PROBLEM 4-27

Statement: **A storage rack is to be designed to hold the paper roll of Problem 4-8 as shown in Figure P4-12. Determine suitable values for dimensions a and b in the figure. Consider bending, shear, and bearing stresses. Assume an allowable tensile/compressive stress of 100 MPa and an allowable shear stress of 50 MPa for both stanchion and mandrel, which are steel. The mandrel is solid and inserts halfway into the paper roll. Balance the design to use all of the material strength. Calculate the deflection at the end of the roll.**

Units: $N := newton$ $\quad kN := 10^3 \cdot newton$ $\quad MPa := 10^6 \cdot Pa$ $\quad GPa := 10^9 \cdot Pa$

Given: Paper roll dimensions $OD := 1.50 \cdot m$ $\quad ID := 0.22 \cdot m$ $\quad L_{roll} := 3.23 \cdot m$

Material properties $S_y := 100 \cdot MPa$ $\quad S_{ys} := 50 \cdot MPa$ $\quad E := 207 \cdot GPa$

Roll density $\rho := 984 \cdot kg \cdot m^{-3}$

Assumptions: 1. The paper roll's weight creates a concentrated load acting at the tip of the mandrel.
2. The mandrel's root in the stanchion experiences a distributed load over its length of engagement

Solution: See Figures 4-27 and Mathcad file P0427.

1. In Problem 3-27, we were concerned only with the portion of the mandrel outside of the stanchion. Therefore, we modeled it as a cantilever beam with a shear and moment reaction at the stanchion. Unfortunately, this tells us nothing about the stress or force distributions in the portion of the mandrel that is inside the stanchion. To do this we need to modify the model by replacing the concentrated moment (and possibly the concentrated shear force) with a force system that will yield information about the stress distribution in the mandrel on that portion that is inside the stanchion. Figure 4-27A shows the FBD used in Problem 3-27. Figure 4-27B is a simple model, but is not representative of a built-in condition. It would be appropriate if the hole in the stanchion did not fit tightly around the mandrel. Figure 4-27C is an improvement that will do for our analysis.

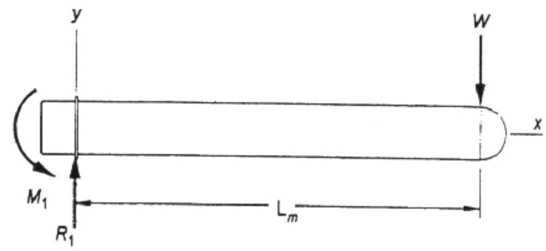


FIGURE 4-27A
Free Body Diagram for Problem 3-27

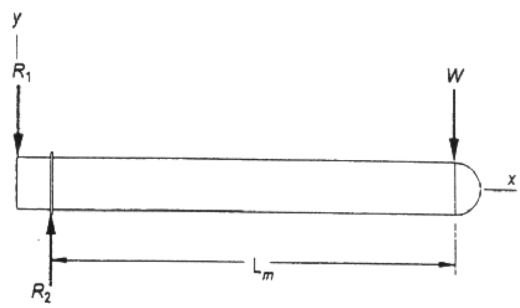


FIGURE 4-27B
Simplified Free Body Diagram, not used

2. Determine the weight of the roll and the length of the mandrel.

$$W := \frac{\pi}{4} \cdot \left(OD^2 - ID^2\right) \cdot L_{roll} \cdot \rho \cdot g \qquad W = 53.9 \, kN$$

$$L_m := 0.5 \cdot L_{roll} \qquad L_m = 1.615 \, m$$

3. From inspection of Figure 4-27C, write the load function equation

$$q(x) = -w\langle x\rangle^0 + w\langle x-b\rangle^0 + R\langle x-b\rangle^{-1} - W\langle x-b-L_m\rangle^{-1}$$

4. Integrate this equation from $-\infty$ to x to obtain shear, $V(x)$

$$V(x) = -w\langle x\rangle^1 + w\langle x-b\rangle^1 + R\langle x-b\rangle^0 - W\langle x-b-L_m\rangle^0$$

5. Integrate this equation from $-\infty$ to x to obtain moment, $M(x)$

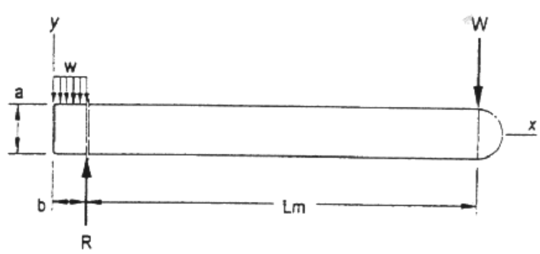


FIGURE 4-27C
Free Body Diagram used in Problem 4-27

$M(x) = -(w/2)\langle x\rangle^2 + (w/2)\langle x - b\rangle^2 + R\langle x - b\rangle^1 - W\langle x - b - L_m\rangle^1$

6. Solve for the reactions by evaluating the shear and moment equations at a point just to the right of $x = b + L_m$, where both are zero.

At $x = (b + L_m)^+$, $V = M = 0$

$$0 = -w\cdot(b+L_m) + w\cdot(L_m) + R - W \qquad R = W + w\cdot b$$

$$0 = -\frac{w}{2}\cdot(b+L_m)^2 + \frac{w}{2}\cdot L_m^2 + R\cdot L_m = -\frac{w}{2}\cdot(b+L_m)^2 + \frac{w}{2}\cdot L_m^2 + (W + w\cdot b)\cdot L_m \qquad w = \frac{2\cdot W\cdot L_m}{b^2}$$

Note that R is inversely proportional to b and w is inversly proportional to b^2.

Your choice for this will result in different final answers

7. To see the value of x at which the shear and moment are maximum, let

$\boxed{b := 400\cdot mm}$ then $w := \frac{2\cdot W\cdot L_m}{b^2}$ and $R := W + w\cdot b$ $L := b + L_m$

8. Define the range for x $x := 0\cdot mm, 0.002\cdot L .. L$

Actually, you would do this in Matlab like in HW #6

9. For a Mathcad solution, define a step function S. This function will have a value of zero when x is less than z, and a value of one when it is greater than or equal to z.

$S(x,z) := if(x \ge z, 1, 0)$

10. Write the shear and moment equations in Mathcad form, using the function S as a multiplying factor to get the effect of the singularity functions.

$$V(x) := -w\cdot S(x, 0\cdot mm)\cdot x + w\cdot S(x,b)\cdot(x-b) + R\cdot S(x,b) - W\cdot S(x,L)$$

$$M(x) := \frac{-w}{2}\cdot S(x, 0\cdot mm)\cdot x^2 + \frac{w}{2}\cdot S(x,b)\cdot(x-b)^2 + R\cdot S(x,b)\cdot(x-b) - W\cdot S(x,L)\cdot(x-L)$$

11. Plot the shear and moment diagrams.

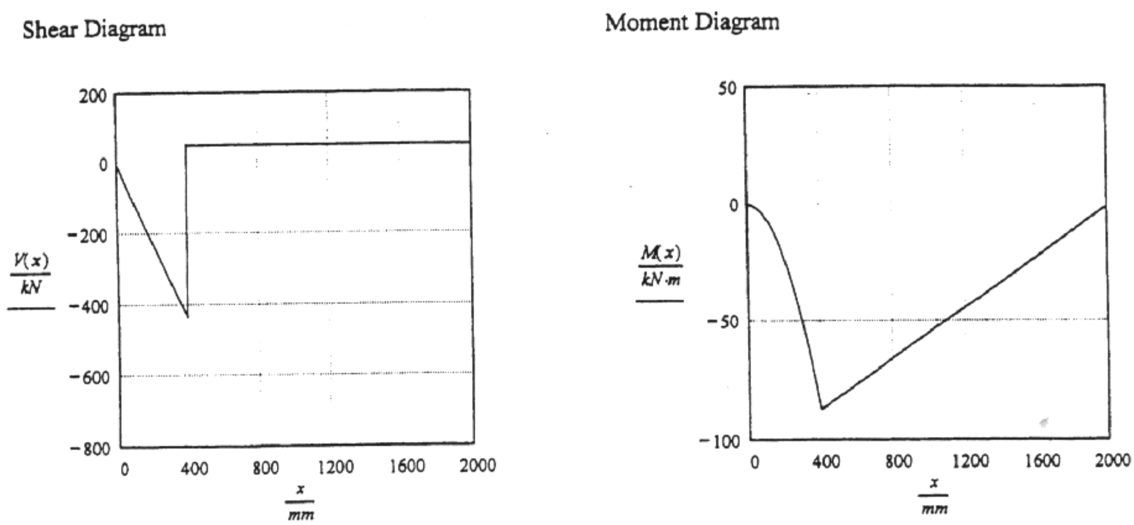


FIGURE 4-27D
Shear and Moment Diagram Shapes for Problem 4-27

12. From Figure 4-27D, the maximum internal shear and moment occur at $x = b$ and are

$$V_{max} = \frac{2\cdot W\cdot L_m}{b} \qquad M_{max} := W\cdot L_m \qquad M_{max} = 87.04 \circ kN\cdot m$$

13. The bending stress will be a maximum at the top or bottom of the mandrel at a section through $x = b$.

$$\sigma_{max} = \frac{M_{max} \cdot a}{2 \cdot I} \qquad \text{where} \qquad I = \frac{\pi \cdot a^4}{64} \qquad \text{so,} \qquad \sigma_{max} = \frac{32 \cdot M_{max}}{\pi \cdot a^3} = S_y$$

Solving for a, $a := \left(\frac{32 \cdot W \cdot L_m}{\pi \cdot S_y} \right)^{\frac{1}{3}}$ $a = 206.97 \, mm$

Round this to $a := 210 \cdot mm$

14. Using this value of a and equation 4.15c, solve for the shear stress on the neutral axis at $x = b$.

$$\tau_{max} = \frac{4 \cdot V_{max}}{3 \cdot A} = \frac{8 \cdot W \cdot L_m}{3 \cdot \left(\frac{\pi \cdot a^2}{4} \right) \cdot b} = S_{ys}$$

Solving for b $b := \frac{8 \cdot W \cdot L_m}{3 \cdot \left(\frac{\pi \cdot a^2}{4} \right) \cdot S_{ys}}$ $b = 134.026 \, mm$

Round this to $b := 134 \cdot mm$

15. These are minimum values for a and b. Using them, check the bearing stress.

Magnitude of distributed load $w := \frac{2 \cdot W \cdot L_m}{b^2}$ $w = 9695 \, \frac{N}{mm}$

Bearing stress $\sigma_{bear} := \frac{w \cdot b}{a \cdot b}$ $\sigma_{bear} = 46.2 \, MPa$

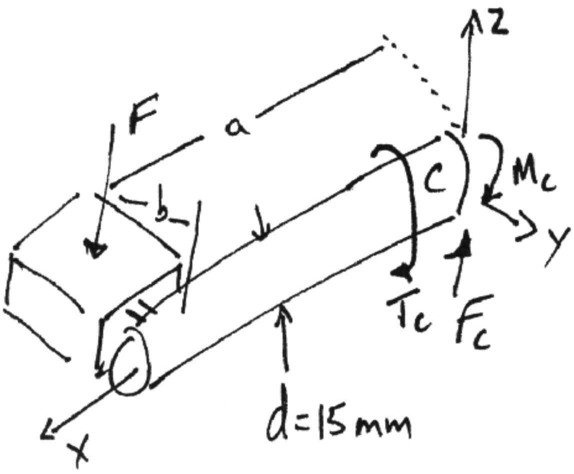
Since this is less than S_y, the design is acceptable for $a = 210 \, mm$ and $b = 134 \, mm$

16. Assume a cantilever beam loaded at the tip with load W and a mandrel diameter equal to a calculated above.

Moment of inertia $I := \frac{\pi \cdot a^4}{64}$ $I = 9.547 \cdot 10^7 \, mm^4$

Deflection at tip (Appendix D) $y_{max} := -\frac{W \cdot L_m^3}{3 \cdot E \cdot I}$ $y_{max} = -3.83 \, mm$

This can be accomodated by the 220-mm inside diameter of the paper roll.



- Find the fluctuating stresses
- Find the fatigue safety factor

$Y = 350$ MPa

$S_{ut} = 500$ MPa

$F_{max} = 1500$ N

$F_{min} = 0$ N

– First determine the worst case stresses. these will occur at C on top of the arm

when F=1500 N

$$\sigma_x = \frac{Mc}{I} = 769.6 \text{ MPa} \qquad \sigma_y = 0 \text{ MPa}$$

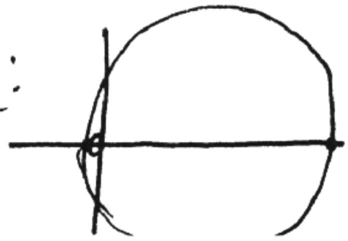
$$\tau_{xy} = \frac{Tc}{J} = 135.8 \text{ MPa}$$

So:
$$\sigma_{I\,max} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 793 \text{ MPa}$$

$$\sigma_{II\,max} = 0 \text{ MPa}$$

$$\sigma_{III\,max} = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = -23 \text{ MPa}$$

Mohr's circle:



Having principal stresses will do you no good in this case. You must determine an "effective" stress or overall stress magnitude. Eq. 5.7c, for von Mises stress is:

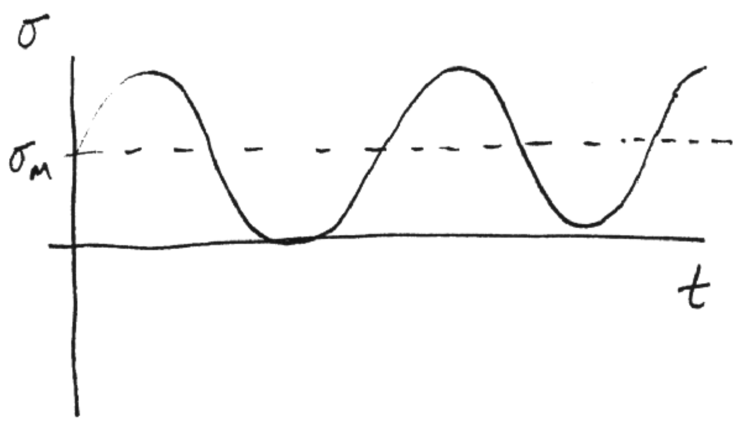
$$\sigma' = \sqrt{\sigma_I^2 - \sigma_I \sigma_{III} + \sigma_{III}^2} \quad \text{(for 2D)}$$

$$\sigma'_{max} = \sqrt{793^2 - (793)(-23) + (-23)^2} = 804.7\,\text{MPa}$$

$$\sigma'_{min} = 0\,\text{MPa} \quad (\text{when } F = 0)$$

Now the alternating and mean stress components can be calculated:

$$\boxed{\begin{aligned} \sigma'_a &= \frac{\sigma'_{max} - \sigma'_{min}}{2} = \underline{\underline{402.4\,\text{MPa}}} \\ \sigma'_m &= \frac{\sigma'_{max} + \sigma'_{min}}{2} = \underline{\underline{402.4\,\text{MPa}}} \end{aligned}}$$



Now handle the fatigue safety factor.

The endurance limit: $S_e' = 0.5\,S_{ut} = 250\text{ MPa}$

The modified endurance limit is:

$$S_e = C_{load}\,C_{size}\,C_{surf}\,C_{temp}\,C_{reliab}\,S_e'$$

where the Cs are modification factors

$C_{load} = 1$ (bending) [eq. 6.7a]

$C_{temp} = 1$ $T \le 450°C$ [eq. 6.7f] (the rider is not on fire)

$C_{size} = 1.189\,d_{eq.}^{-0.097}$ for $8\text{mm} \le d \le 250\text{mm}$ [eq. 6.7b]

$$= 1.189\left(\sqrt{\frac{A_{95}}{0.0766}}\right)^{-0.097}$$

$$= 1.189\left(\sqrt{\frac{2.354}{.0766}}\right)^{-0.097}$$

$A_{95} = .01046\,d^2$ (non rotating) from Fig. 6-25

$C_{size} = 1.007 \Rightarrow 1$

$C_{surf} = A\,(S_{ut})^b$

$A = 4.51$, $b = -0.265$ machined from Table 6-3

based on choices so they may be different

$C_{surf} = 0.869$

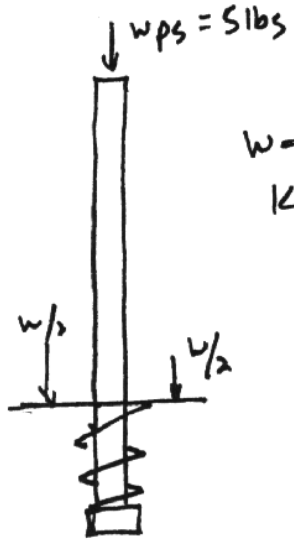
$C_{reliab} = 0.753$

99.9% Table 6-4

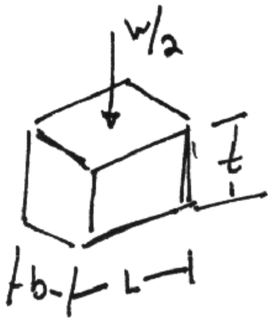
So: $\boxed{S_e = 163.56\text{ MPa}}$

and: $N_f = \dfrac{S_e\,S_{ut}}{\sigma_a'\,S_{ut} + \sigma_m'\,S_e} = 0.31$

6-14



Using 2000 series Aluminum, design the cantilever beam sections to survive jumps of 2in w/ a dynamic safety factor of 2 for a life of 5e4 cycles



First assume: 2024 Al

$\Rightarrow$ weight apllied @ $L/2$

$\Rightarrow b = 1.5$ to give adequet support

$\Rightarrow L/2 = 5in$

$\Rightarrow S_{ut} = 64$ Ksi for 2024

Table C-2 ; p994

* Note: your values will be different based on your assumptions

- with the above assumptions all that remains is to solve for t

Begin by finding F_{max} as in HW 9 using Energy Method

$$F_{max} = ky = K\left[\frac{mg}{K} + \frac{1}{2}\sqrt{\left(\frac{2mg}{K}\right)^2 + \frac{8mgh}{K}}\right] = \underline{\underline{238lb}}$$

recall: $U_h = U_s \Rightarrow$ from $E_{max\ height} = E_{min\ height}$

$$(mgy + mgh) = \frac{1}{2}Ky^2$$

So : $P_{max} = \frac{F_{max}}{2} = 119$ lbs

$P_{min} = 0$ lbs

$M_{max} = P_{max}(L/2) = 595$ lb·in

$M_{min} = 0$ lb·in

• Find the alternating and mean components of the moment.

$M_a = \frac{M_{max} - M_{min}}{2} = \underline{297.5 \text{ lb·in}}$

$M_m = \frac{M_{max} + M_{min}}{2} = \underline{297.5 \text{ lb·in}}$

• Find the Endurance limit : $S_e' = 20$ ksi $\boxed{p994}$ @ 5e8 cycles

• Find modification factors $S_e = C_{load} C_{size} C_{surf} C_{temp} C_{reliab} S_e'$

• $C_{load} = 1$ (bending) • $C_{temp} = 1$ $\boxed{T < 450°F \text{ eq 6.7f}}$

• $C_{reliab} = 1$ $\boxed{50\% \text{ Table 6-4}}$

$C_{surf} = A(S_{ut})^b$ $A = 2.7$ $b = -.265$ $\boxed{\text{machined Table 6-3}}$

• $C_{surf} = 0.897$

$C_{size} = 0.869 \left(d_{eq}\right)^{-.097}$ $d_{eq} = \sqrt{\frac{0.05(w)(t)}{0.0766}}$

• $C_{size} = 0.87(t)^{0.403}$

• $S_e = (1)(0.87\, t^{0.403})(0.897)(1)(1)(20 \text{ ksi}) = \underline{\underline{15.61\, t^{0.403}}}$

· Find fatigue strength at $N = 10^3$ cycles (S_m)

$S_m = 0.9\, S_{ut} = 57.6\,\text{ksi}$ (eq 6.9)

· Use eq 6.10 (a), $S_f = a N^b$, to find corrected fatigue strength

where $a(t) = \dfrac{S_m}{(10^3)^{b(t)}}$, $b(t) = \dfrac{1}{z} \log \dfrac{S_m}{S_e(t)}$

$\boxed{N = 5e4}$

$z = -5.699$ for 5e8 from table 6-5

$$S_f(t) = \left[\frac{57.6}{(10^3)^{-0.175 \log\left(\frac{3.69}{t^{0.403}}\right)}} \right] N^{\left[-0.175 \log\left(\frac{3.69}{t^{0.0403}}\right)\right]}$$

Now: $N_{fd} = 2 = \dfrac{wt^2}{6} \dfrac{S_f(t)\, S_{ut}}{M_a S_{ut} + M_m S_f(t)}$

plug numbers in and solve for t

$\underline{\underline{t = 0.304 \text{ in}}}$