Spring Design example. M&A E 325.
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Ball launcher for pin ball machine

Given stiffness **K** , free lenght of spring **L**, plunger travel **y**, initial deflection **yo** mass of ball **m**, and spring average diameter **Da.**

Geometric design specifications: Find **d** and **Na** for a round stock music wire compression spring that can launch 10^5 balls with a dynamic margin of safety **Nsf** of at least 1.

Analysis of performance: Find the initial velocity **v** of the balls, and the natural resonant frequency **fn** of the spring in the barrel.

Finding spring geometry to meet design specifications

```
F := K y + K yo
(* F is spring force;
  don' t forget to include the initial deflection yo *)
d := Da / SI (Da = average diameter of spring; SI is spring index;
    d is diameter of material , round cross section *)
Kw := (4 SI - 1) / (4 SI - 4) + 0.615 / SI
(*stress concentration in inner side of coil*)
tw := Kw (8 F Da) / (Pi d^3) (* maximal shear stress *)
Sut := A d^b (* ultimate strength in tension ; Equation 13-3*)
Sfw := 0.36 Sut (* Ultimate strength in torsion for 10^5 cycles;
 Table 13-7, unpeened ASTM 228; Example 13-1; Figure 13-15*)
Nsf := Sfw / tw (* dynamic factor of safety *)
```

- **Parameters specified**

```
K = 50 (*lbs/in*);
Da = 2 (*in*);
y = 1 (* in*);
m  = 0.1 (* lbm, about 45.3 grams *);
yo = 0.5 (*in*);
L = 5 (* in*);
```

- **from Table 13-4, for music wire:**

```
A = 184649 (*psi*);
b = -0.1625;
G = 1.150 10^7 (*psi*);
gamma = 0.285 (* lbs/ in^3*);
g = 386 (* in/sec^2*);
```

- **First iteration**

- **Assume SI=12; SI = Da/d**

```
SI = 12;
```

```
F
```

```
75.
```

```
Kw
```

```
1.11943
```

```
tw
```

```
92359.8
```

```
Sut
```

```
247057.
```

```
Sfw
```

```
88940.4
```

```
Nsf
```

```
0.962978
```

- **Second iteration**

Assume a lower spring index to increase dynamic safety margin above 1.

Assume SI=10; SI = Da/d

```
SI = 10;
```

```
F
```

```
75.
```

```
Kw
```

```
1.14483
```

```
tw
```

```
54661.8
```

```
Sut
```

```
239844.
```

```
Sfw
```

```
86344.
```

```
Nsf
```

```
1.5796
```

Done. Dynamic safety margin is now above 1.

- **Determine number of active coils, Na**

From K = G d^4/(8Da^3 Na)

```
Na := G d^4 / (8 Da^3 K)
```

```
Na
```

```
5.75
```

Determine solid length

```
Ls := d * Na
```

```
Ls (*in*)
```

```
1.15
```

Ls + y + yo = 2.15 in. Which is less than the free lenght of the spring. OK.

Solutions to geometric design specifications

```
d (* wire diameter, in inches d := Da/SI *)
```

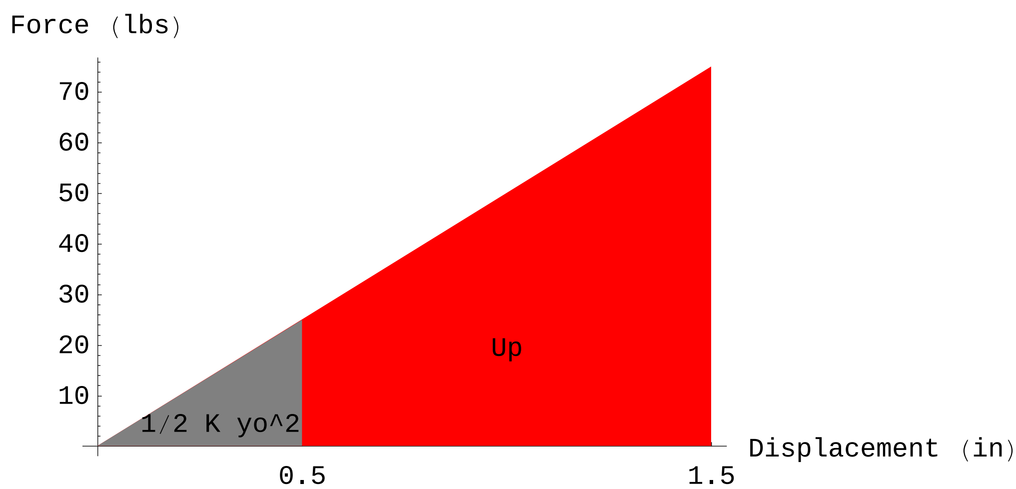
```
0.2
```

```
Na (* number of active coils*)
```

```
5.75
```

Analysis of performance

```
Up := 1 / 2 K (y + yo) ^2 - 1 / 2 K yo^2
 (* potential energy stored in cocked spring when moving
   plunger from yo to y. See area under curve labeled Up*)
```




```
Up (*in lbf*)
```

```
50.
```

- **Kinetic energy of ball when ejected is Uk = 1/2 m v^2;**
  Conservation of energy requires that Up = Uk

```
v := (2 Up / m) ^.5
```

```
v (* in/s*)
```

```
31.6228
```

```
Wa := (Pi^2 d^2 Da Na gamma ) / 4
(* weight of active coils, from eq. 13-11 b*);
fn := 1 / 2 (K g / Wa) ^.5; (* natural resonant frequency ,
 assuming both ends fixed. eq. 13-11 a*)
```

```
Wa (* lbf*)
```

0.323476

```
fn (* Hertz*)
```

122.131