MAE 325 HW 6 Solutions
Due: 10/15/99

1/5

PROBLEM 3-26a

Statement: **A beam is supported and loaded as shown in Figure P3-11*d*. Find the reactions, maximum shear, and maximum moment for the data given in row *a* from Table P3-1.**

Units: $N := newton$

Given:

Beam length	$L := 1 \cdot m$
Distance to distributed load	$a := 0.4 \cdot m$
Distance to reaction load	$b := 0.6 \cdot m$
Distributed load magnitude	$w := 200 \cdot N \cdot m^{-1}$
Concentrated load	$F := 500 \cdot N$

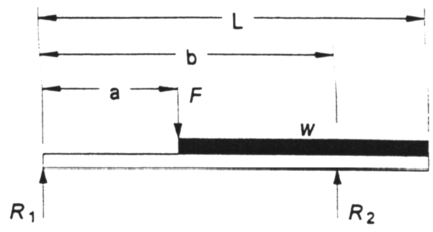


FIGURE 3-26A
Free Body Diagram for Problem 3-26

Solution: See Figures 3-26 and Mathcad file P0326a.

1. From inspection of Figure 3-26aA, write the load function equation

$$q(x) = R_1\langle x - 0\rangle^{-1} - w\langle x - a\rangle^{0} + R_2\langle x - b\rangle^{-1} - F\langle x - a\rangle^{-1}$$

2. Integrate this equation from $-\infty$ to x to obtain shear, $V(x)$

$$V(x) = R_1\langle x - 0\rangle^{0} - w\langle x - a\rangle^{1} + R_2\langle x - b\rangle^{0} - F\langle x - a\rangle^{0}$$

3. Integrate this equation from $-\infty$ to x to obtain moment, $M(x)$

$$M(x) = R_1\langle x - 0\rangle^{1} - w\langle x - a\rangle^{2}/2 + R_2\langle x - b\rangle^{1} - F\langle x - a\rangle^{1}$$

4. Solve for the reactions by evaluating the shear and moment equations at a point just to the right of $x = L$, where both are zero.

At $x = L^+$, $V = M = 0$

$$V = R_1 - w\cdot(L-a) + R_2 - F = 0$$

$$M = R_1\cdot L - \frac{w}{2}\cdot(L-a)^2 + R_2\cdot(L-b) - F(L-a) = 0$$

$$R_1 := \frac{1}{b}\cdot\left[\frac{w}{2}\cdot(L-a)^2 + F\cdot(b-a) - w\cdot(L-a)\cdot(L-b)\right] \qquad R_1 = 147\, N$$

$$R_2 := w\cdot(L-a) + F - R_1 \qquad R_2 = 473\, N$$

5. Define the range for x $\quad x := 0\cdot m, 0.005\cdot L .. L$

6. For a Mathcad solution, define a step function S. This function will have a value of zero when x is less than z, and a value of one when it is greater than or equal to z.

$$S(x,z) := if(x \ge z, 1, 0)$$

7. Write the shear and moment equations in Mathcad form, using the function S as a multiplying factor to get the effect of the singularity functions.

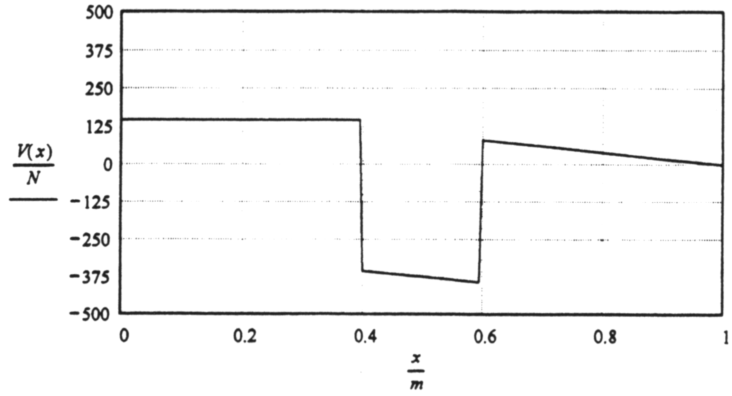
$$V(x) := R_1\cdot S(x, 0\cdot m) - w\cdot S(x,a)\cdot(x-a) + R_2\cdot S(x,b) - F\cdot S(x,a)$$

$$M(x) := R_1\cdot S(x, 0\cdot m)\cdot x - \frac{w}{2}\cdot S(x,a)\cdot(x-a)^2 + R_2\cdot S(x,b)\cdot(x-b) - F\cdot S(x,a)\cdot(x-a)$$

2/5

8. Plot the shear and moment diagrams.

Shear Diagram



Moment Diagram

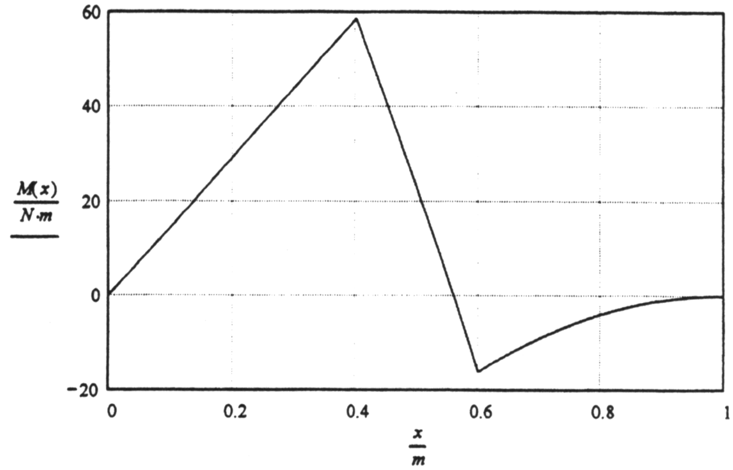


FIGURE 3-26aB

Shear and Moment Diagrams for Problem 3-26a

9. Determine the maximum shear and maximum moment from inspection of the diagrams.

Maximum shear: $V_{max} := |V(b - 0.001 \cdot mm)|$ $\qquad V_{max} = 393 \cdot N$

Maximum moment occurs where V is zero, which is $x = a$:

$M_{max} := |M(a)|$ $\qquad M_{max} = 58.7 \cdot N \cdot m$

PROBLEM 4-26a

Statement: **A beam is supported and loaded as shown in Figure P4-11d. Find the reactions, maximum shear, maximum moment, maximum slope, maximum bending stress, and maximum deflection for the data given in row *a* from Table P4-2.**

Units: $N := newton$ $\quad MPa := 10^6 \cdot Pa$ $\quad GPa := 10^9 \cdot Pa$

Given:

Beam length	$L := 1 \cdot m$
Distance to distributed load	$a := 0.4 \cdot m$
Distance to R_2	$b := 0.6 \cdot m$
Distributed load magnitude	$w := 200 \cdot N \cdot m^{-1}$
Concentrated load	$F := 500 \cdot N$
Moment of inertia	$I := 2.85 \cdot 10^{-8} \cdot m^4$
Distance to extreme fiber	$c := 2.00 \cdot 10^{-2} \cdot m$
Modulus of elasticity	$E := 207 \cdot GPa$

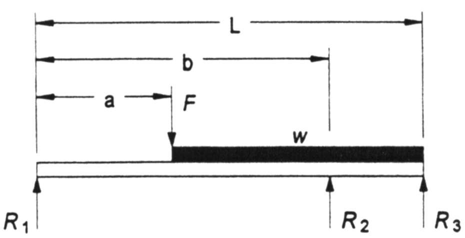


FIGURE 4-26A
Free Body Diagram for Problem 4-26

Solution: See Figures 4-26 and Mathcad file P0426a.

1. From inspection of Figure P4-11d, write the load function equation

$$q(x) = R_1\langle x\rangle^{-1} - F\langle x-a\rangle^{-1} - w\langle x-a\rangle^{0} + R_2\langle x-b\rangle^{-1} - R_3\langle x-L\rangle^{-1}$$

2. Integrate this equation from $-\infty$ to x to obtain shear, $V(x)$

$$V(x) = R_1\langle x\rangle^{0} - F\langle x-a\rangle^{0} - w\langle x-a\rangle^{1} + R_2\langle x-b\rangle^{0} - R_3\langle x-L\rangle^{0}$$

3. Integrate this equation from $-\infty$ to x to obtain moment, $M(x)$

$$M(x) = R_1\langle x\rangle^{1} - F\langle x-a\rangle^{1} - w\langle x-a\rangle^{2}/2 + R_2\langle x-b\rangle^{1} - R_3\langle x-L\rangle^{1}$$

4. Integrate the moment function, multiplying by $1/EI$, to get the slope.

$$\theta(x) = [R_1\langle x\rangle^{2}/2 - F\langle x-a\rangle^{2}/2 - w\langle x-a\rangle^{3}/6 + R_2\langle x-b\rangle^{2}/2 + R_3\langle x-L\rangle^{2}/2 + C_3]/EI$$

5. Integrate again to get the deflection.

$$y(x) = [R_1\langle x\rangle^{3}/6 - F\langle x-a\rangle^{3}/6 - w\langle x-a\rangle^{4}/24 + R_2\langle x-b\rangle^{3}/6 + R_3\langle x-L\rangle^{3}/6 + C_3 x + C_4]/EI$$

6. Evaluate R_1, R_2, R_3, C_3 and C_4

   At $x = 0$, $x = b$, and $x = L$; $y = 0$, therefore, $C_4 = 0$.

   At $x = L^+$, $V = M = 0$

Guess $\quad R_1 := 100 \cdot N \quad R_2 := 100 \cdot N \quad R_3 := 100 \cdot N \quad C_3 := -5 \cdot N \cdot m^2$

Given

$$\frac{R_1}{6} \cdot b^3 - \frac{F}{6} \cdot (b-a)^3 - \frac{w}{24} \cdot (b-a)^4 + C_3 \cdot b = 0 \cdot N \cdot m^3$$

$$\frac{R_1}{6} \cdot L^3 - \frac{F}{6} \cdot (L-a)^3 - \frac{w}{24} \cdot (L-a)^4 + \frac{R_2}{6} \cdot (L-b)^3 + C_3 \cdot L = 0 \cdot N \cdot m^3$$

$$R_1 - F - w \cdot (L-a) + R_2 + R_3 = 0 \cdot N$$

$$R_1 \cdot L - F \cdot (L-a) - \frac{w}{2} \cdot (L-a)^2 + R_2 \cdot (L-b) = 0 \cdot N \cdot m$$

4/5

$$\begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ C_3 \end{bmatrix} := find(R_1, R_2, R_3, C_3)$$

$R_1 = 112.33 \cdot N$ $\quad R_2 = 559.17 \cdot N$ $\quad R_3 = -51.50 \cdot N$ $\quad C_3 = -5.607 \cdot N \cdot m^2$

7. Define the range for x $\qquad x := 0 \cdot in, 0.002 \cdot L .. L$

8. For a Mathcad solution, define a step function S. This function will have a value of zero when x is less than z, and a value of one when it is greater than or equal to z.

   $S(x,z) := if(x \ge z, 1, 0)$

9. Write the shear and moment equations in Mathcad form, using the function S as a multiplying factor to get the effect of the singularity functions.

$$V(x) := R_1 \cdot S(x, 0 \cdot in) - F \cdot S(x,a) - w \cdot S(x,a) \cdot (x-a) + R_2 \cdot S(x,b) + R_3 \cdot S(x,L)$$

$$M(x) := R_1 \cdot S(x, 0 \cdot in) \cdot x - F \cdot S(x,a) \cdot (x-a) - \frac{w}{2} \cdot S(x,a) \cdot (x-a)^2 \ldots$$
$$+ R_2 \cdot S(x,b) \cdot (x-b)$$

10. Plot the shear and moment diagrams.

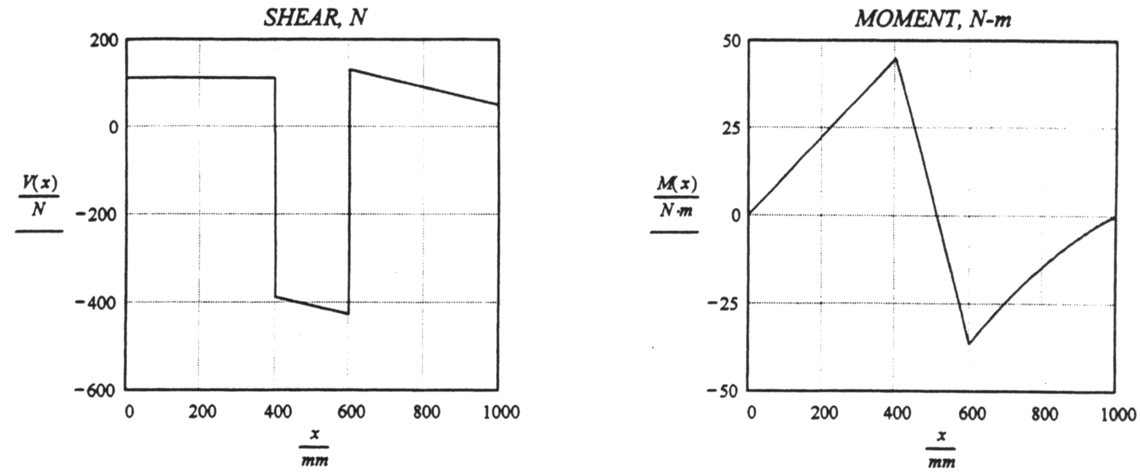


FIGURE 4-26aB
Shear and Moment Diagrams for Problem 4-26a

11. From the diagram, we see that maximum shear occurs at $x = b^-$,

$V_{max} := V(b - 0.001 \cdot mm)$ $\qquad V_{max} = -428 \cdot N$

12. The maximum moment occurs at $x = a$,

$M_{max} := M(a)$ $\qquad M_{max} = 44.9 \cdot N \cdot m$

13. Write the slope and deflection equations in Mathcad form, using the function S as a multiplying factor to get the effect of the singularity functions. See Figure 4-26aB where these functions are plotted.

$$\theta(x) := \frac{1}{E \cdot I} \cdot \left[\frac{R_1}{2} \cdot S(x, 0 \cdot in) \cdot x^2 - \frac{F}{2} \cdot S(x,a) \cdot (x-a)^2 - \frac{w}{6} \cdot S(x,a) \cdot (x-a)^3 \ldots \atop + \frac{R_2}{2} \cdot S(x,b) \cdot (x-b)^2 + \frac{R_3}{2} \cdot S(x,L) \cdot (x-L)^2 + C_3 \right]$$

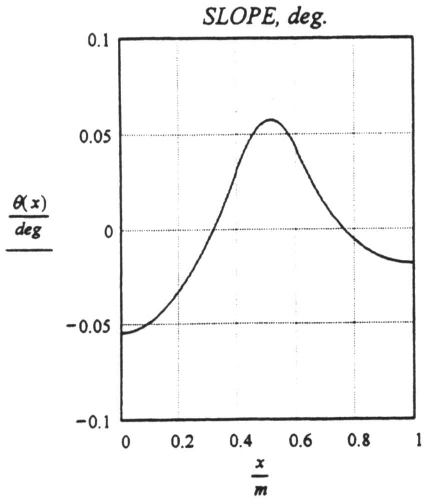
$$y(x) := \frac{1}{E \cdot I} \cdot \left[\frac{R_1}{6} \cdot S(x, 0 \cdot in) \cdot x^3 - \frac{F}{6} \cdot S(x,a) \cdot (x-a)^3 - \frac{w}{24} \cdot S(x,a) \cdot (x-a)^4 \ldots \atop + \frac{R_2}{6} \cdot S(x,b) \cdot (x-b)^3 + \frac{R_3}{6} \cdot S(x,L) \cdot (x-L)^3 + C_3 \cdot x \right]$$

14. Maximum slope occurs between $x = a$ and $x = b$ $\qquad \theta_{max} := 0.0576 \cdot deg$

15. Maximum deflection occurs between $x = 0$ and $x = a$ $\qquad y_{max} := -0.200 \cdot mm$

16. The maximum bending stress occurs at $x = a$, where the moment is a maximum. For $\quad c = 20 \cdot mm$

$$\sigma_{max} := \frac{M_{max} \cdot c}{I} \qquad \sigma_{max} = 31.5 \cdot MPa$$



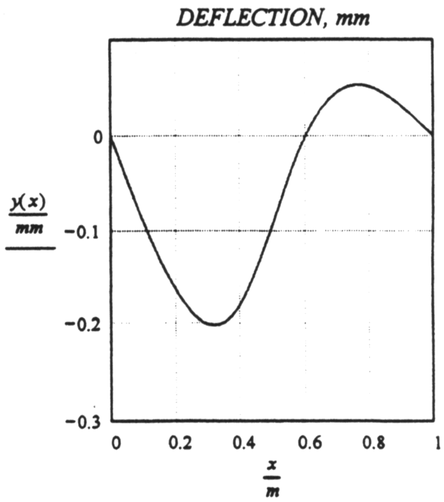


FIGURE 4-26aC

Slope and Deflection Diagrams for Problem 4-26a