MAE 4770/5770 Engineering Vibrations   Spring 2012   Prof.R.Rand

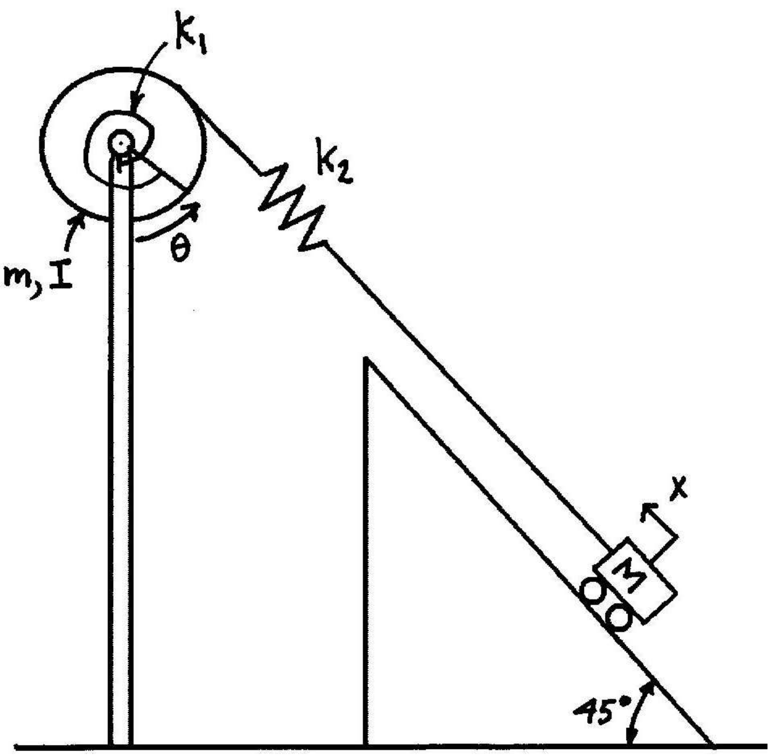
Prelim No.1

in class, Wednesday Feb.29, 2012

RULES: Closed book, closed notes, no computers, no calculators.

1. A rigid pole supports a wheel at its center as shown. The wheel has mass m and moment of inertia I about its center. The wheel is restrained by a rotational spring with spring constant k_1. A cable is attached to the wheel such that there is no slip. The other end of the cable is attached to a spring with spring constant k_2 which is itself attached to a mass M which moves on an inclined plane. Generalized coordinates x and θ are measured from equilibrium.

Use Lagrange's equations to compute the equations of motion for this system. Include gravity, neglect friction,



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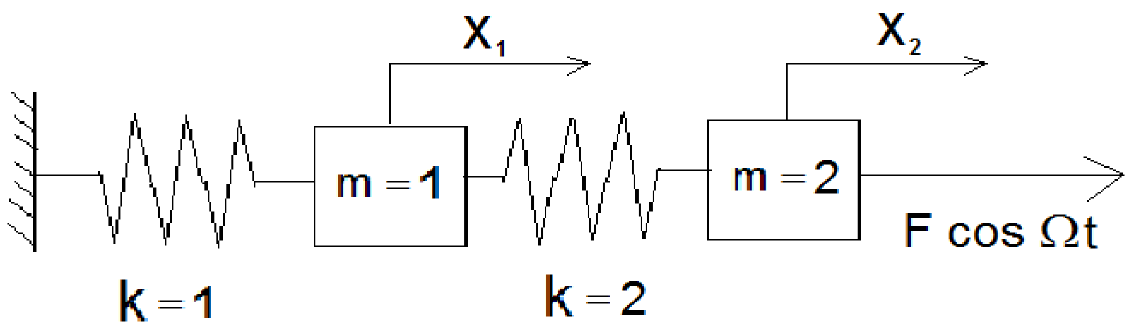
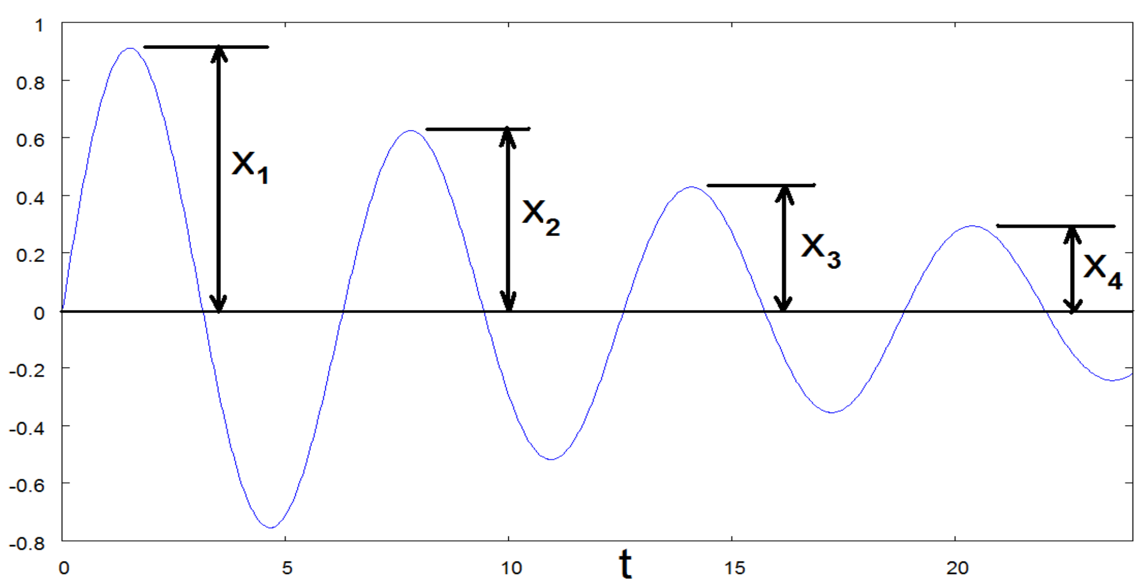
2. A 2 DOF system is driven by a sinusoidal forcing function as shown. What forcing frequency Ω gives the smallest amplitude of the steady state response of x_2? (Assume very light damping eliminates the complementary solution at steady state.)



3. An underdamped 1 DOF oscillator has EOM

$$\ddot{x} + 2n\dot{x} + x = 0$$

a) Find the form of $x(t)$ for the initial condition $x(0) = 0$.
b) Let x_i (for $i = 1, 2, 3, ...$) be the value at the i^{th} successive peak of $x(t)$. Given that $x_1/x_3 = 2$, find an expression for the damping coefficient n.



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Prelim No.2

in class, Wednesday April 4, 2012

RULES: Closed book, closed notes, no computers, no calculators.

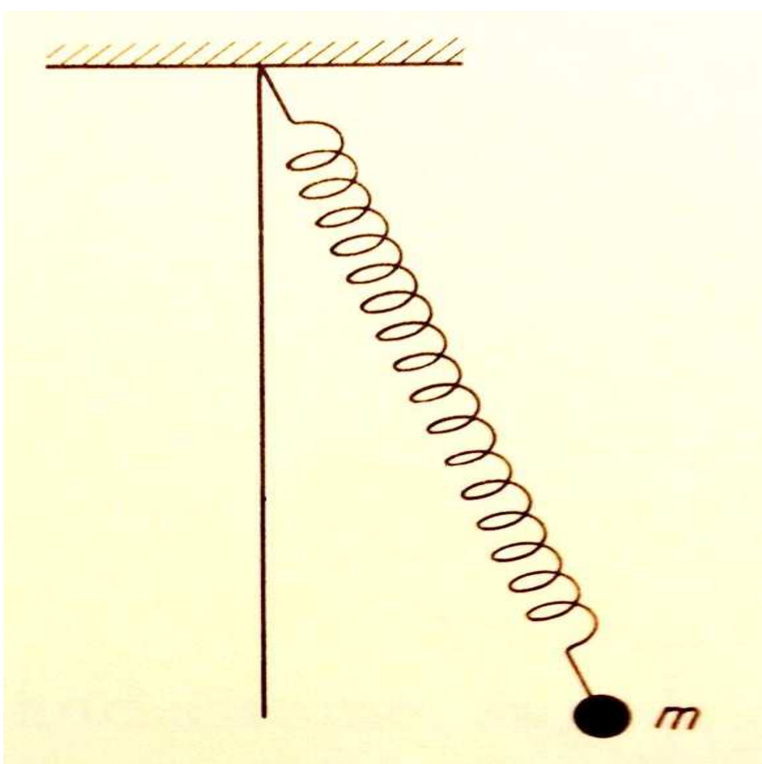
1. Determine the natural frequencies of a rod undergoing longitudinal vibrations if one end ($x = 0$) is held fixed, and the other end ($x = \ell$) is free.
E= Young's modulus
A= cross sectional area
ρ= mass per unit length
ℓ= length of rod

2. Same problem as 1 above, by Rayleigh's method. Determine a bound on the lowest natural frequency using

$$V(x) = a + bx + cx^2$$

Choose a,b,c such that the B.C. are satisfied.

3. An "elastic pendulum" consists of a mass m suspended under gravity g by a weightless elastic spring of unstretched length ℓ and having spring constant k. Derive the equations of motion using Lagrange's equations. Define all variables and coordinates.



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Final Exam

Tuesday May 15, 2012

RULES: Closed book, closed notes, no computers, no calculators.

1. A vehicle moves at velocity v over a bumpy surface modeled as a sine wave:

$$y = A \sin px$$

The vehicle's suspension system is modeled as a frictionless mass-spring system with equivalent mass m and spring constant k.

What speed v_{res} causes resonance?

Hint: Draw a free body diagram, use it to determine a governing differential equation, solve the equation.

2. Use the method of harmonic balance to determine an approximate expression for any limit cycles which are exhibited by the following nonconservative system:

$$\frac{d^2x}{dt^2} + x = 0.1 \left[\frac{dx}{dt} - \left(\frac{dx}{dt} \right)^3 \right]$$

You may find the following trig identities helpful:

$$\sin^3 t = \frac{3 \sin t - \sin(3t)}{4}$$

$$\cos t \sin^2 t = \frac{\cos t - \cos(3t)}{4}$$

$$\cos^2 t \sin t = \frac{\sin(3t) + \sin t}{4}$$

$$\cos^3 t = \frac{\cos(3t) + 3 \cos t}{4}$$

3. A point mass moves without friction on a curve given by $y = x^4$ where y is vertically upward. Find the governing ODE via Lagrange's equations. Include gravity, omit friction.

4. Use Rayleigh's quotient with the trial function

$$\phi(x) = \sin\frac{\pi x}{L}$$

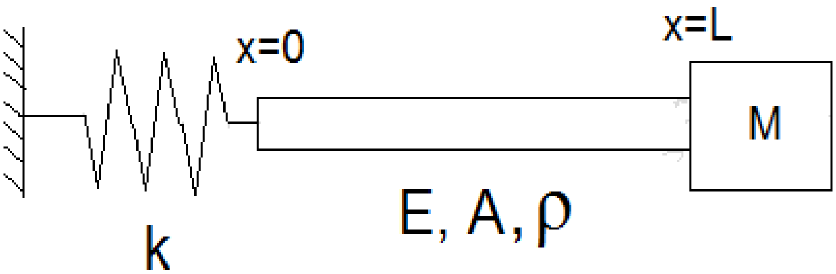
to approximate the lowest natural frequency of a simply supported (pinned-pinned) beam (transverse motion) with a concentrated mass m at its midspan.

Hint:

$$R(f) = \frac{\int_0^L EI\left(\frac{d^2 f}{dx^2}\right)^2 dx}{\int_0^L \rho A f(x)^2 dx + \sum_{i=1}^{n} m_i f(x_i)^2}$$

5. The longitudinal motion of a rod
a) is restrained by a spring of stiffness k at $x = 0$, and
b) is attached to a mass M at $x = L$.

Find the B.C. at $x = 0$ and at $x = L$.



6. Find the natural frequencies of the following system: