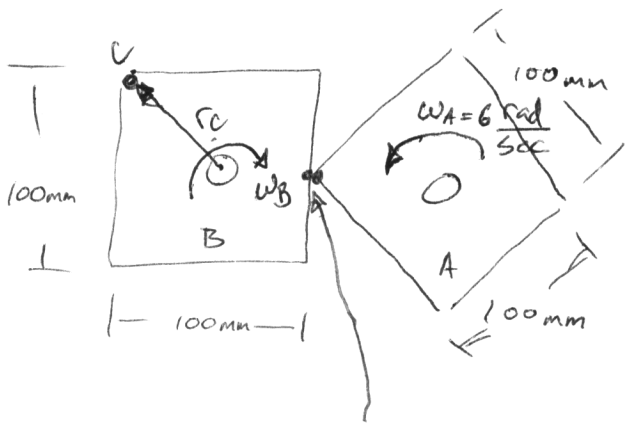
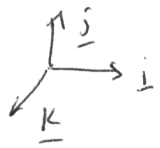
MAE 325 Fall 1999
Assignment 4: Due Wed 9/22/99

(1) Find the largest ω_B and v_c





at point of contact the velocities of the two gears are equal.

$$r_A \omega_A = r_B \omega_B$$

$$\omega_B = \frac{r_A}{r_B} \omega_A$$

$$r_B = 50$$

$$r_A = 50\sqrt{2}$$

So $\omega_B = \frac{50\sqrt{2}}{50} 6$

$$\underline{\omega}_B = 8.485 \frac{rad}{sec} \underline{k}$$

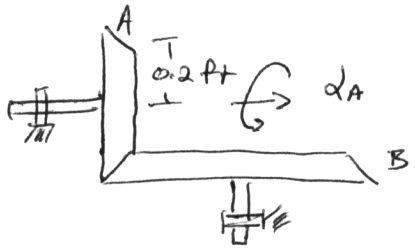
$$\underline{v}_c = \underline{\omega}_B \times \underline{r}_c$$

$$\underline{v}_c = -(8.485)(50\sqrt{2})(-\cos 45\, \underline{i} + \sin 45\, \underline{j})$$

$$\underline{v}_c = 424.25\, \underline{i} + 424.25\, \underline{j}$$

$$\|\underline{v}_c\| = 600 \text{ mm/sec}$$

(2) Find t for B to attain an angular velocity of $50 \frac{rad}{sec}$ if $\alpha_A = 2 \frac{rad}{sec^2}$ and it starts from rest.



1— 0.5 ft —1

$$\alpha = \frac{d\omega}{dt}$$

$$\int_0^t d\omega = \int_0^t \alpha \, dt$$

$$\omega_A(t) - \omega_A(t=0) = \int_0^t 2 \, dt$$

$$\omega_A(t) = 2t \ \frac{rad}{sec} \qquad (1)$$

$$r_A \omega_A = r_B \omega_B$$

$$\omega_B = \frac{r_A}{r_B} \omega_A$$

$$\omega_B = \frac{0.2}{0.5} \omega_A \qquad (2)$$

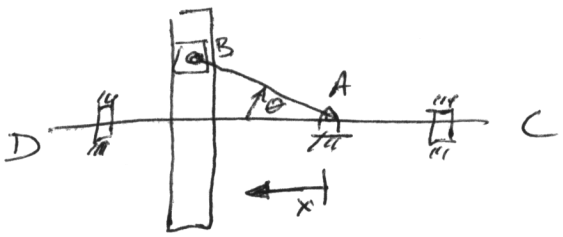
subs (1) into (2):

$$50 = \omega_B = \frac{0.4}{0.5} t$$

so: $\boxed{t = 62.5 \text{ sec}}$

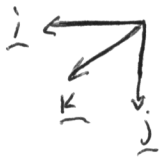
(3) Find V_{CD} and a_{CD} for any $\theta, \dot{\theta}, \ddot{\theta}$

$\overline{AB} = 1.5$ ft.



Key points $\Rightarrow$ $\underline{V}_{CD} = V_B \cdot \underline{i}$

$\underline{a}_{CD} = a_B \cdot \underline{i}$



$$V_{CD} = \underline{V}_B \cdot \underline{i}$$

$$V_{CD} = (\underline{\omega}_{AB} \times \underline{r}_{AB}) \cdot \underline{i}$$

$$V_{CD} = \{-\omega_{AB}\,\underline{k} \times 1.5(\cos\theta\,\underline{i} - \sin\theta\,\underline{j})\} \cdot \underline{i}$$

$$V_{CD} = \{-1.5\,\omega_{AB}(\cos\theta\,\underline{j} + \sin\theta\,\underline{i})\} \cdot \underline{i}$$

$$\boxed{V_{CD} = -1.5\,\omega_{AB}\sin\theta \text{ ft/sec}}$$

if $\omega_{AB} = 4$ rad/sec

$V_{CD} = -6\sin\theta$ ft/sec

$$a_{CD} = \underline{a}_B \cdot \underline{i}$$

$$a_{CD} = \{\underline{\alpha}_{AB} \times \underline{r}_{AB} - \omega_{AB}^2 \underline{r}_{AB}\} \cdot \underline{i}$$

$$a_{CD} = \{-\alpha_{AB}\,\underline{k} \times 1.5(\cos\theta\,\underline{i} - \sin\theta\,\underline{j}) - 1.5\,\omega_{AB}^2(\cos\theta\,\underline{i} - \sin\theta\,\underline{j})\} \cdot \underline{i}$$

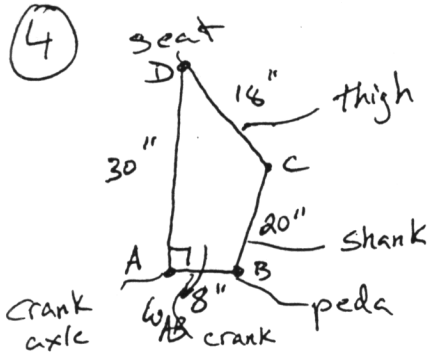
$$a_{CD} = \{-1.5\alpha_{AB}(\cos\theta\,\underline{j} + \sin\theta\,\underline{i}) - 1.5\,\omega_{AB}^2(\cos\theta\,\underline{i} - \sin\theta\,\underline{j})\} \cdot \underline{i}$$

$$\boxed{a_{CD} = -1.5(\alpha_{AB}\sin\theta + \omega_{AB}^2\cos\theta) \text{ ft/sec}^2}$$

if $\omega_{AB} = 4$ rad/sec and $\alpha_{AB} = 0$ rad/sec^2

$a_{CD} = -24\cos\theta$ ft/sec^2

(4)



Find: ω_{BC} – ang. vel. of shank

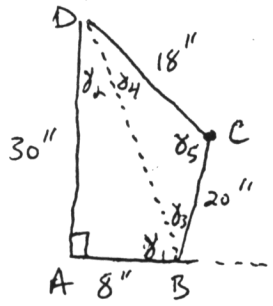
ω_{CD} – ang. vel. of thigh

$V_{C/D}$ – vel. knee relative to bike

α_{BC} – ang. acc. of shank

α_{CD} – ang. vel. of thigh

First solve geometry problem:



$l_{BD}^2 = 30^2 + 8^2$

$l_{BD} = 31''$

$\sin \gamma_1 = \frac{30}{31}$

$\gamma_1 = 75°$

$\gamma_1 + \gamma_2 + 90 = 180$

$\gamma_2 = 15°$

using law of cosines: $a^2 = b^2 + c^2 - 2bc\cos A$

$\gamma_3 = \cos^{-1} \frac{20^2 + 31^2 - 18^2}{2(20)(31)}$

$\gamma_3 = 53.25°$

$\gamma_4 = \cos^{-1} \frac{18^2 + 31^2 - 20^2}{2(18)(31)}$

$\gamma_4 = 37.5°$

$\gamma_5 = 180 - \gamma_3 - \gamma_4$

$\gamma_5 = 109.25$

So: $\angle ADC = \gamma_2 + \gamma_4 = 52.5$

$\angle BCD = \gamma_5 = 109.25$

$\angle ABC = \gamma_1 + \gamma_3 = 108.25$

$\angle BAD = 90°$

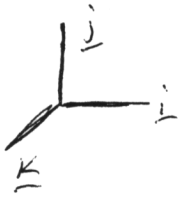
Check: $\Sigma \angle s = 360°$

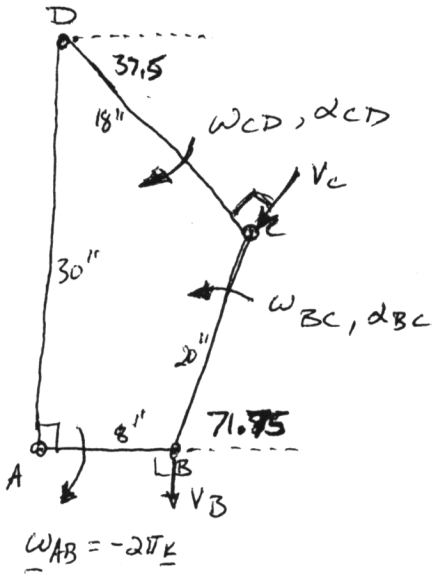
Now work on ω's and α's:

$$\underline{\omega}_{AB} = -60 \frac{rev}{min} \cdot \frac{2\pi \text{ rad}}{rev} \cdot \frac{min}{60 sec} \underline{k}$$

$$\underline{\omega}_{AB} = -2\pi \underline{k} \quad \frac{rev}{sec}$$

FBD

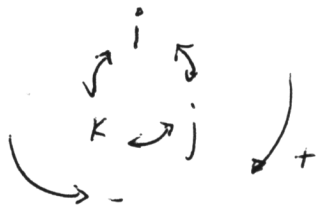




Note: $V_B \perp AB$

$V_C \perp CD$

cross product rule:



$$\underline{V}_C = \underline{V}_B + \underline{V}_{C/B}$$

$$\underline{\omega}_{CD} \times \underline{r}_{DC} = \underline{\omega}_{AB} \times \underline{r}_{AB} + \underline{\omega}_{BC} \times \underline{r}_{BC}$$

$$-\omega_{CD}\underline{k} \times 18(\cos 37.5\,\underline{i} - \sin 37.5\,\underline{j}) = -2\pi \underline{k} \times 8\,\underline{i} + \omega_{BC}\underline{k} \times 20(\cos 71.75\,\underline{i} + \sin 71.75\,\underline{j})$$

$$-18\omega_{CD}(\sin 37.5\,\underline{i} + \cos 37.5\,\underline{j}) = -16\pi\,\underline{j} + 20\omega_{BC}(-\sin 71.75\,\underline{i} + \cos 71.75\,\underline{j}) \quad (1)$$

$(1) \cdot \underline{i} \Rightarrow$ $-18\omega_{CD}\sin 37.5 = -20\omega_{BC}\sin 71.75$ (2)

$$\omega_{CD} = \frac{20}{18} \cdot \frac{\sin 71.75}{\sin 37.5}\omega_{BC} = 1.733\,\omega_{BC} \quad (3)$$

$(1) \cdot \underline{j} \Rightarrow$ $-18\omega_{CD}\cos 37.5 = -16\pi + 20\omega_{BC}\cos 71.75$ (4)

Substitute (3) into (4) and solve for ω_{BC}

$$\omega_{BC} = 1.620 \text{ rad/sec}$$

using (3)

$$\omega_{CD} = 2.809 \text{ rad/sec}$$

$$\underline{V}_{C/D} = \underline{\omega}_{CD} \times \underline{r}_{CD} \qquad (5)$$

$$= -2.809\,\underline{k} \times 18(\cos 35\,\underline{i} - \sin 35\,\underline{j})$$

$$\boxed{\underline{V}_{CD} = -(29.004\,\underline{i} + 41.418\,\underline{j})\ \text{in/sec}}$$

$$\underline{a}_C = \underline{a}_B + \underline{a}_{B/C}$$

$$\underline{\alpha}_{CD} \times \underline{r}_{DC} - \omega^2_{CD}\,\underline{r}_{DC} = -\omega^2_{AB}\,\underline{r}_{AB} + \underline{\alpha}_{BC} \times \underline{r}_{BC} - \omega^2_{BC}\,\underline{r}_{BC}$$

$$-\alpha_{CD}\,\underline{k} \times 18(\cos 37.5\,\underline{i} - \sin 37.5\,\underline{j}) - \omega^2_{CD} \cdot 18(\cos 37.5\,\underline{i} + \sin 37.5\,\underline{j})$$

$$= -\omega^2_{AB} \cdot 8\,\underline{i} + \alpha_{BC}\,\underline{k} \times 20 \cdot (\cos 71.75\,\underline{i} + \sin 71.75\,\underline{j})$$

$$- \omega^2_{BC} \cdot 20(\cos 71.75\,\underline{i} + \sin 71.75\,\underline{j}) \qquad (6)$$

$(6)\cdot\underline{i} \Rightarrow \quad +18\alpha_{CD}\sin 37.5 + 18\omega^2_{CD}\cos 37.5 = 8\omega^2_{AB} + 20\alpha_{BC}\sin 71.75 + 20\omega^2_{BC}\cos 71.75$

$(6)\cdot\underline{j} \Rightarrow \quad -18\alpha_{CD}\cos 37.5 + 18\omega^2_{CD}\sin 37.5 = 20\alpha_{BC}\cos 71.75 - 20\omega^2_{BC}\sin 71.75$

Subs. ω_{AB}, ω_{BC}, ω_{CD} and solving as before:

$$\boxed{\alpha_{BC} = -4.888\ \text{rad/sec}^2}$$

$$\boxed{\alpha_{CD} = 11.692\ \text{rad/sec}^2}$$