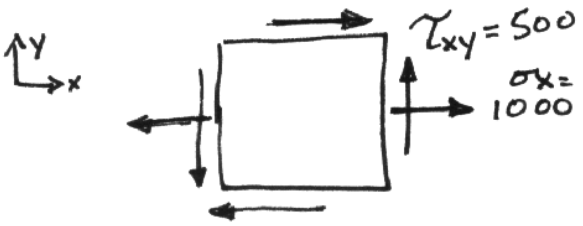
MAE 325 Fall 1999

HW 7 Solutions Due 10/20

(1) p 4.1 (a) Find principal stresses and τ_{max} analytically and verify w/ Mohr's circle

Stress Element



Using principal stress on right:

$$\boxed{\begin{aligned} \tau_{13} &= \frac{|\sigma_1 - \sigma_3|}{2} = 707.1 \\ \tau_{12} &= \frac{|\sigma_1 - \sigma_2|}{2} = 603.55 \\ \tau_{23} &= \frac{|\sigma_2 - \sigma_3|}{2} = 103.55 \end{aligned}}$$

$\leftarrow \tau_{max}$

Solving eq. 4.4 (c) analytically:

$$\sigma^3 - C_2\sigma^2 - C_1\sigma - C_0 = 0$$

$$C_1 = \sigma_x + \sigma_y + \sigma_z = 1000$$

$$C_2 = \tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2 - \sigma_x\sigma_y - \sigma_y\sigma_z - \sigma_z\sigma_x = (500)^2$$

$$C_0 = \sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{yz}\tau_{zx} - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 - \sigma_z\tau_{xy}^2 = 0$$

$$\sigma^3 - 1000\sigma^2 - 250{,}000\sigma = 0$$

factoring:

$$\sigma(\sigma^2 - 1000\sigma - 250{,}000) = 0$$

$\boxed{\sigma = 0}$ is one solution (principal stress)

$$\sigma = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = +500 \pm \sqrt{1000^2 + 4(500)^2}$$

$$\boxed{\sigma = 1207.1, -207.1}$$

Note: For the convention where $\sigma_1 \geq \sigma_2 \geq \sigma_3$ $\Longrightarrow$ so: $\sigma_1 = 1207.1$, $\sigma_2 = 0$

$\sigma_3 = -207.1$

Mohr's Circle

$$[\sigma] = \begin{bmatrix} 1000 & 500 & 0 \\ 500 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Center of Mohr's circle:

$$C = \frac{\sigma_x + \sigma_y}{2} = 500$$

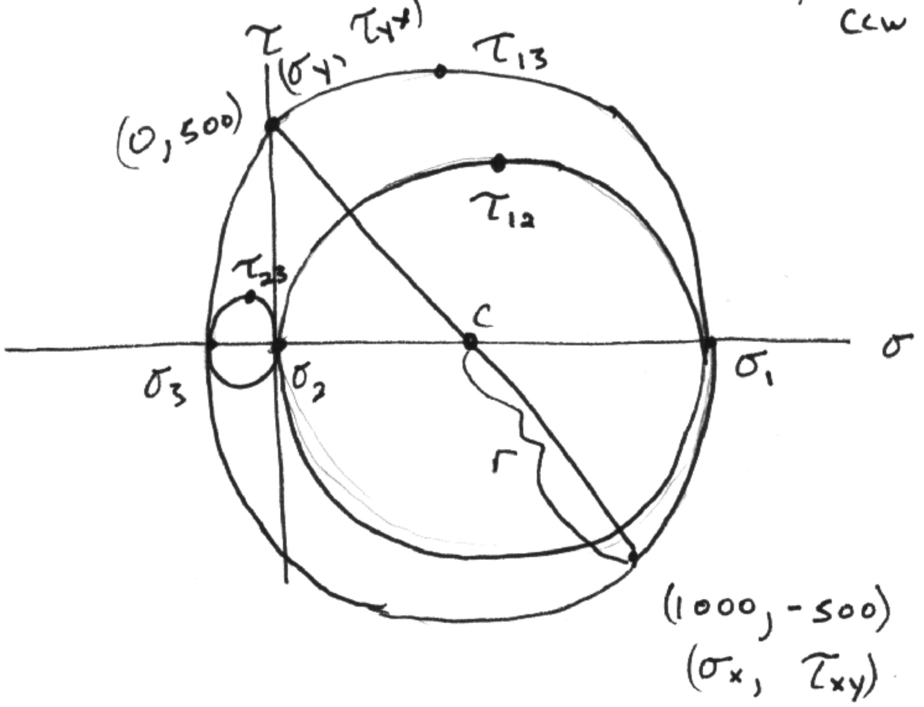
radius of Mohr's Circle:

$$r = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$r = 707.1$$

$$\boxed{\begin{aligned} \sigma_1 &= C + r = 1207.1 \\ \sigma_3 &= C - r = -207.1 \\ \sigma_2 &= 0 \end{aligned}}$$

plotting: recall Mohr's convention $\tau_{yx} \Rightarrow (+)$ cw, $\tau_{xy} \Rightarrow (-)$ ccw



$$\boxed{\begin{aligned} \tau_{13} &= r = 707.1 \\ \tau_{12} &= \frac{|\sigma_1 + \cancel{\sigma_2}^{\,0}|}{2} = 603.55 \\ \tau_{23} &= \frac{|\cancel{\sigma_2}^{\,0} - \sigma_3|}{2} = 103.55 \end{aligned}}$$

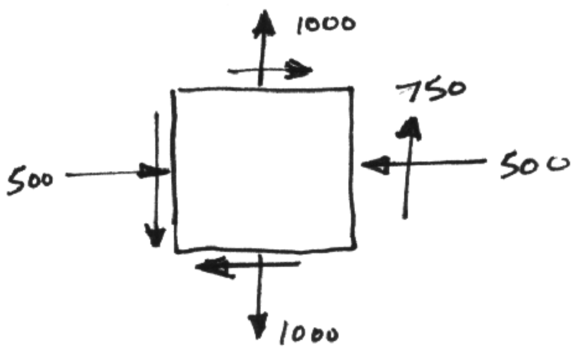
Answers check.

② p 4.1 (f) Find principal stresses and τ_{max} analytically and verify w/ Mohr's Circle

similar to (a)

$$[\sigma] = \begin{bmatrix} -500 & 750 & 0 \\ 750 & 1000 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Stress Element.



Solving analytically:

$$\sigma^3 - C_2\sigma^2 - C_1\sigma - C_0 = 0$$

$C_2 = -500 + 1000 = 500$

$C_1 = 750^2 - (500)(1000) = 1062500$

$C_0 = 0$

$$\sigma(\sigma^2 - 500\sigma - 1062500) = 0$$

$\boxed{\sigma = 0}$ $\boxed{\begin{aligned}\sigma &= 1310.7 \\ \sigma &= -810.7\end{aligned}}$

so: $\sigma_1 = 1310.7$

$\sigma_2 = 0$

$\sigma_3 = -810.7$

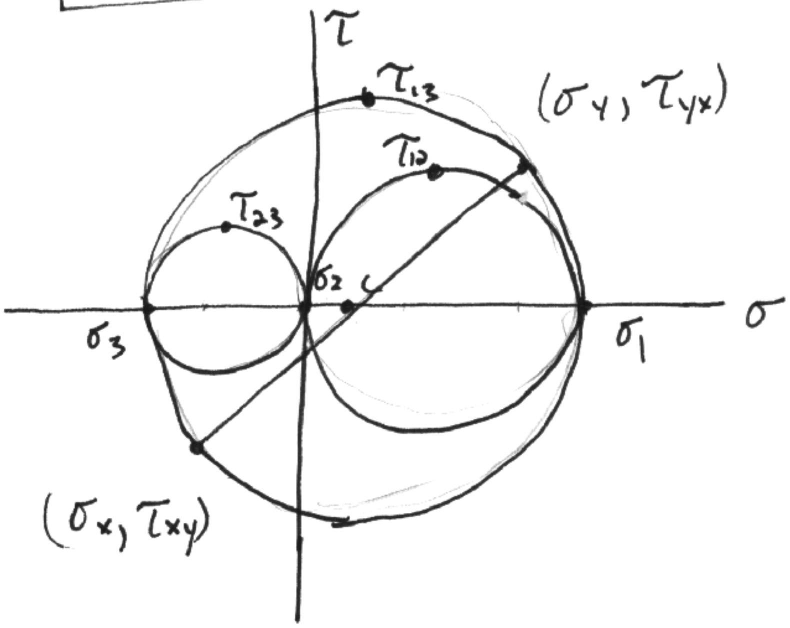
and $\boxed{\begin{aligned}\tau_{12} &= 655.35 \\ \tau_{13} &= 1060.7 \\ \tau_{32} &= 405.35\end{aligned}}$ ← τ_{max}

Mohr's Circle

$$C = \frac{\sigma_x + \sigma_y}{2} = 250$$

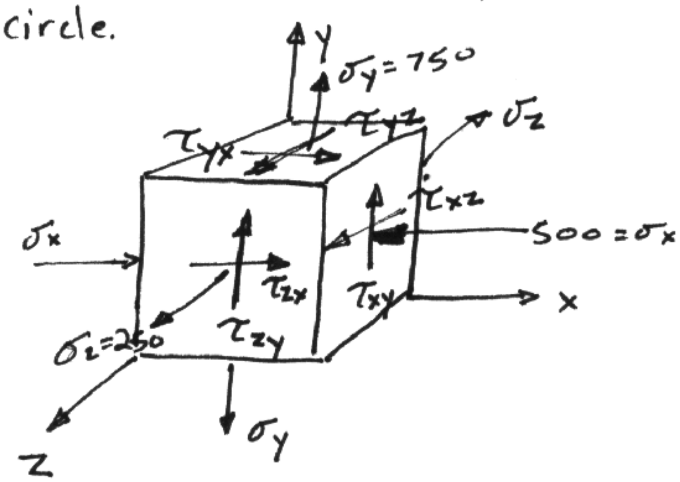
$$r = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 1060.7$$

$\boxed{\begin{aligned}&\sigma_1 = C + r = 1310.7 \quad && \tau_{12} = 655.35 \\ &\sigma_2 = 0 && \tau_{13} = 1060.7 \\ &\sigma_3 = -810.7 && \tau_{32} = 405.35\end{aligned}}$



(3) p 4.1(j) Find principal stresses and τ_{max} analytically and Verify w/ Mohr's circle.

$$[\sigma] = \begin{bmatrix} -500 & 100 & 1000 \\ 100 & 750 & 250 \\ 1000 & 250 & 250 \end{bmatrix}$$



solving analytically:

$$\sigma^3 - C_2\sigma^2 - C_1\sigma - C_0 = 0$$

$$C_2 = -500 + 750 + 250 = 500$$

$$C_1 = 100^2 + 1000^2 + 250^2 - (-500)(750) - (750)(250) - (250)(-500) = 1{,}385{,}000$$

$$C_0 = (-500)(750)(250) + 2(100)(1000)(250) - (-500)(250)^2 - (750)(1000)^2 - 250(100)^2 = -765{,}000{,}000$$

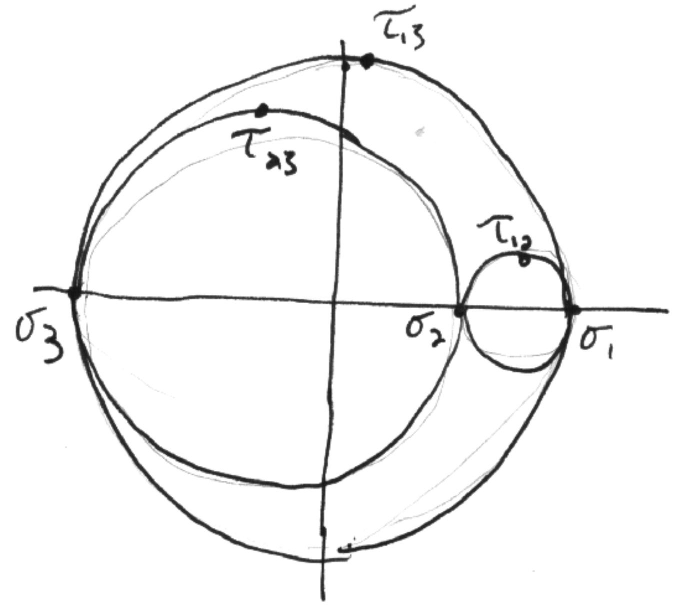
using MATLAB:

```
>> roots([1, -500, -1,385,000, 765,000,000])
```

$\sigma_1 = 1126.6$	$\tau_{13} = 1160.8 \leftarrow \tau_{max}$
$\sigma_2 = 568.3$	$\tau_{21} = 279.15$
$\sigma_3 = -1194.9$	$\tau_{32} = 890.6$

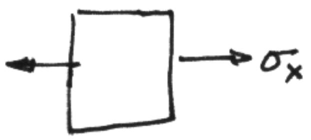
3 D Mohr's circle

- you have to know at least 1 principal stress to draw Mohr's circle
- since we don't know any from inspection we cannot solve this problem using 3D Mohr's circle
- the final circle is below



(4) Find τ_{max}, $\sigma_1, \sigma_2, \sigma_3$ analytically and verify w/ Mohr's circle

$$[\sigma] = \begin{bmatrix} \sigma_x & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



From inspection we see that σ_x is a principal Stress (no shear)

analytically:

$C_2 = \sigma_x \quad C_1 = 0 \quad C_0 = 0$

so: $\sigma^3 - \sigma_x \sigma^2 = 0$

$\sigma^2(\sigma - \sigma_x) = 0$

$$\boxed{\sigma_{2,3} = 0 \qquad \sigma_1 = \sigma_x}$$

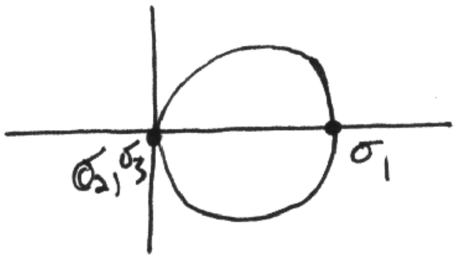
$$\boxed{\begin{aligned} \tau_{12} = \tau_{13} &= \frac{\sigma_x}{2} \\ \tau_{23} &= 0 \end{aligned}}$$

Mohr's circle:

$C = \frac{\sigma_x}{2} \qquad r = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + 0} = \frac{\sigma_x}{2}$

$\sigma_1 = C + r = \sigma_x \qquad \sigma_2 = 0$

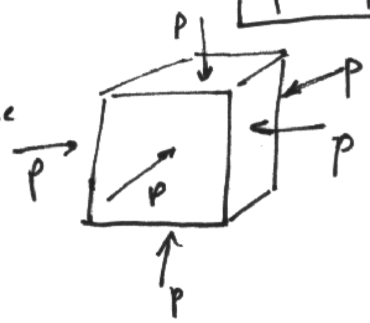
$\sigma_3 = C - r = 0$



Special cases: A tensile stress is already a principal stress. Likewise, hydrostatic pressure is a principal stress state

(5) $$[\sigma] = \begin{bmatrix} -p & 0 & 0 \\ 0 & -p & 0 \\ 0 & 0 & -p \end{bmatrix}$$

Again - no shear, so from inspection these are principal stresses



$C_2 = -3p \quad C_1 = -3p^2 \quad C_0 = -p^3$

$\sigma^3 + 3\sigma^2 p + 3\sigma p^2 + p^3 = 0$

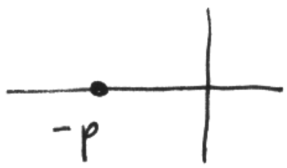
factoring:

$(\sigma_1 + p)(\sigma_2 + p)(\sigma_3 + p) = 0$

$$\boxed{\sigma_1 = \sigma_2 = \sigma_3 = -p}$$

$$\boxed{\tau_{13} = \tau_{12} = \tau_{32} = 0}$$ No shears!

Mohr's Circle:



$C = \frac{-2p}{2} = -p \qquad r = \sqrt{\left(\frac{\cancel{-p+p}}{2}\right)^2 + 0}$

$r = 0$

A point