"SOLUTIONS"

Your Name: Staff

Your TA & section day: ____________

# MAE 325 Final Exam

## Wednesday December 15, 1999, 9:00 AM - 11:30 AM

This version: December 15, 1999

5 problems, 100 points, and 150 minutes.

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|---|
| Last chance: you have till midnight tonight to get a bonus point by completing the MAE 325 Course Evaluation on the WWW at courseval.cornell.edu. |

**Please follow these directions to ease grading and to maximize your score.**

a) No books, notes, or calculators allowed. Ask for extra scrap paper if you need it.

b) Put *scrap* work not to be graded on left hand pages, *neat work to be graded* on right hand pages. If you need the space, clearly mark work to be graded that is on left hand pages.

c) Full credit if

- work is I. ) neat,
  II. ) clear, and
  III.) well organized;
- reasonable justification, enough to distinguish an informed answer from a guess, is given (unless otherwise stated). That is, all answers and work need some explanation;
- your answers are TIDILY REDUCED; and
- your answers are $\boxed{\text{boxed}}$ in.

d) If you base your answer on an unreasonable intermediate result we will not track your reasoning.

e) If you remember large appropriate formulas you may use them (at your own risk should they not apply or not be efficient).

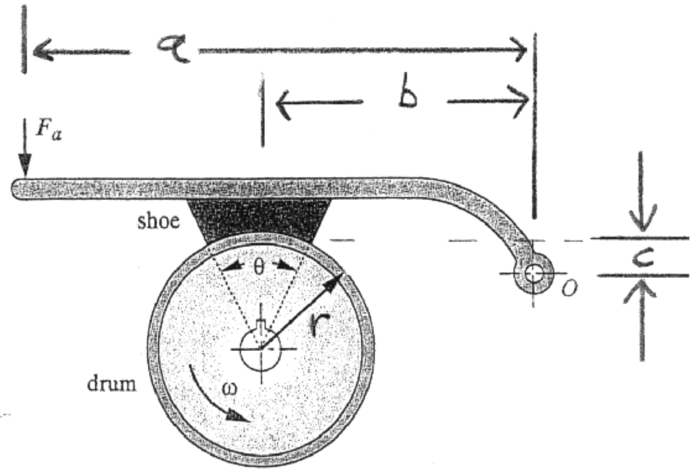
f) Unless otherwise stated, you will get full credit for, instead of doing a calculation, presenting Matlab code that would generate the desired answer. To ease grading and save space, your Matlab code can use shortcut notation like "$\dot\theta_7$ = 18" instead of, say, "theta7dot = 18".

| | | |
|---|---|---|
| Problem | 1: | /20 |
| Problem | 2: | /20 |
| Problem | 3: | /20 |
| Problem | 4: | /20 |
| Problem | 5: | /20 |
| TOTAL: | | /100 |

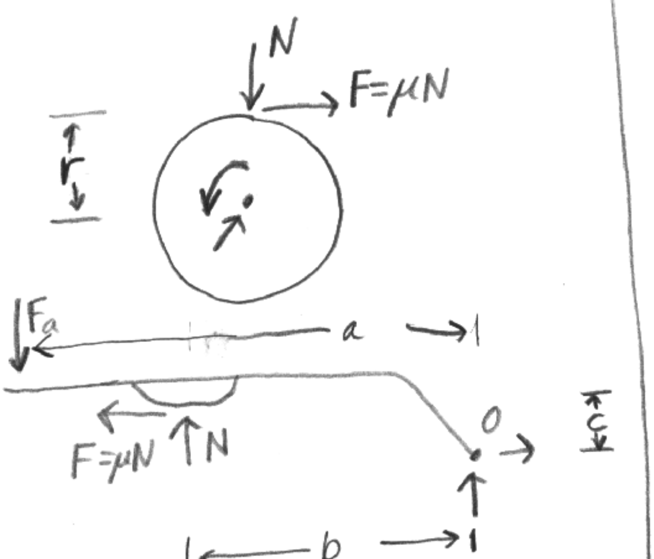
1) **(20 pt)** The "short shoe" (means $\theta$ is small) drum brake is shown below.
$\Leftarrow$ *Please put scrap work for problem 1 on the page to the left* $\Leftarrow$.
$\Downarrow$ *Put neat, clear work to be graded for problem 1 below.* $\Downarrow$
*( If you need the space, clearly mark work to be graded on the scrap page.)*

a) (12 points) Assuming $\omega > 0$ find the braking torque in terms of (some or all of) $F_a, a, b, c, r$ and $\mu$.

b) (8 points) Write a single inequality in terms of (some or all of) $a, b, c, r$ and $\mu$ that gives the conditions for this brake to be self locking (self locking means the drum is jammed to a stop as soon as the shoe touches the drum, even if gently).




FBDs




Braking Torque = Moment on drum due to friction

$$M_B = Fr$$

AMB/O of brake arm (Statics)

$$\Sigma M_{/O} = 0 = (-F_a a - Fc + Nb)\,\mathbf{k}$$

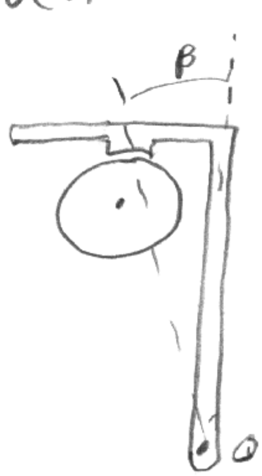
$$\Rightarrow \quad F_a a = -Fc + \frac{F}{\mu} b$$

$$= F\left(\frac{b}{\mu} - c\right) \Rightarrow F = \frac{F_a a}{b/\mu - c}$$

$$\Rightarrow \boxed{M_B = \frac{F_a a r}{b/\mu - c}} \quad (a)$$

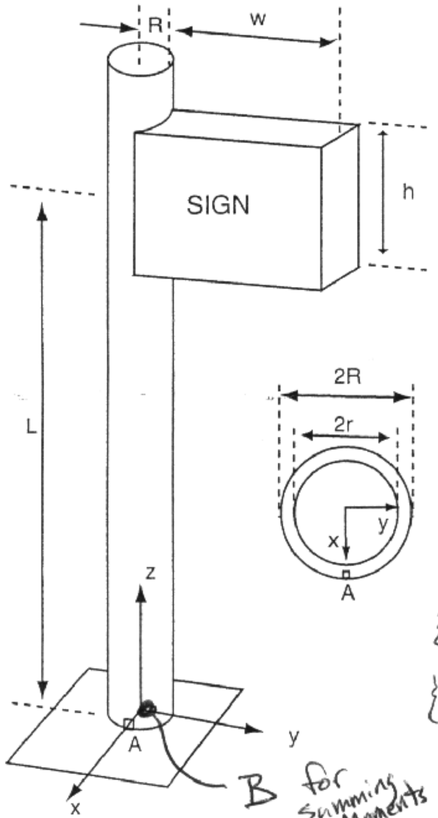
Self locking when F is positive even for negative $F_a \Rightarrow \quad c > b/\mu$

$$\boxed{\mu > b/c \text{ self locking or } \beta < \phi \quad \angle \phi = \text{friction angle}} \quad (b)$$

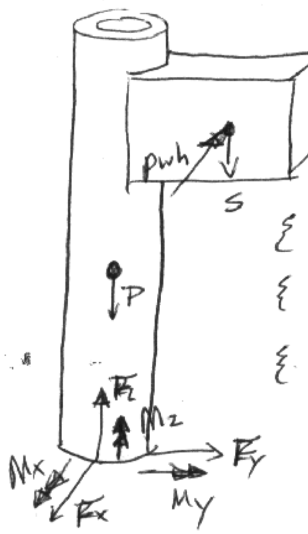


2) (20 pt) A sign of dimensions w and h is supported by a hollow circular pole having outer ~~diameter~~ Radius $R$ and inner ~~diameter~~ radius $r$. The sign is attached flush to the pole and its middle is at a distance $L$ above the ground. The wind pressure against the sign is $p$ (assume wind pressure is strictly perpendicular to the face of the sign in the negative $x$ direction). The cross-sectional moments of inertia of the pole are $I = \pi(R^4 - r^4)/16$ and $J = 2 * I$. The weight of the pole is $P$. The weight of the sign is $S$. Point A is at the base of the pole, on the outer edge of the front of the pole (a distance $R$ along the $x$-axis, see cross section detail). The sign is on the $y-z$ plane.

$$I = \frac{\pi(R^4 - r^4)}{4}$$




B for summing moments




$$\Sigma F = 0 = -pwh\,\underline{i} - S\,\underline{k} - P\,\underline{k} + F_x\,\underline{i} + F_y\,\underline{j} + F_z\,\underline{k}$$

$\{\ \}\cdot \underline{j} \Rightarrow$ $\boxed{F_y = 0}$

$\{\ \}\cdot \underline{k} \Rightarrow$ $\boxed{F_z = S + P}$

$\{\ \}\cdot \underline{i} \Rightarrow$ $\boxed{F_x = pwh}$

$$\Sigma M_{/B} = 0 = \left(L\underline{k} + \tfrac{w}{2}\underline{j}\right) \times (-pwh\,\underline{i}) + \left(\tfrac{w}{2}\underline{j}\right) \times (-S\underline{k}) + M_x\underline{i} + M_y\underline{j} + M_z(\underline{k})$$

$$\{\{0 = -pwhL\,\underline{j} + pwh\tfrac{w}{2}\underline{k} - S\tfrac{w}{2}\underline{i} + M_x\underline{i} + M_y\underline{j} + M_z\underline{k}\}\}$$

$\{\{\ \}\}\cdot \underline{i} \Rightarrow$ $\boxed{M_x = S\frac{w}{2}}$

$\{\{\ \}\}\cdot \underline{j} \Rightarrow$ $\boxed{M_y = pwhL}$

$\{\{\ \}\}\cdot \underline{k} \Rightarrow$ $\boxed{M_z = -pwh\frac{w}{2}}$

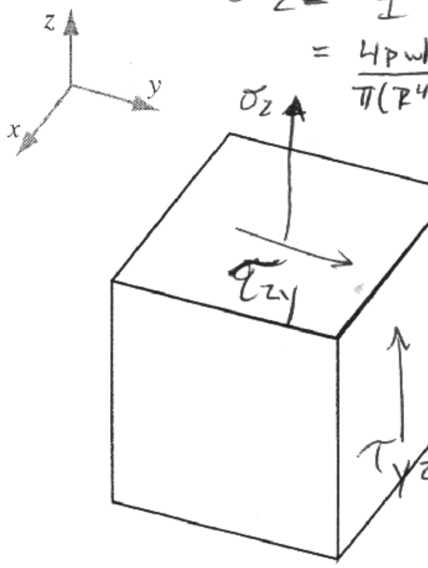
a) (10 points) In the space above draw a free-body diagram of the structure at a cut at its base. Use statics to find the forces and moments at this cut in terms of (some or all of) $P, S, L, R, r, h, w$ and $p$.

b) (10 points) Fill in the components of the stress matrix at point A, assign values to all of the tractions shown. Answer in terms $P, S, L, R, r, h, w$ and $p$.

Stress is symmetric

$$[\sigma] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \dfrac{pw^2hR}{\pi(R^4 - r^4)} \\ 0 & \dfrac{pw^2hR}{\pi(R^4-r^4)} & \dfrac{4pwhLR}{\pi(R^4-r^4)} - \dfrac{S+P}{\pi(R^2-r^2)} \end{bmatrix}$$

$$\sigma_z = \frac{My}{I} - \frac{P}{A} = \frac{4pwhLR}{\pi(R^4-r^4)} - \frac{S+P}{\pi R^2 - \pi r^2}$$

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⇓ *Put neat, clear work to be graded for problem 2 below.* ⇓
*( If you need the space, clearly mark work to be graded on the scrap page.)*

$$\tau_{yz} = \tau_{zy} = \frac{Tc}{J} = \frac{pw^2hR}{\pi(R^4 - r^4)}$$

3) (20 pt) Two pinned-end columns of the same material have the same length and the same cross-sectional area. The columns are free to buckle in any direction. One has a circular cross section, the other an equilateral triangle cross section. Please note the inertia tables on the last page.

a) (15 points)Which of the two will have the largest critical load? Express you answer as the ratio of the buckling load for the cicular cross section to the buckling load for the triangular cross section (That is $\frac{P_\triangle}{P_\circ} = ?$ ). [Note: you only need to know the rudiments of the buckling formulas to answer this question (e.g., factors of $\pi^2$, etc. drop out).]

b) (5 points) The number you get in part (a) is either bigger than one or less than one. Explain why it comes out the way it does.

*⇐ Please put scrap work for problem 3 on the page to the left ⇐.*
*⇓ Put neat, clear work to be graded for problem 3 below. ⇓*
*( If you need the space, clearly mark work to be graded on the scrap page.)*

a) $P_{cr} = \frac{\pi^2 E I}{L^2}$

$I_\circ = \frac{\pi r^4}{4}$

$I_\triangle = \frac{bh^3}{36} = \frac{hb^3}{48} = \frac{\sqrt{3}\, b^4}{96}$

$h = \frac{\sqrt{3}\, b}{2}$

Equal areas

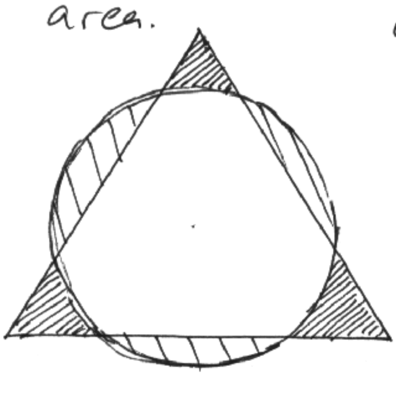
$\pi r^2 = \frac{bh}{2}$

$r = \sqrt{\frac{bh}{2\pi}}$

$$\frac{P_{cr\triangle}}{P_{cr\circ}} = \frac{I_\triangle}{I_\circ} = \frac{\frac{\sqrt{3}\, b^4}{96}}{\frac{\cancel{\pi}}{4}\frac{b^2h^2}{4\pi^{\cancel{2}}}} = \frac{16\pi}{96}\frac{\sqrt{3}\, b^4}{b^2\frac{3b^2}{4}} = \frac{\pi\, 16\cdot 4\cdot\sqrt{3}}{96\cdot 3}\ \frac{64}{\cancel{64}}$$

$$\boxed{\frac{P_{cr\triangle}}{P_{cr\circ}} = \frac{\pi 2\sqrt{3}}{9}}$$

b) Bigger than one. The buckling load for the triangular cross section ~~will~~ is greater because the moment of inertia of an equal area triangle is greater than that of a circle of the same area.

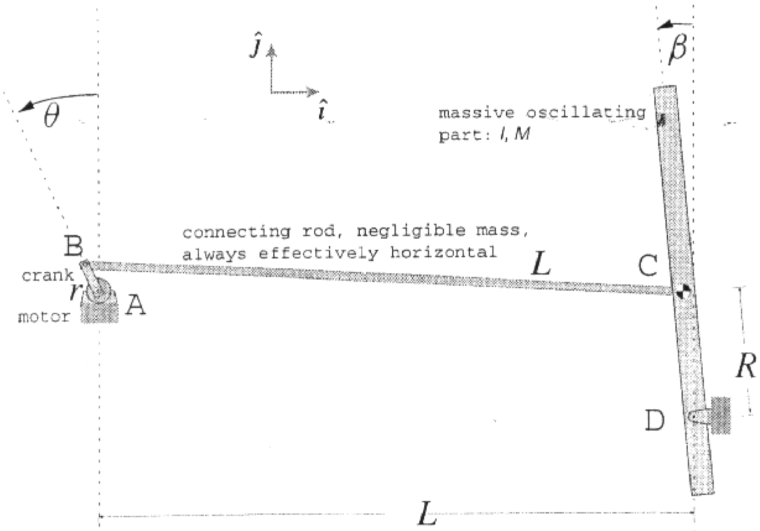


 regions add more to I than  regions take away because $I = \int r^2 dA$ and the outter regions have larger r.

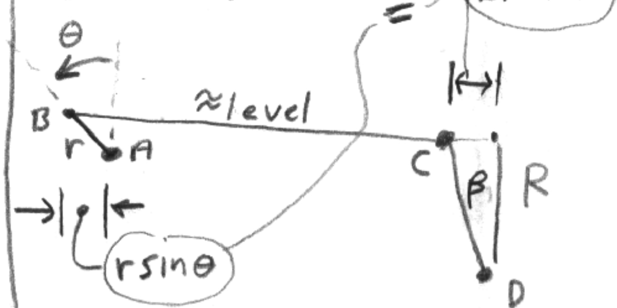
4) (20 pt) A motor is used to oscillate a massive part and we are concerned about the tension in the connecting rod. The motor at A turns at constant rate so that $\theta = \omega t$ with $\dot{\omega} = 0$. Short crank AB has length $r$ and is connected at B to connecting rod BC. The negligible-mass connecting rod has length $L$ and connects to massive machine part DC at its center of mass at C. Part DC is hinged to the ground at D a distance $R$ from C. The horizontal distance between A and D is also $L$ and when $\beta = 0$ and $\theta = 0$ point A is level with point C. To greatly simplify the geometry, you can assume that $L \gg R$ and that $R \gg r$ so that BC is essentially horizontal at all times and the angle $\beta$ is small enough so that you can assume that $\sin\beta = \tan\beta = \beta$. Neglect gravity. Part CD has mass $M$ and moment of inertia about its center of mass $I$. Find the maximum tension $T_{\max}$ in the connecting rod BC over a cycle in terms of some or all of $r, L, R, \omega, I$ and $M$ and the angle $\theta$ at which it occurs.

[Hint: don't do any mechanics calculations until you have drawn good free body diagrams and have figured out the (small angle based) relation between $\theta$ and $\beta$.]

*⇐ Please put scrap work for problem 4 on the page to the left ⇐.*
*⇓ Put neat, clear work to be graded for problem 4 below. ⇓*
*( If you need the space, clearly mark work to be graded on the scrap page.)*



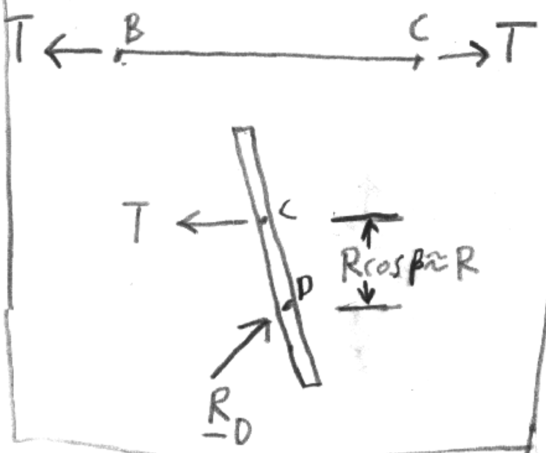

Geometry & Kinematics




$\beta \approx \tan\beta = \dfrac{r\sin\theta}{R} \Rightarrow \boxed{\beta = \dfrac{r\sin\theta}{R}}$

[small angle approx]

FBDs




AMB for CD about D

$\sum \underline{M}_{/D} = \dot{\underline{H}}_{/D}$

$\{TR\underline{k} = \ddot{\beta}(I + MR^2)\underline{k}\}$

$\{\}\cdot\underline{k} \Rightarrow T = \dfrac{I+MR^2}{R}\ddot{\beta}$

$T_{max} = \dfrac{I+MR^2}{R}\ddot{\beta}_{max}$

$T_{max} = \dfrac{I+MR^2}{R}\dfrac{r}{R}\dot{\theta}^2$ at $\theta = \frac{3\pi}{2}$

$$\boxed{T_{max} = Mr\omega^2\left[1+\frac{I}{MR^2}\right] \text{ at } \theta = 3\pi/2}$$

$\dot{\beta} = \dfrac{r\dot{\theta}\cos\theta}{R}$

$\ddot{\beta} = \dfrac{-r\dot{\theta}^2\sin\theta}{R} + \dfrac{r}{R}\ddot{\theta}\cos\theta$

but $\omega = \text{const} = \dot{\theta}$
$\Rightarrow \ddot{\theta} = 0$

$\Rightarrow \ddot{\beta} = \dfrac{-r}{R}\dot{\theta}^2\sin\theta$

$\boxed{\ddot{\beta}_{max} = \dfrac{r}{R}\dot{\theta}^2}$

when $\theta = \dfrac{3\pi}{2}$

5) (20 pt) A low-carbon steel, machined cantilever beam of rectangular cross section (see the Figure) is subjected to fully reversed alternating load, $F$, as show in Figure 1. Disregard stress concentrations and gravity.

a) (5 points) What is the corrected endurance limit, $S_e$, of the material if the beam to last $10^9$ cycles without failing in tension. Take into account correction factors for loading, surface, temperature and reliability. Use a safety factor of 2 in your calculation. Assume 90% reliability, and air temperature of 1000°F. Write down the equation for $S_e$ first, and then substitute values. Set up a formula ready for calculator calculation, but don't do the arithmetic.

$C_{\text{load}} = 1$ for bending
$C_{\text{temp}} = 1 - 0.0032(T - 840)$ for $T$ in °F
Use text Table 6-4 copied on the last page
Use text Figure 6-26 copied on the last page

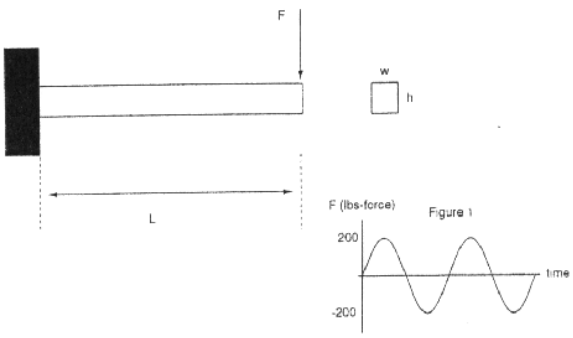
Material properties:
$S_{\text{ut}} = 80,000$ psi
$S_{\text{yield}} = 67,000$ psi
$S_{e'} = 0.5\ S_{ut}$
$E = 30 \cdot 10^6$ psi

b) (5 points) Assuming the answer to part (a) is "$S$", specify what the height of the beam, $h$, should for the beam not to fail in $10^9$ cycles. (See Figure below for other beam dimensions). Again, write down the equation for $h$ first, and then substitute values. You need not do the arithmetic. Assume: $w = 1$ inch $L = 10$ inches

c) (5 points) Find the maximal deflection at the free end of the beam for the solution in part b). Again, write down the equation for the deflection first, and then substitute values. You need not do the arithmetic.

d) (5 points) Using the graph paper on the next page draw the constant-life diagram for the material in tension. Assume that $S_e = 40,000$ psi, and $\sigma_a$ due to the fully reversed loading shown in Figure 1 is 10,000 psi. Graphically calculate the maximal positive mean stress $\sigma_m$ that can be added to the alternating stress $\sigma_a$. Use only the modified-Goodman line and the yield-line (you draw them) in your calculation.

Machined
$S_{ut} = 80$ kpsi




(a) $S_e = C_{load}\, C_{temp}\, C_{reliab}\, C_{surf}\, S_e' \left(\frac{1}{SF}\right)$

$S_e = \dfrac{(1)(1-0.0032[1000-840])(0.897)(0.78)(0.5\cdot 80000)}{2}$

($S_e =$ 6 828.68 psi) ← not desired

⇐ *Please put scrap work for problem 5 on the page to the left* ⇐.
⇓ *Put neat, clear work to be graded for problem 5 below.* ⇓
*( If you need the space, clearly mark work to be graded on the scrap page.)*

(b) $S = \sigma_{max} = \dfrac{Mc}{I} = \dfrac{FL(h/2)}{\frac{1}{12}wh^3} = \dfrac{6FL}{wh^2}$

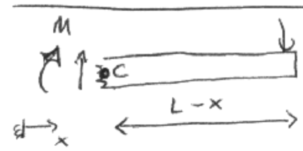
$h = \sqrt{\dfrac{6FL}{wS}} = \sqrt{\dfrac{6\,(200\ lb)(10\ in)}{(1\ in)(S)(lb/in^2)}} = \sqrt{\dfrac{12000}{S}\ in^2}$

$\boxed{h = \sqrt{\dfrac{12,000}{S}\ in^2}}$

(c) $\delta = \dfrac{FL^3}{3EI} = \dfrac{FL^3}{3E\frac{wh^3}{12}} = \dfrac{4FL^3}{Ew\left(\frac{6FL}{wS}\right)^{3/2}}$

$\delta = \dfrac{4(200\ lb)(10\ in)^3}{(30\times10^6 psi)\left(\frac{12000}{S}\ in^2\right)^{3/2}}$

Derivation for part C




$\Sigma M/c \Rightarrow \quad M = -F(L-x)$

$u'' = \dfrac{M}{IE} = \dfrac{-F(L-x)}{EI}$

$u' = -\dfrac{F}{EI}\left(Lx - \dfrac{x^2}{2}\right) + C_1$

$u'(0) = 0 \quad \Rightarrow C_1 = 0$

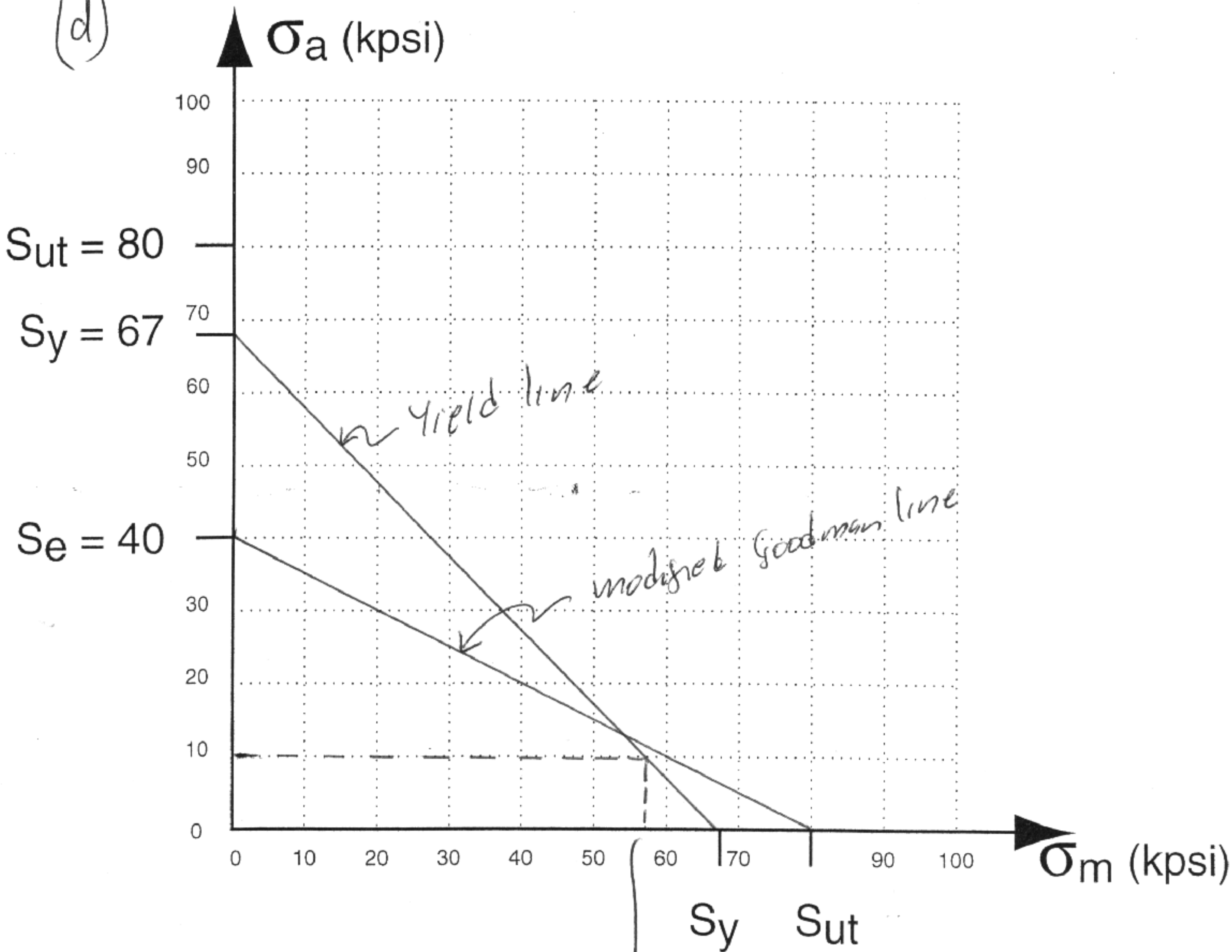
$u = -\dfrac{F}{EI}\left(\dfrac{Lx^2}{2} - \dfrac{x^3}{6}\right) + C_2$

$u(0) = 0 \quad C_2 = 0$

$u(L) = -\dfrac{F}{EI}\left[\dfrac{L^3}{2} - \dfrac{L^3}{6}\right]$

$u(L) = -\dfrac{FL^3}{3EI}$

(d)




$$\sigma_m = S_y - 10{,}000 = 67{,}000 - 10{,}000$$

$$\sigma_m = 57{,}000 \text{ psi}$$

**Table 6-4**

Reliability Factors for $S_d = 0.08\ \mu$

| Reliability % | $C_{reliab}$ |
|---|---|
| 50 | 1.000 |
| 90 | 0.897 |
| 99 | 0.814 |
| 99.9 | 0.753 |
| 99.99 | 0.702 |
| 99.999 | 0.659 |

6

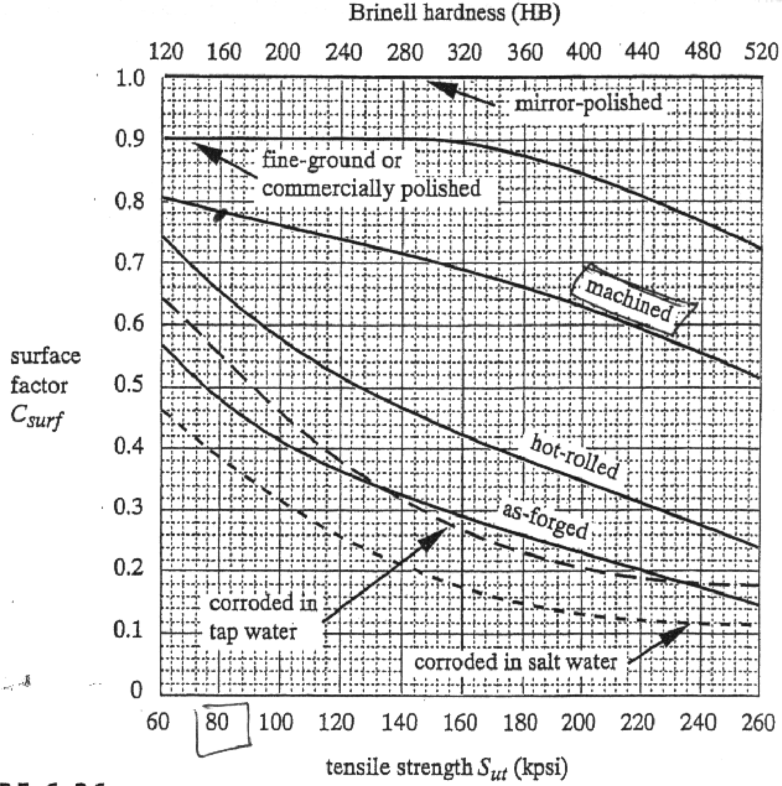
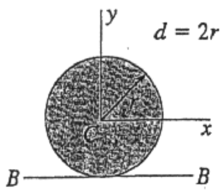



**FIGURE 6-26**

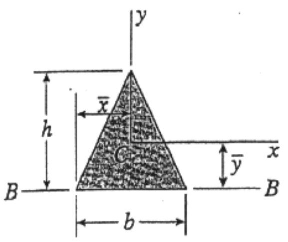
Surface Factors for Various Finishes on Steel (From Fig. 12.6, p. 234, R. C. Juvinall, 1967, *Stress, Strain, an Strength*, McGraw-Hill, New York, with permission)




**Circle** (Origin of axes at center)

$$A = \pi r^2 = \frac{\pi d^2}{4} \qquad I_x = I_y = \frac{\pi r^4}{4} = \frac{\pi d^4}{64}$$

$$I_{xy} = 0 \qquad I_p = \frac{\pi r^4}{2} = \frac{\pi d^4}{32} \qquad I_{BB} = \frac{5\pi r^4}{4} = \frac{5\pi d^4}{64}$$




**Isosceles triangle** (Origin of axes at centroid)

$$A = \frac{bh}{2} \qquad \bar{x} = \frac{b}{2} \qquad \bar{y} = \frac{h}{3}$$

$$I_x = \frac{bh^3}{36} \qquad I_y = \frac{hb^3}{48} \qquad I_{xy} = 0$$

$$I_p = \frac{bh}{144}(4h^2 + 3b^2) \qquad I_{BB} = \frac{bh^3}{12}$$

(*Note:* For an equilateral triangle, $h = \sqrt{3}\, b/2$.)