to find $y(1)$ for the above equation, starting with two different initial values: $y(0) = 1$ *and* $y(0) = 0.1$. Three-place accuracy is acceptable. Show your "numerical" solutions on a (x, y) plot such as that used in part a).

MATH 293 FALL 1994 PRELIM 1 # 6 293FA94P1Q6.tex

3.1.38 Use Euler's Method to find an approximate solution of the Initial Value Problem given below at $t = 0.2$. Do two steps with $h = 0.1$

$$\frac{dy}{dt} = y^2 + t, \qquad y(0) = 1$$

MATH 293 SPRING 1995 PRELIM 1 # 6 293SP95P1Q6.tex

3.1.39 Consider the differential equation

$$\frac{dy}{dx} = x^2 + y^2, \qquad with \qquad y(0) = 0$$

Euler's method is applied to solve the equation with uniform step size h.
Let $y_1 = y(h),\ \ y_2 = y(2h), \ldots\ y_n = y(nh)$. Find y_1, y_2, y_3, y_4

MATH 294 SPRING 1991 PRELIM 1 # 4 294SP91P1Q4.tex

3.1.40 a) Consider the initial value problem

$$\frac{d^2y}{dx^2} + xy^2 - 3x^3\frac{dy}{dx} - x = 0 \qquad with \qquad y(1) = 0 \; and \; \frac{dy}{dx}(1) = 1$$

b) Write the equation as a first order system.

c) Use Euler's method with a step size of $\frac{1}{4}$ on the system to obtain the approximate solution for y at $x = \frac{3}{2}$

MATH 294 SPRING 1991 FINAL # 3 294SP91FQ3.tex

3.1.41 Consider the initial value problem

$$x^3y' - 4x^4(1 - y) = 0, \; with \; y(0) = 2$$

a) Obtain its analytic solution and,

b) Use Euler's method with a step size of 0.50 to obtain an approximate solution at $x = 1.5$

MATH 294 FALL 1990 MAKE UP FINAL # 3 294FA90MUFQ3.tex

3.1.42 Consider the initial value problem

$$x^2 - 2x^3(1-y) = 0, \ with \, y(0) = 2$$

a) Obtain an analytic solution of the initial value problem
b) Use Euler's method with a step size of 0.50 to obtain an approximate solution at $x = 1.5$
c) Write the computer program to implement Euler's method for this problem

MATH 294 FALL 1992 FINAL # 1 294FA92FQ1.tex

3.1.43 **a)** Solve the initial-value problem $y' + ty^2 = 0, \ y(0) = 1$
b) Compute two steps, at $\Delta t \equiv h = 0.1$ each, of an approximate solution to the equation in part (a) using Euler's method.

MATH 294 SUMMER 1990 PRELIM 2 # 1 294SU90P2Q1.tex

3.1.44 **a)** Solve the following *exact* differential equation

$$(x+y)dx + (x+y^2)dy = 0$$

b) Solve the following linear differential equation subject to the initial conditions $y(\pi) = \frac{2}{\pi}$
c) Use Euler's method with $h = 0.5$ to estimate $y(2)$ if $y(1) = 2$ and

$$y' = -xy$$

MATH 294 SPRING 1992 PRELIM 2 # 2 294SP92P2Q2.tex

3.1.45 Consider the initial value problem $\dfrac{d^2y}{dt^2} - ty\dfrac{dy}{dt} + y = t, \ y(0) = 0 \ \ and \ \ \dfrac{dy}{dt}(0) = 1$
a) Rewrite this problem as an initial value problem for a first order system of two differential equations.
b) Write down Euler's method for the system in part a) using step size h.
c) Use part b) with h = .1 to calculate the approximate solution at t = .1 and t = .2

MATH 294 SPRING 1996 MAKE UP PRELIM 2 # 1 294SP96MUP2Q1.tex

3.1.46 Solve the initial value problem

$$y' + \frac{1}{2}xy^3 = 0, \qquad y(0) = 1$$

What is $y(1)$?
a) Use Euler's method to compute an approximate solution $y_2(1)$ in two steps ($h = \Delta x = 0.5$) to the above equation.
b) Solve

$$(1 - 4xy^2)\frac{dy}{dx} = y^3$$

MATH 294 FALL 1995 FINAL # 1 294FA95FQ1.tex

3.1.47 Carbon-14 has a half-life of 6000 years, in this problem. Assume that the amount of carbon-14 in a sample satisfies the differential equation $y'(t) = -ry(t)$.

a) Determine r (leave your answer in exact form).

b) Find an expression for $y(t)$ if $y(0) = y_o$.

c) If some remains are discovered in which the current residual amount of carbon-14 is 10% of the original amount, give an (exact) expression for the age of the remains.

MATH 293 FALL 1993 PRELIM 1 # 2 293FA93P1Q2.tex

3.1.48 A certain process is governed by the ODE

$$\frac{dy}{dx} = 3 + x - x^2, \qquad y(0) = 1.$$

a) Using two steps in the Euler Method, with step length h = 0.2, find an approximate value for $y(0.4)$.

b) Calculate the difference between the exact value of y at $x = 0.4$ and your approximate value found in part a.

MATH 293 FALL 1996 PRELIM 1 # 2 293FA96P1Q2.tex

3.1.49 Numerical methods. Assume that $\dfrac{dx}{dt} = 2x + 1$ and $x(0) = 5$.

a) How would you modify the MATLAB script file and function file below to find $x(3)$ using Euler's method with a step size of .2. Clearly mark up the text below.

```
% Script file to solve (2a)...almost.
% This script file runs without error with the function 'math_is_fun'.
% It just doesn't quite solve the problem at hand.
x = .2 ;
for k=1 : 3 ;
x = x + math_is_fun(x);
end
'The answer is:', k
function xdot = math_is_fun(x);
% This function is in a separate file called 'math_is_fun.m'
xdot = .5 * (2 * x + 1) / 15 ;
```

b) Find, using arithmetic accurate to at least .0001, what Euler's method with a step size of .5 predicts for $x(1)$. (The hand arithmetic is all reasonably simple.)

c) If you have done part(b) correctly should your answer agree to within .0001 with the exact solution for x(1)? Why or why not? (A short clear sentence will suffice here.)

MATH 294 SPRING 1996 MAKE UP FINAL # 3 294SP96MUFQ3.tex

3.1.50 Peter S. is said to have paid \$24 for a piece of property 350 years ago. Compute the average rate of interest over the period such that the property is now worth \$7.5x10^{15}. If New Yorkers had invested an additional dollar each year, what would the investment be worth today? What if they withdrew \$1 each year?

MATH 294 SPRING 1996 FINAL # 3 294SP96FQ3.tex

3.1.51 A population y(t) changes at a rate proportional to the *square* of the number present. The initial population is 200 and the population at time 3 is 500.

a) Find the form of the differential equation for y as well as y itself.

b) Find all the constants required, both in the differential equation and in y.

c) At what value of t does the population approach ∞?

MATH 294 SUMMER 1995 PRELIM 2 # 4 294SU95P2Q4.tex

3.1.52 A population, $y(t)$, of bacteria grows according to the law

$$\frac{dy}{dt} = e^{-t}y.$$

a) Sketch the direction field for this equation over the range

$$0 \leq t \leq 10, \qquad 0 \leq y \leq 500$$

b) Solve for $y(t)$, $t \geq 0$, if the initial population is 100 individuals.

c) Graph the result of part b, and find the population limit, $lim_{t\to\infty}y(t)$

MATH 294 FALL 1993 MAKE UP PRELIM 2 # 2 294FA93MUP2Q2.tex

3.1.53 A certain college graduate borrows \$8000 to buy a car. Th lender charges interest at annual rate of 10%. Assuming that interest is compounded continuously and that the borrower makes payments continuously at a constant annual rate k, determine the payment rate k that is required to pay off the loan in three years. Also determine how much interest is paid during the three-year period.

MATH 293 FALL 1996 PRELIM 1 # 3 293FA96P1Q3.tex

3.1.54 **Cooling coffee.** A 145^o cup of coffee is left to cool in a 65^o room at 10AM. After 10 minutes (At 10:10 AM) it has cooled to 85^o. What was its temperature after 5 minutes (at 10:05 AM)?
You many quote any laws you remember for how things cool or you may state a reasonable law if you don't remember the standard one.
Remember to clearly define the variables you use.
[There are a variety of ways of doing this problem involving more or less mathematical reasoning. In the end, whatever the reasoning, all of the arithmetic that is needed can be done easily by hand.]

MATH 294 FALL 1994 PRELIM 2 # 4 294FA94P2Q4.tex

3.1.55 Assume that the rate of cooling of an object is proportional to the difference between the object's temperature and the surrounding temperature. If a pizza is removed from an oven at 600^oF, and its temperature at 6 pm is 500^oF, and at 6:15 pm is 400^oF, determine the time that the pizza came out of the oven. The kitchen is 100^o. (You may leave your answer in exact form.)

MATH 294 SPRING 1996 MAKE UP PRELIM 2 # 3 294SP96MUP2Q3.tex

3.1.56 It is observed experimentally that the surface temperature T of an object cools at a rate approximately proportional to the difference between T and θ, the fixed temperature of the surrounding environment.

a) Express this observation as a differential equation; if your equation includes a constant, mention the units and sign of this parameter.

b) Find an expression for temperature T at time t, assuming that initially the temperature was T_o.

c) Show that a second measurement of the temperature (say T_1) at t_1 can be used to evaluate the proportionality constant mentioned in a).

d) Using these facts, the time of a homicide can be estimated by measuring the temperature of the corpse at several times. Explicitly find an expression for the time t_d elapsed since death in terms of T_o, T_1, T_d (the temperature at t_d),t_1 and θ.

MATH 294 SPRING 1993 PRELIM 1 # 4 294SP93P1Q4.tex

3.1.57 A 30^oF frozen pizza is placed in a 600^o oven. At 6:35 pm the pizza temperature is 520^o and at 6:47 it is 560^o. At what time was the pizza first put into the oven? (You can leave your answer in exact but messy form.)

MATH 294 SPRING 1996 PRELIM 2 # 3 294SP96P2Q3.tex

3.1.58 The nasty architects decide to turn Beebe Lake (volume $Vmeters^3$) green for St. Patrick's Day festivities. At midnight on March 17 ($t = 0$), they dump an amount $G_o(kg)$ of pure green dye into the lake. Starting at the same time, engineers introduce red dye, having concentration r $(\frac{kg}{m^3})$, with a flow rate of $F(\frac{m^3}{sec})$, at the upper inlet to the lake. Water drains from the lake at the same rate. Assuming that the dyes are instantaneously and uniformly mixed throughout the lake's volume,

a) Write the differential equation whose solution represents the amount of green dye in the lake at any time $t \geq 0$. What initial condition(s) must be satisfied by the solution to this equation?

b) Do the same for the red dye, writing its differential equation and its initial condition(s).

c) Solve these equations. Plot the solutions to a) and b) as functions of t.

d) Find the time when the lake contains the same amount of red dye as green dye (i.e., when Beebe Lake returns to its nondescript brown color).

MATH 294 FALL 1993 PRELIM 2 # 2 294FA93P2Q2.tex

3.1.59 A tank which initially holds 1000 liters of salt water is stirred and drained at the rate of 3 liters per minute, while 3 liters of pure water are poured in per minute. If the salt concentration is 20 grams/liter after 5 hours, what was it initially?

MATH 294 FALL 1990 PRELIM 2 # 3 294FA90P2Q3.tex

3.1.60 Determine the analytic solution of $\dfrac{dy}{dx} = 2y^2 sinx, \;\; y(0) = 1$

MATH 294 FALL 1991 PRELIM 2 # 1 294FA91P2Q1.tex

3.1.61 **a**) Find the general solution of

$$(y + 2xsiny)dx + (x + x^2cosy)dy = 0$$

b) Determine the solution of the initial-value problem

$$(x^2 + 4)\frac{dy}{dx} + 2xy = x, \quad with \;\; y(0) = 0.$$

MATH 293 FALL 1998 PRELIM 1 # 1 293FA98P1Q1.tex

3.1.62 Consider the differential equation

$$\frac{dy}{dx} = x^2 + y^2$$

a) Sketch the curves (isoclines) in the xy plane on which the slope of the solutions are 0, 1, and 4. Show the direction field at one or more points on each curve.

b) Sketch the solution curve for which $y(0) = 0$.

MATH 293 FALL 1998 PRELIM 1 # 2 293FA98P1Q2.tex

3.1.63 Consider the initial value problem

$$\frac{dy}{dt} = \frac{2}{t} - y^2, \quad with \;\; y(1) = 0.$$

Use Euler's method to find an approximate value of the solution at t = 4 by taking three equals steps with step size h = 1.

MATH 293 FALL 1998 PRELIM 1 # 3 293FA98P1Q3.tex

3.1.64 **a**) Determine the solution of the initial value problem

$$\frac{1}{cos3x}\frac{dy}{dx} = 1 + y^2, \quad with \;\; y(0) = 1.$$

Hint:

$$\frac{dtan\theta}{d\theta} = sec^2\theta = 1 + tan^2\theta.$$

b) A more realistic model of air drags leads to a differential equation for the velocity that in its simplest form is

$$\frac{dv}{dt} = 1 - v^2.$$

Obtain the general solution to this equation in explicit form. Hint:

$$\frac{1}{1 - v^2} = \frac{1}{1 + v} + \frac{1}{1 - v}.$$

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3.1.65 Find the solution of the initial value problem

$$\frac{dy}{dx} = -2xy + e^{-x^2}, \;\; with \;\; y(0) = 2.$$

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3.1.66 A 120 liter tank initially contains 90 grams of salt dissolved in 90 liters of water. Brine, containing 2 grams per liter of salt, flows into the tank at the rate of 4 liters per minute. The mixture in the tank is well stirred and flows out of the tank at the rate of 3 liters per minute.

a) Find the differential equation governing the amount M(t) of salt in the tank while it is filling.

b) How much salt does the tank contain when it is full?

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3.1.67 Solve the following initial value problem

$$\frac{dy}{dx} = x^2e^{-4x} - 4y \qquad\qquad y(0) = -5$$

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3.1.68 Find explicit solutions to the equation

$$\frac{dy}{dx} = \frac{y}{1-x^2}$$

that satisfy these initial conditions:

a) $y(0) = 1$

b) $y(0) = -1$

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3.1.69 The following questions relate to the equation

$$\frac{dy}{dx} = -siny$$

a) Sketch the direction field for the equation in the region $-1 < y < 4$ *and* $0 < t < 5$.

b) In the same picture, sketch the graph of the particular solution of the equation which satisfies $y(0) = 1$. Do *not* solve the equation.

c) Let $y(t)$ be the solution such that $y(0) = 1$. Is the function $y(t)$ a bounded function of t? Justify your conclusion.

d) Let $y(t)$ be the solution such that $y(0) = 1$. Is the function $y(t)$ an increasing function of t? Is it a decreasing function of t? Or does it oscillate (increasing for some t values and decreasing for others)? Justify your conclusion.

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3.1.70 Set up Euler's method for the numerical solution of the equation

$$\frac{dy}{dx} = t^2 - y$$

With $y(1) = 3$. Use a stepsize of 2 and obtain an approximate value for $y(5)$.

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3.1.71 Use the following information to set up initial value problems for the amount of salt in the large tank as a function of time. Set up the equations but *do not solve* the equations.

a) A brine solution of salt flows at a constant rate of 6 L/min into a large tank that initially held 200 L of brine solution in which was dissolved 10 kg of salt. The solution inside the tank is kept well stirred and flows out of the tank at 6 L/min. If the concentration of the salt in the brine entering the tank is 0.5 kg/L, determine the mass of salt in the tank after t minutes.

b) Assume that the rate of brine flowing into the tank remains 6 L/min but that the brine flows out of tank at 8 L/min instead of 6 L/min.