MAE 325 Fall 1999

HW 9 Due 11/3/99

1/5

PROBLEM 13-12

Statement: **Repeat Problem 13-10 using the cantilevered diving board design in Figure P13-1*b*.**

Units: $GPa := 10^{9}\cdot Pa$

Given:

Beam length $L := 1300\cdot mm$

Weight at free end $P := 100\cdot kgf$

Cross-section $w := 305\cdot mm$

$t := 32\cdot mm$

Young's modulus $E := 10.3\cdot GPa$

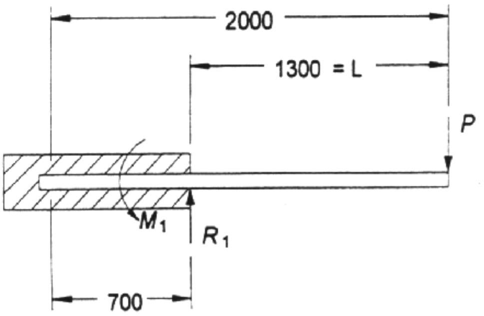


FIGURE 13-12
Free Body Diagram for Problem 13-12

Assumptions: The weight of the board is negligible compared to the applied load and so can be ignored.

Solution: See Figure 13-12 and Mathcad file P1312.

1. The area moment of inertia of the board is $I := \frac{w\cdot t^{3}}{12}$ $I = 8.329\cdot 10^{5}\,\cdot mm^{4}$

2. The spring rate (stiffness) of the board can be found from the deflection equation in Figure D-1(*a*) in Appendix D. When the load is at the end of the beam, the maximum deflection is

$$y_{max} = \frac{F\cdot L^{3}}{3\cdot E\cdot I} \qquad k = \frac{F}{y} = \frac{3\cdot E\cdot I}{L^{3}}$$

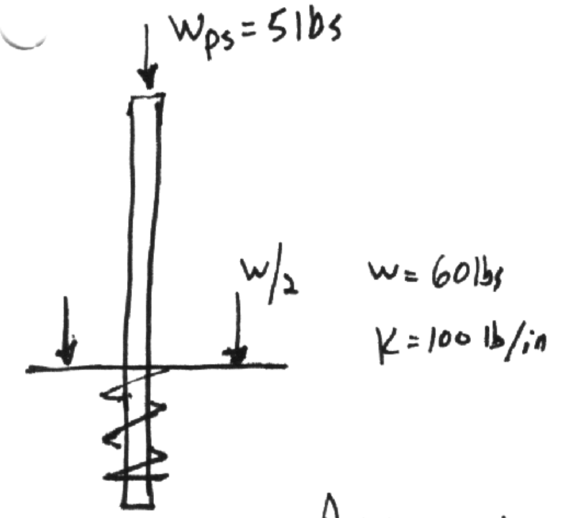
Solving for *k*, $k := \frac{3\cdot E\cdot I}{L^{3}}$ $k = 11.71\,\frac{N}{mm}$

3. Use equations 3.4 to find the natural frequency of the system.

$$\omega_n := \sqrt{\frac{k\cdot g}{P}} \qquad \omega_n = 10.82\,\frac{rad}{sec}$$

$$f_n := \frac{\omega_n}{2\cdot \pi} \qquad f_n = 1.72\,\cdot Hz$$

$\boxed{13-14}$



Design a spring for a pogo stick for a 60 lb girl. K = 100lb/in

2 in jumps, $N_{sf} = 1$, 5E4 cycles

- Assume: C $\quad c = \frac{D}{d}$ $(5 < c < 15)$

  L_f

- Find load at max. deflection ⇒ use Conservation of energy ($\Delta E_{tot} = 0$)

$$E_{tot} = U_{kin} + \underbrace{U_p} \longrightarrow (U_{height} + U_{spring})$$

$$\overset{U_s = 0}{E_{tot}} / \text{max height} = \overset{U_h = 0}{E_{tot}} / \text{min height} \quad \rightarrow \text{no } U_K \text{ @ either}$$

$$U_h = U_s$$

$$(mgy + mgh) = \frac{1}{2}Ky^2$$

$$y^2 - \frac{2mg}{K}y - \frac{2mgh}{K} = 0 \Rightarrow y = \frac{\frac{2mg}{K} \pm \sqrt{\frac{4m^2g^2}{K^2} + \frac{8mgh}{K}}}{2}$$

$$F_{max} = Ky = K\left[\frac{mg}{K} + \frac{1}{2}\sqrt{\left(\frac{2mg}{K}\right)^2 + \frac{8mgh}{K}}\right]$$

y can't be negative so choose this one.

- Find the max shear stress

$$K_w = \frac{4c-1}{4c-4} + \frac{0.615}{c} \quad (\text{eq. } 13.9a) \qquad c = 12$$

$$\boxed{K_w = 1.1227}$$

$$\tau_{max} = K_w \frac{8 F_{max} D}{\pi d^3} \qquad \text{but } D = cd$$

$$\tau_{max} = K_w \frac{8 F_{max} c \cancel{d}}{\pi d^{\cancel{3}2}} = K_w \frac{8 F_{max} C}{\pi d^2}$$

- Find Max torsional fatigue strength

$$S_{fw} = 0.36\, S_{ut} \qquad \text{for music wire} \qquad (\text{Table } 13\text{-}7)$$

$$= 0.36 \left[A d^b \right] \qquad \text{where } A = 184649 \text{ psi}$$

$\hookrightarrow$ (eq 13.3) $\qquad b = -0.1625$ (Table 13.4)

- Use (eq 13.14)

$$N_{fs} = \frac{S_{fw}}{\tau_{max}} \Rightarrow 1 = \frac{0.36 A d^b}{\left[K_w \frac{8 F_{max} C}{\pi d^2} \right]}$$

rearranging: $8 K_w F_{max} C = 0.36 \pi A d^{(b+2)}$

subs ①: $8 K_w \left(k \sqrt{\frac{2mgh}{k}} \right) c = 0.36 \pi A d^{(b+2)}$ ②

Subs. knowns into ②:

$K_w = 1.1227$ $K = 100\ \text{lb/in}$ $h = 2\text{in}$

$A = 184649\ \text{psi}$ $b = -0.1625$ $mg = (\underset{\text{girl}}{\underset{\uparrow}{60}} + \underset{\text{pogo stick}}{\underset{\uparrow}{5}})\ \text{lbs}$

③ $$8\left[1.1227\right]\left(100\sqrt{\frac{2(65)(2)}{100}}\right)C = 0.36\pi\left(184649\right)d^{(1.8375)}$$

④ $$D = Cd$$

Solve ③ w/ your C for d and plug into ④ Note: every different choice of C will give a different answer

- Find N_a: $$N_a = \frac{6d^4}{8KD^3}$$

- Find $$f_n = \frac{1}{2}\sqrt{\frac{Kg}{W}} = \frac{1}{2}\sqrt{\frac{Kg}{\pi D N_a \frac{\pi d^2}{4}\gamma_{\leftarrow \text{density}}}}$$

- Calculate take off velocity

Again use energy methods.

$$E_{take\,off} = E_{max\,height} = E_{min\,height}$$

$$U_{kin} = mgh$$

(@ take off all E is Kinetic)

$$\frac{1}{2}mv^2 = mgh$$

$$v^2 = 2gh$$

$$\boxed{V = \sqrt{2gh}} = \sqrt{2(32.2\,ft/s^2)(\tfrac{2}{12}\,ft)}$$

$$= 3.28\,ft/s$$