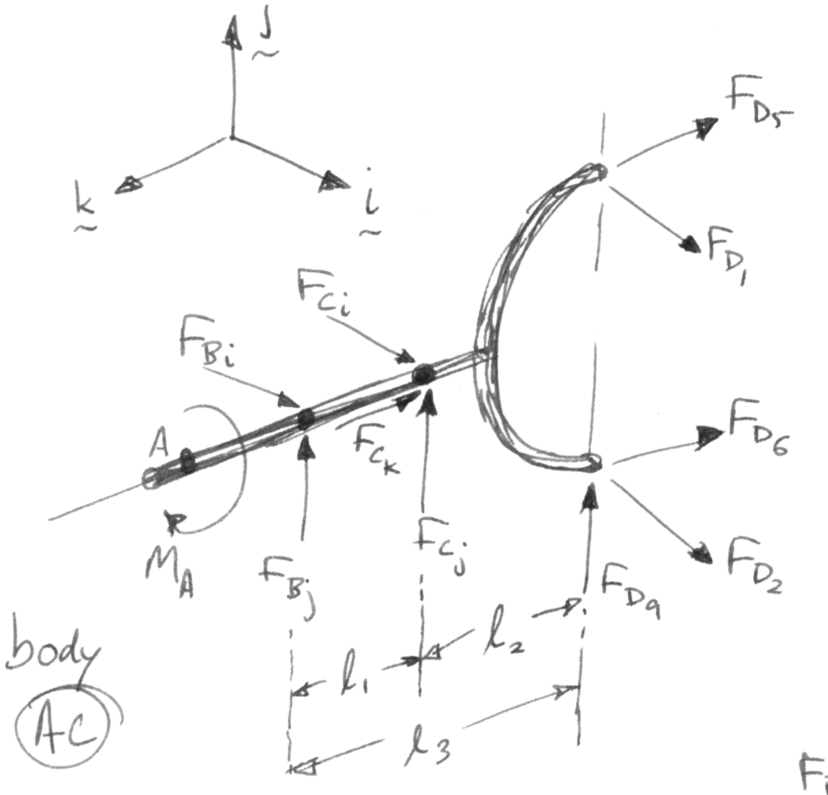
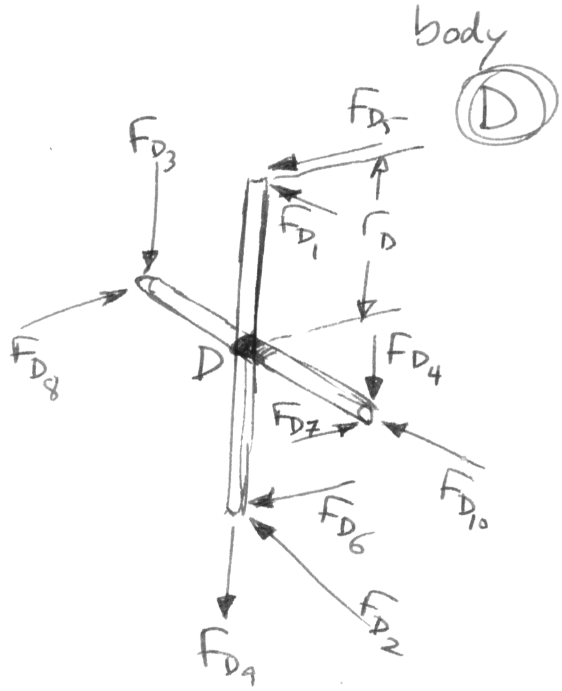
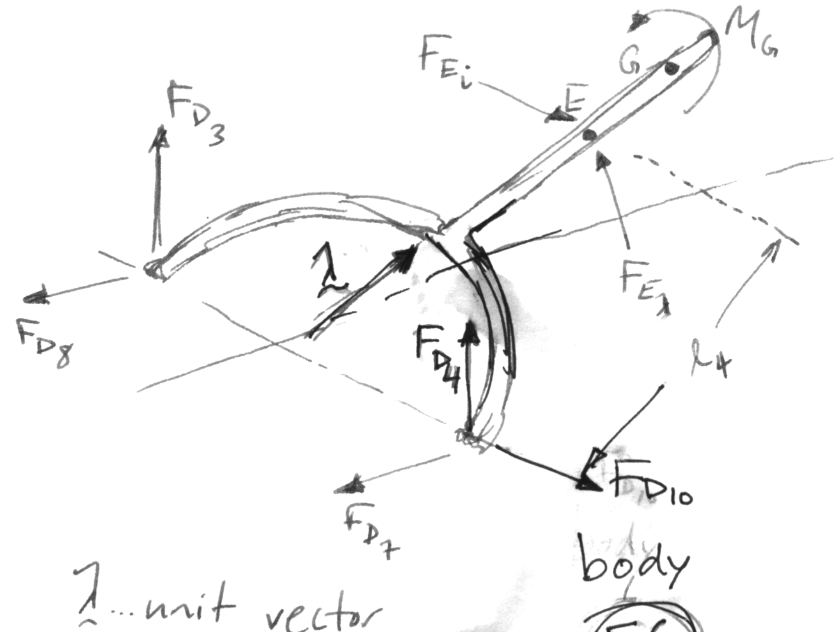
Problem #1, parts (a) and (b)

Strategy: FBD, derive all equations, reduce equations by choosing particular components with order of reduction chosen wisely







λ ... unit vector along shaft of EG

$M_A = 30\,Nm$

$l_1 = 0.1\,m$

$l_2 = 0.125\,m$

$l_3 = l_1 + l_2 = 0.225\,m$

$l_4 = 0.15\,m$

$\lambda = \sin\gamma\, j - \cos\gamma\, k$

$\gamma = 25°$

Now have FBD of all components and 18 unknowns, need 18 equations ... three from each of angular and linear momentum balance on the three components. Moment equations come from vector cross products, with only the result given below.

in (AC)

$$\sum \underset{\sim}{M}_{/B} = \underset{\sim}{0} = \underset{\sim}{i}\left(l_1 F_{C_j} + l_3 F_{D_9} + r_D F_{D_6} - r_D F_{D_5}\right)$$
$$+ \underset{\sim}{j}\left(-l_1 F_{C_i} - l_3 F_{D_1} - l_3 F_{D_2}\right)$$
$$+ \underset{\sim}{k}\left(-M_A - r_D F_{D_1} + r_D F_{D_2}\right)$$

$$\sum \underset{\sim}{F} = \underset{\sim}{0} = \underset{\sim}{i}\left(F_{B_i} + F_{C_i} + F_{D_1} + F_{D_2}\right) + \underset{\sim}{j}\left(F_{B_j} + F_{C_j} + F_{D_9}\right)$$
$$+ \underset{\sim}{k}\left(-F_{C_k} - F_{D_5} - F_{D_6}\right)$$

in (D)

$$\sum \underset{\sim}{M}_{/D} = \underset{\sim}{0} = r_D \underset{\sim}{k}\left(F_{D_1} - F_{D_2} + F_{D_3} - F_{D_4}\right) + r_D \underset{\sim}{i}\left(F_{D_5} - F_{D_6}\right)$$
$$+ r_D \underset{\sim}{j}\left(F_{D_7} - F_{D_8}\right)$$

$$\sum \underset{\sim}{F} = \underset{\sim}{0} = \underset{\sim}{i}\left(-F_{D_1} - F_{D_2} - F_{D_{10}}\right) + \underset{\sim}{j}\left(-F_{D_3} - F_{D_4} - F_{D_9}\right)$$
$$+ \underset{\sim}{k}\left(F_{D_5} + F_{D_6} - F_{D_7} - F_{D_8}\right)$$

(EG)

$$\sum \underset{\sim}{M}_{/E} = \underset{\sim}{0} = \underset{\sim}{i}\left[-l_4 \cos\gamma \left(F_{D_3} + F_{D_4}\right) - l_4 \sin\gamma \left(F_{D_7} + F_{D_8}\right)\right]$$
$$+ \underset{\sim}{j}\left[-M_G \sin\gamma + l_4 \cos\gamma\, F_{D_{10}} + r_D\left(F_{D_8} - F_{D_7}\right)\right]$$
$$+ \underset{\sim}{k}\left[M_G \cos\gamma + r_D\left(F_{D_4} - F_{D_3}\right) + l_4 \sin\gamma\, F_{D_{10}}\right]$$

$$\sum \underset{\sim}{F} = \underset{\sim}{0} = \underset{\sim}{i}\left(F_{D_{10}} + F_{E_i}\right) + \underset{\sim}{j}\left(F_{D_3} + F_{D_4} + F_{E_\lambda}\cos\gamma\right) + \underset{\sim}{k}\left(F_{D_7} + F_{D_8} + F_{E_\lambda}\sin\gamma\right)$$

Choice of which equations to consider first can make it not to difficult to do by hand:

$$\text{AC} \quad \sum \underset{\sim}{M}_{/B} \cdot \underset{\sim}{k} = 0 = -M_A + r_D\left(F_{D_2} - F_{D_1}\right) \quad \therefore M_A = r_D\left(F_{D_2} - F_{D_1}\right) \quad (1)$$

$$\text{EG} \quad \sum \underset{\sim}{M}_{/E} \cdot \underset{\sim}{\lambda} = 0 = -M_G + r_D\left(F_{D_3} - F_{D_4}\right)\cos\gamma + r_D\left(F_{D_8} - F_{D_7}\right)\sin\gamma \quad (2)$$

$$\text{D} \quad \sum \underset{\sim}{M}_{/D} \cdot \underset{\sim}{j} = 0 = r_D\left(F_{D_7} - F_{D_8}\right) \quad \therefore F_{D_7} = F_{D_8} \quad (3)$$

$$\text{D} \quad \sum \underset{\sim}{M}_{/D} \cdot \underset{\sim}{k} = 0 = r_D\left(F_{D_1} - F_{D_2} + F_{D_3} - F_{D_4}\right)$$

$$\therefore F_{D_1} - F_{D_2} = F_{D_3} - F_{D_4} \quad (4)$$

substitute (3) & (4) into (2) to give

$M_G = r_D \cos\gamma \left(F_{D_2} - F_{D_1}\right)$ and substitute (1) to get

$M_G = M_A \cos\gamma$ or $\boxed{M_G = 27.19\,\text{Nm}}$

Note: this result could have been gotten without writing the full equations on all components, but we'll be putting them to use momentarily for part (b) so it was not wasted effort.

For part (b), use the remaining equations to determine the unknowns:

$$\text{(EG)} \quad \sum \underset{\sim}{M}_{/E} \cdot \underset{\sim}{j} = 0 = -M_G \sin\gamma + l_4 \cos\gamma F_{D_{10}} + r_D \underbrace{\cancel{(F_{D_8} - F_{D_7})}}_{\to 0 \text{ by } (3)}$$

$$\therefore F_{D_{10}} = \frac{M_G \sin\gamma}{l_4 \cos\gamma} \cong 84.52\text{N}$$

$$\text{(EG)} \quad \sum \underset{\sim}{F} \cdot \underset{\sim}{i} = 0 = F_{D_{10}} + F_{Ei} \quad \therefore \boxed{F_{Ei} = -84.52\text{N}}$$

$$\text{(EG)} \quad \sum \underset{\sim}{M}_{/E} \cdot \underset{\sim}{i} = 0 = l_4 \cos\gamma (F_{D_4} + F_{D_3}) - l_4 \sin\gamma (F_{D_7} + F_{D_8})$$

$$\left.\begin{aligned} &\text{(EG)} \quad \sum \underset{\sim}{F} \cdot \underset{\sim}{j} = 0 = F_{D_4} + F_{D_3} + F_{E\lambda} \cos\gamma \\ &\text{(EG)} \quad \sum \underset{\sim}{F} \cdot \underset{\sim}{k} = 0 = F_{D_7} + F_{D_8} + F_{E\lambda} \sin\gamma \end{aligned}\right\} \begin{aligned} F_{E\lambda} &= -\frac{1}{\cos\gamma}(F_{D_4} + F_{D_3}) \\ &= -\frac{1}{\sin\gamma}(F_{D_7} + F_{D_8}) \end{aligned}$$

these may be written in matrix form

$$\begin{bmatrix} l_4 \cos\gamma & -l_4 \sin\gamma \\ -\frac{1}{\cos\gamma} & +\frac{1}{\sin\gamma} \end{bmatrix} \begin{bmatrix} F_{D_4} + F_{D_3} \\ F_{D_7} + F_{D_8} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

and because the matrix is non-singular, the solution must be

$$F_{D_4} + F_{D_3} = 0 \quad (6) \qquad \text{and} \qquad F_{D_7} + F_{D_8} = 0$$

thus $\boxed{F_{E\lambda} = 0}$

and from (3), $F_{D_7} = F_{D_8}$ so $F_{D_7} = F_{D_8} = 0$ (7)

(D) $\sum \underset{\sim}{F} \cdot \underset{\sim}{j} = 0 = -F_{D9} - \overbrace{\cancel{(F_{D4} + F_{D3})}}^{0 \text{ by } (6)}$ $\quad \therefore F_{D9} = 0 \quad (8)$

(AC) $\sum \underset{\sim}{M}_{/B} \cdot \underset{\sim}{j} = 0 = -l_1 F_{Ci} - l_3 (F_{D1} + F_{D2}) = 0$

(D) $\sum \underset{\sim}{F} \cdot \underset{\sim}{i} = 0 = -(F_{D1} + F_{D2}) - F_{D10} \quad \therefore F_{D1} + F_{D2} = -F_{D10}$ (11) $\Bigg\} F_{Ci} = \frac{l_3}{l_1} F_{D10}$ (12)

$\therefore \boxed{F_{Ci} \cong 190.2\,N}$

(AC) $\sum \underset{\sim}{M}_{/B} \cdot \underset{\sim}{i} = 0 = l_1 F_{Cj} + l_3 F_{D9} + r_D (F_{D6} - F_{D5})$ (10)

(D) $\sum \underset{\sim}{M}_{/D} \cdot \underset{\sim}{i} = 0 = (F_{D5} - F_{D6}) r_D \quad \therefore F_{D5} = F_{D6}$ (9)

substitute (8) & (9) into (10) $\Rightarrow \boxed{F_{Cj} = 0}$ (13)

(D) $\sum \underset{\sim}{F} \cdot \underset{\sim}{k} = 0 = F_{D5} + F_{D6} - \overbrace{\cancel{(F_{D7} + F_{D8})}}^{0 \text{ by } (7)}$

combine with (9) to get $F_{D5} = F_{D6} = 0$ (14)

(AC) $\sum \underset{\sim}{F} \cdot \underset{\sim}{k} = 0 = -F_{Ck} \overbrace{\cancel{- F_{D5} - F_{D6}}}^{0 \text{ by } (14)} \quad \therefore \boxed{F_{Ck} = 0}$

(AC) $\sum \underset{\sim}{F} \cdot \underset{\sim}{i} = 0 = F_{Bi} + F_{Ci} + F_{D1} + F_{D2}$

using (11) & (12), $F_{Bi} = F_{D10} \left(1 - \frac{l_3}{l_1}\right) \quad \therefore \boxed{F_{Bi} \cong -105.7\,N}$

(AC) $\sum \underset{\sim}{F} \cdot \underset{\sim}{j} = 0 = F_{Bj} + \overbrace{\cancel{F_{Cj}}}^{0 \text{ by } (13)} + \overbrace{\cancel{F_{D9}}}^{0 \text{ by } (8)} \quad \therefore \boxed{F_{Bj} = 0}$

part ⓒ

$$P_{in} = P_{out}$$

$$M_A \omega_{AC} = M_G \omega_{EG}$$

$$\omega_{EG} = \frac{M_A \omega_{AC}}{M_G} = \boxed{11.03 \ \frac{rev}{sec}}$$

part ⓓ

$$\underset{\sim}{\omega}_{EG} = \underset{\sim}{\omega}_{AC} + \underset{\sim}{\omega}_{D/AC} + \underset{\sim}{\omega}_{EG/D}$$

① $\omega_{EG} \underset{\sim}{\lambda} = \omega_{AC} \underset{\sim}{k} + \omega_{D/AC} \underset{\sim}{j} + \omega_{EG/D} \underset{\sim}{i}$

①$\cdot \underset{\sim}{k} \Rightarrow \omega_{EG} \cos\gamma = \omega_A$

$$\therefore \ \omega_{EG} = \frac{\omega_{AC}}{\cos\gamma} = \boxed{11.03 \ \frac{rev}{sec}}$$

note that this is exactly the same result as for part ⓒ

part ⓔ

combining results from parts ⓒ and ⓓ

$M_G = M_A \cos\gamma$ in current configuration

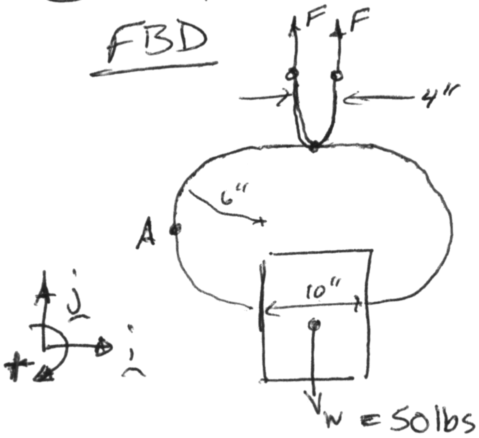
and for a rotation of 90°

$$\boxed{M_A = M_G \cos\gamma}$$

as the geometry of the problem as it relates to parts ⓒ and ⓓ is unchanged except for a reversal of the positions of input and output shafts

② p 3-17

FBD

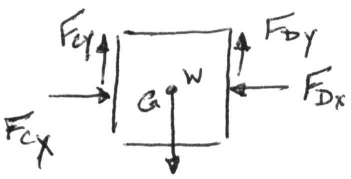


Find all Forces on the tongs.
Find the bending moment at A.

$\Sigma F_y = 0$: $2F = W$ (1)

$\underline{\underline{F = 25 \text{ lbs}}}$

FBD



ΣF_y: $F_{cy} + F_{Dy} = W$ (2)

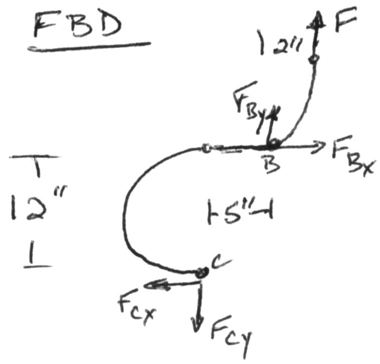
ΣM_G: $F_{cy} = F_{Dy}$ (3)

using (2), (3)

$2F_{cy} = W$

$\underline{\underline{F_{cy} = \frac{1}{2}W = 25\text{lbs}}}$

FBD



ΣF_x: $F_{Bx} - F_{cx} = 0$

$F_{Bx} = F_{cx}$ (4)

ΣF_y: $F_{By} - F_{cy} + F = 0$ (5)

$F_{By} = F_{cy} - F$

$\underline{\underline{F_{By} = 25 - 25 = 0}}$

ΣM_B: $+12F_{cx} - 5F_{cy} - 2F = 0$ (6)

$F_{cx} = \frac{1}{12}\left[5F_{cy} + 2F\right]$

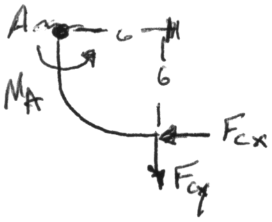
$\underline{\underline{F_{cx} = 14.6 \text{ lbs}}}$

from (4) $F_{cx} = F_{Bx}$

$\underline{\underline{F_{Bx} = 14.6\text{lbs}}}$

FBD w/ cut

Summing moments

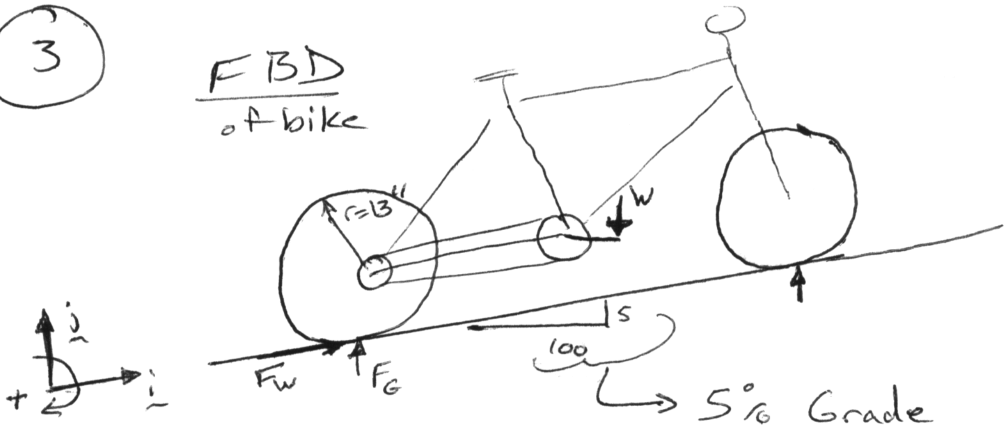


$M_A = 6F_{cx} + 6F_{cy}$

$\underline{\underline{M_A = 237.6 \text{ in lbs}}}$

③

FBD of bike



Assume All of the riders weight is on the front pedal when it is horizontal and the bike + rider is in equilibrium going up a 5% grade. what is the ratio of teeth on the sprockets?

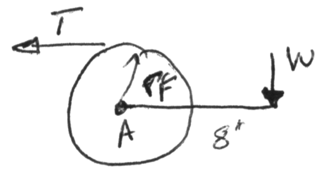
$\tan\theta = \frac{5}{100}$

$\Rightarrow \theta = \tan^{-1}\frac{5}{100} = 2.86°$

$\Sigma F_x: \; F_W - W\sin\theta = 0$

$F_W = W\sin\theta \quad (1)$

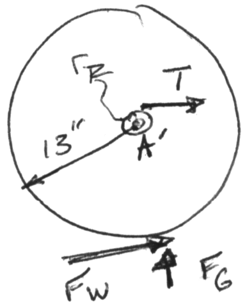
FBD of Front Sprocket



$\Sigma M_A: \; 8W = r_F T$

$T = \frac{8W}{r_F} \quad (2)$

FBD of rear wheel/sprocket



$\Sigma M_{A'}: \; 13F_W = r_R T \quad (3)$

Subs. (2) into (3)

$13 F_W = \frac{r_R}{r_F} 8W \quad (4)$

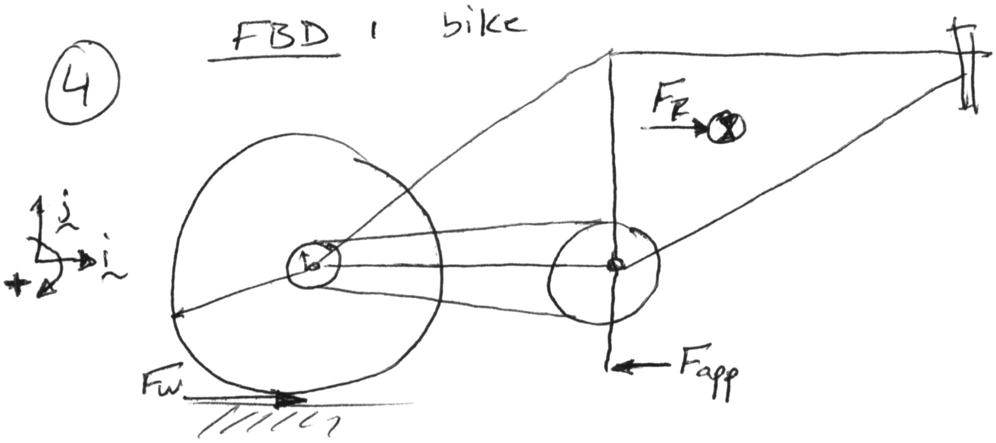
Subs (1) into (4) and solve for tooth ratio

$\frac{r_R}{r_F} = \frac{13\sin\theta \cancel{W}}{8\cancel{W}} = 0.081$ $\qquad \frac{r_F}{r_R} = 12.3$

i.e. if the front sprocket has 100 teeth the rear sprocket has 8 teeth.

(4)

FBD 1 bike

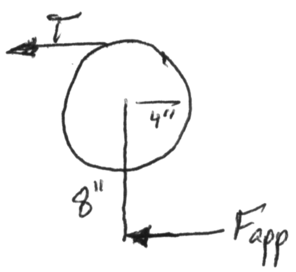


A person walks up to the bike and pushes on the bottom pedal w/ the crank vertical

If the person pushes backwards which way does the bike go?

wheel	$r_w = 13''$
rear sprocket	$r_{rs} = 1''$
front sprocket	$r_{fs} = 4''$
crank	$r_c = 8''$

FBD 2 Front sprocket

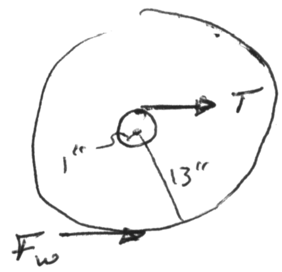


$\Sigma M = 0$

$8F_{app} - 4T = 0$

$T = 2F_{app} \quad (1)$

FBD 3 Rear spocket



$\Sigma M = 0$

$T = 13F_w \quad (2)$

Now $\Sigma F_x = 0 \Rightarrow -F_{app} + F_w + F_R = 0 \quad (3)$

Using (1),(2) $\frac{2}{13}F_{app} = F_w \quad (4)$

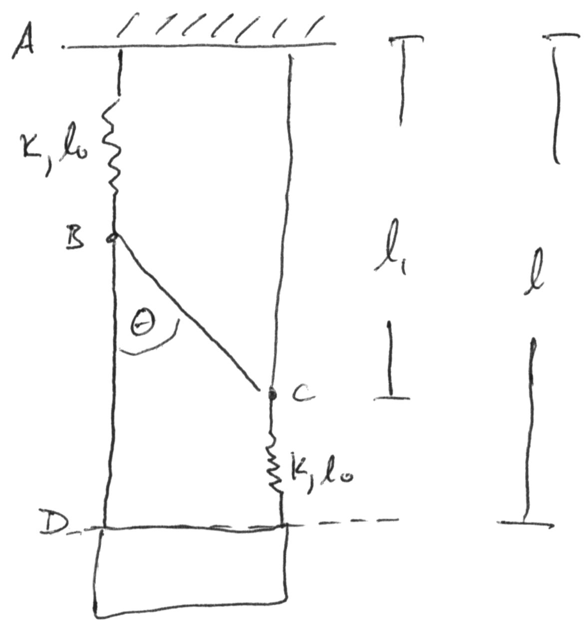
Subs: (4) into (3) $-F_{app} + \frac{2}{13}F_{app} + F_R = 0$

$$F_R = \frac{11}{13}F_{app}$$

This result means that a force of $\frac{11}{13}F_{app}$ (for this choice of dimensions has to be applied to keep the bike in equilibrium without it, the bike would move backwards!

5

10/10



If all segments are taught, what happens when the string is cut?

For a spring:

$F = kx$

so $l_0 = \frac{F}{k}$ (1)

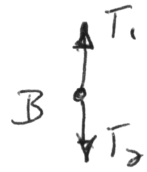
Look @ pt. B:

Assume $T > 0$ and $0 < \theta < 90$

$\Sigma F_y: \quad T_1 - T_2 - T\sin\theta = 0$

$T_1 = T_2 + \underbrace{T\sin\theta}_{\text{some positive quantity}}$ (2)

When the string is cut:



$T_1 = T_2$ (3)

So with the string T_1 is greater than without it. This means that the springs would be stretched more w/ the ~~spring~~ crossmember BC than without (eq 1.). When the string is cut, the weight goes up (b) because the force on the springs is lower.