Practice Dynamics and Control Q Exam Questions

SiGMA

v.3. Editing history: May 2007 (Michael Sherback), March 2008 (Allen Hurst, Nisar Ahmed),

May 2009 (Mark McClelland, Allen Hurst, Nisar Ahmed)

Abstract

This document contains practice questions for the Q exam in dynamics and control. The origins of these questions are indicated. Some questions were asked by professors on actual Q-exams; some were invented by professors and passed down by word of mouth. The rest were invented by many of your fellow MAE (or affiliated) grad students. All questions were inserted by the above editors at various points in the space-time continuum.

I. Intro

It seems that students typically see one question each from dynamics, linear systems, and control. The following sections address these topics. If you took classes from certain professors, they will probably ask things related to the classes. Questions may straddle categories. If Prof. Lipson is involved he will come at you out of left field. Answering a question will result in follow-up questions addressing perceived weaknesses or deeper aspects.

A. Dynamics Questions

1) Rear Wheel Drive up a hill: This was asked by Prof. Psiaki in ???.

You live in Ithaca and need to drive up steep icy hills. Would you rather have a front or rear wheel drive car? Why?

2) Tipping over a swing set: This was asked by Prof. Psiaki in ???.

A child of mass m swings on a swing set of mass M. Under what conditions will the swing set tip over if it is not affixed to the ground?

(Note: there are two other variants of the swing question:

a. at would point in the swing's path would you apply a finite force to get the child the most height?

b. at what point in the swing's path should the child jump off to get the most horizontal distance?)

3) Flag on a bike: This was asked by Prof. Psiaki in Spring 2003.

You are riding a bike. It is windy. Which way will a flag on your bike point? (there must have been more to this)

4) Throwing dirt vs. carrying it: This was asked by Prof. Psiaki in Spring 2004.

You need to move dirt from the bottom of hill to the top. Should you toss dirt with a shovel, or use a wheelbarrow? Why? If you throw dirt, at what angle should it be thrown? Why?

5) Pushing on a bike pedal: This was asked by Prof. Psiaki in ???.

If you push backwards on the pedal of a bicycle when the crank points down, will the bike move forward or backwards? Why?

6) Pedaling twice as hard: Invented by us.

You are pedaling a single-speed bike up a constant hill at a steady speed. You are suddenly invigorated and start to exert twice the torque. What will happen? Is this physically reasonable? What is the effect of drag?

7) Disc on a peg: This was asked by Prof. Peck in Spring 2004.

Derive the equations of motion for a disc suspended from a peg through a hole in an arbitrary location on the disc.

8) Vibrations in a flimsy dumbbell: This was asked by Prof. Garcia in Spring 2003.

Draw a thin circular beam with masses at each end. What will the modes be? If it is a satellite, which ones would you care about?

9) Robot arm: This was asked by Prof. Valero-Cuevas in Spring 2007.

Planar two-link robot arm. Write the Jacobian from joint torques to endpoint forces. This question was not remembered in any more detail.

10) Vibrations in a 2D airplane: This was asked by Prof. Garcia in 2007.

Suppose you modeled an airplane as a large center mass for the fuselage, linked to two smaller masses for the wings. How many modes are there and what are the directions of the modes? (Note: this question also appears somewhere in the vibrations textbook by Inman).

11) Air hockey table: This was asked by Prof. Peck in ???

You have a 2D air hockey table with a rectagular puck on it. Suppose you can place four thrusters on the puck to move it around. How should you arrange the thrusters on the puck so that you can move it anywhere on the table? Find the equations of motion for the puck. Find the Jacobian to go from the thruster space to the x-y table position space.

12) Letting air out of a balloon: This was invented by Prof. Peck

Suppose you fill up a balloon with air and hold the air hole pinched shut. Find the EOM for the balloon once you release it.

13) Plank on two rollers: Invented by us

A plank of wood sits horizontally on top of two adjacent circular rollers (which are initially idle). The rollers are made of rubber and their centers are aligned. The roller on the left can spin clockwise and the roller on the right can spin counterclockwise. The rollers are suddenly turned on, with a very tiny delay between activation of the left and right rollers. Find the equation of motion for the plank of wood. How would the motion change if the plank were made of marble?

14) Pizza in a baker's oven: Invented by us

You are putting a pizza in an oven with a giant wooden pizza paddle. Suppose the oven is made of stone and has some known coefficient of friction. What percentage of the pizza needs to be on the paddle to remove it from the paddle and place it successfully in the oven?

15) Springy double pendulum: Invented by Prof. Garcia

Find the EOM and mode shapes for a double pendulum that is constructed as follows: a stiff massless rod hinged to the ceiling at one end, and attached by a hinge at its other end to a spring, which has a mass M attached at its other end.

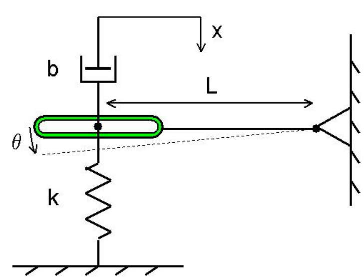
16) Space potato: Asked by Prof. Peck in Spring 2007

You are given a 3D potato that is hinged to a ceiling, such that it can pivot in any direction. Find an expression for the angular momentum of the potato if you are an observer who is not sitting on the potato.

17) Soup cans rolling down an incline: Invented by us

You have 2 soup cans of equal size and mass. One can contains thin and watery chicken noodle soup; the other can contains thick and chunky chili. You roll both cans down a slope at the same time. Which soup can 'wins' the race?

18) Math mechanism: Invented by us.



You are given the machine shown above. The input is x, the output is θ. For small θ and negligible masses, find the transfer function (assume zero initial conditions). Based on the transfer function, what mathematical operator does this system represent? What type of function is its operand in the time-domain? Find the response if x is a unit-step.

19) Two masses and a spring with friction: Invented by Prof. Ruina

There are two masses sitting on a rough surface; the masses are attached by a spring. You apply a horizontal force to one of the masses. Describe the motion of the masses and find their EOM. Compare/contrast the motion of the system under these conditions with its motion as $k \to \infty$? Does this make sense? Can you explain why this occurs?

20) Two hinged rods: This was asked by Prof. Moon in 2004.

Draw two rods that are hinged. Draw a ground plane. Describe what happens when you let go of the them. How can you find equations of motion? How many degrees of freedom exist? Are your assumptions realistic? Would the system rotate before it hits the ground?

21) Blocks with single spring: Invented by us.

You have a two masses with a spring between them. Each mass has an external force. Describe how to find the equations of motion. What are the eigenvalues and eigenvectors? What do these mean? How can you tell if the system is controllable or observable? What if instead of using coordinates x_1 and x_2 to model the motions of the blocks, you used the coordinate q (the distance between the blocks)?

22) Shopping cart: Invented by Prof. Ruina.

When you push a shopping cart forward it (should) go straight, but when you push it backwards it flips around. Why does this happen? Describe reasonable assumptions.

23) Rolling ball: Invented by us.

Draw a bowl. Imagine you release a ball into the bowl, find the equations of motion. What if the bowl was not spherical?

24) Angled inverted pendulum: You are given an inverted pendulum on a cart, with a force applied on the cart. How would you find the dynamics? What if the pendulum started at 25 degrees? What guarantees can you make about the system?

25) Monkeys on a rope: Invented by us.

Three monkeys are on a rope attached to a ceiling. The monkey farthest up the rope has a constant downward acceleration; the monkey in the middle of the rope has a constant upward acceleration; the monkey on the bottom has a constant upward acceleration. Find an expression for the tension at the top of the rope. Now suppose the rope is attached to the ceiling by the spring. What is the natural frequency of the system?

26) Funky see-saw: This was asked by Prof. Peck in Spring 2009 (this is similar to the space potato).

Given an arbitrarily shaped 3D seesaw with a universal joint at the center of mass. How many degrees of freedom are there in the system. What is the kinetic energy of the system, what is the potential energy? Derive the equations of motion. Now consider a traditional pendulum with a pivot point at the center of mass but with both a torsional spring and damper at the pivot. Write the equations of motion. What is the Lagrangian for the system? What is the total energy? Given the above, how can you write the damping coefficient c in terms of energy. What are the necessary conditions for this system to be stable?

B. Controls Questions

1) Control of a resonant plant:
This was asked by Prof. Campbell in Spring 2004.

Draw a Bode plot of a plant with many lightly damped resonances. What would you do to control this?

2) Delay and feedback.:
Asked by Prof. Campbell in 2007.

Given a plant $1/(s^2+1)$, and a delay approximated by (s-10)/(s+10), draw the bode plot for a series connection. Discuss feedback issues.

3) Delay, phase, and instability.:
Invented by us, similar to the Campbell 2007 question.

Draw a mass-spring-damper system attached to a wall. Give the transfer function. Suppose you want to control it. What do you do? If feed-forward is proposed, what might go wrong? If proportional negative feedback is proposed, can it ever go unstable? If you implement it and it actually does go unstable, speculate why. Could delay cause instability? Explain why using Bode or Nyquist plots.

4) Feedback connection of stable plants:
This was asked by Prof. D'Andrea in Spring 2005.

You have two stable plants. You connect the output of the first to the input of the second, and the output of the second to the input of the first. Is this stable?

5) Riding a bike backwards:
This was asked by Prof. Psiaki in Spring 2007.

The dynamics of a backwards bicycle are given as follows: $\ddot{x}+\dot{x}-12x=\dot{u}-2u$. Find the transfer function for the system. Is this system stable? Explain why this system is controllable/observable? Describe how to stabilize with the root locus method. Is the open loop system 'more unstable' with the stabilizing controller in it? Does this make sense?

6) Moving a block:
Invented by us.

You have a block of mass M sitting on a flat, frictionless surface. You want to move it to a point r on the horizontal surface using a horizontal input F on the block. Design a feedback control scheme to do this. How would your design change if you were asked to move the block to point r in minimum time versus if you were asked to move the block to point r with minumum error? Discuss steady-state and transient performance, as well as stability and controllability.

7) Servomotor control design:
This was asked by Prof. Campbell in ???.

Write the equation of motion for a simple motor (no need to consider any circuitry). Draw the Bode plot for this open loop plant. Is this plant stable? Design a feedback controller to stabilize this, and draw the Bode plot of your compensated open-loop system (i.e. the Bode plot of the effective feedforward plant with the controller in place). Discuss stability/performance issues. Intuitively explain what happens to the dynamics of the motor when the loop is closed.

8) Poles and zeros:
Invented by us.

Draw a basic block diagram showing a system with feedback control (no sensor gain or dynamics). Suppose I know that my plant has one or more unstable poles, and I decide to control the system by canceling these poles with zeros in the controller. Why is this a good or bad idea? Write equations to demonstrate your answer. What if your plant model is slightly wrong? What if there is a disturbance or noise in the system? Now assume that you only have stable poles in your plant, but you still decide to cancel some of them with controller zeros. Is this okay? Are there any negative side effects? What can you say about controllability or observability?

9) Control of a robotic arm: This was asked by Prof. Campbell in Spring 2009.

Consider a robot arm of length L moving in the plane around a fixed joint. You have control over the torque provided at the joint and have an overhead camera that provides you with a measurement for the arc length traveled, y. Find the equations of motion for the system. Write the transfer function between torque and y. Design a feedback control system using proportional feedback. Draw the root locus and bode plots for this closed loop system. Is it stable? If not, how would you modify your controller to stabilize the system. If the robot arm is flexible then the system becomes a continuous one, thus it has an infinite number of modes, represented by an alternating sequence of poles and zeros radiating out along the jw-axis (given by Dr. Campbell). For such a system, draw the Bode response.

C. Linear Systems Questions

1) The eigenvalue problem: This was asked by Prof. Valero-Cuevas in 2004 (?) and in 2006.

What exactly are you doing when you solve an eigenvalue problem? What is the motivation for the mechanics? For example, why do you take $\det(A - I\lambda) = 0$?

2) Discrete ring plant: This was asked by Prof. D'Andrea in Spring 2001.

You have a ring of nodes that each have a single state variable. At each time step, each node averages the state variables of itself and its neighbors, and takes this as its new state variables. Write a (matrix) equation to describe this. Can you say anything about the state transition matrix eigenvalues?

3) Spring pendulum, then two masses in space: This was asked by Prof. Garcia in 2007.

You have a pendulum where the rod is a spring. Derive the EOM. Then... two masses in space are connected by a spring. What are the mode shapes and eigenvalues (apologies for vagueness)?

4) Stamp problem: Invented by us.

You have an envelope with some amount of pre-paid postage on it, along with an infinite supply of 3 cent and 5 cent stamps. Model the state of the total postage on the envelope as a linear system. What set of numbers do your state vector and control input live in? Explain whether controllability and reachability mean the same thing for this type of linear system; what does the controllability matrix tell you here? Is this system *fully* state-reachable/controllable within the set of numbers your state lives in? How would you specify the controllable/reachable sets of postages for the system? Are these sets finite or infinite? Under what conditions are the controllable and reachable sets the same?

5) ODEs: Invented by us.

Find the solution to the following scalar ODE using any method: $\dot{u} = au$, $u(0) = u_0$. Sketch the solution. What does the value of a tell you about this system? Now consider the following system of ODEs:

$$\dot{v} = 4v - 5w, \quad v(0) = 8$$

$$\dot{w} = 2v - 3w, \quad w(0) = 5$$

Write this in matrix form. What do you expect the solution to be? Suppose you assume a solution of this form and substitute back into your matrix ODE. What will you obtain? If you rearrange terms in this expression, what does its form imply about the matrix term? Solve this expression; what does the solution tell you about the original matrix ODE? Compare/contrast this solution to the scalar ODE solution?

6) SVD analysis: Invented by us.

You are given an LTI model for some arbitrary system: $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}(\rho)\mathbf{u}$, where the control input matrix depends on some scalar parameter ρ. You are told that some system states do not respond to any $\mathbf{u}$ for some values of ρ. Devise a mathematical test for determining whether a value of ρ is 'good' or not. Now suppose someone asks you to find the 'best' ρ, such that *all* system states are reachable with a minimum amount of 'control effort'. Use a singular value decomposition to devise a reasonable 'control effort' cost function that depends on ρ. Similarly, devise an 'observation effort' cost function to find the 'best' κ, if someone hands you the system's sensor model: $\mathbf{y} = \mathbf{C}(\kappa)\mathbf{x}$. Finally, give physical interpretations of ρ and κ.

7) System identification: This was asked by Prof. Psiaki in Spring 2009.

Given the discrete time system $y(k+2) = a * y(k+1) + b * y(k)$ and the following values of y: [16 32 24 8 -4 -8], find the values of a and b via matrix methods. Given a system of the same form, which may have different values for a and b but with y=[512 -256 128 -64 32 -16], can you determine the values for a and b? If not, why not? What is one eigenvalue for this system?

D. Lipson Questions

1) Convex Hull: This was asked by Prof. Lipson in Spring 2003.

Draw a bunch of points on the board. Draw an ellipse that contains 95% of the data. Then draw the smallest rectangle that contains the points. What are its properties? Is there some sort of symmetry involved? What is the order of computations involved (e.g. N, N^2, N^N, etc.)?

2) Catching errors: This was asked by Prof. Lipson in Spring 2007.

You send out a paper and get back two reviews. Each reviewer finds n errors, with m of them in common. Do you want m to be large or small and why? What is a good estimate of the total number of errors?

3) Walking in the rain: This was asked by Prof. Lipson in ???.

If I'm out in the rain, will I get wet faster running or walking?

4) Mechanical Integrator: This was asked by Prof. Lipson in ???.

The Reuleaux Collection contains a mechanical integrator that you particularly fancy. You want to design this 1D mechanical integrator for yourself. Its input is the curve of the function you are trying to integrate (some more details needed on the actual mechanism). Find the equations from the input to the output.

5) Determining masses: Imagine that Upson Hall has an unknown, integer weight. What is the best set of weights to use to find the weight of Upson? If you were to use a computer to help, what would the algorithm look like? Does it matter if you were allowed to use a continuous base instead of only integers[1]?

6) Traveling tourist: You want to visit all the capitols of the US states. How would you create an algorithm to find the shortest amount of time your trip will take? What are the properties of this best path? Is this algorithm efficient? Imagine you wanted to visit each zip code, will your method scale well?

[1]See for better discussion: B. Hayes, *American Scientist*, 89(6):490, Nov-Dec 2001.