Proceedings, Bicycle and Motorcycle Dynamics 2019
Symposium on the Dynamics and Control of Single-Track Vehicles,
9–11 September 2019, University of Padova, Italy

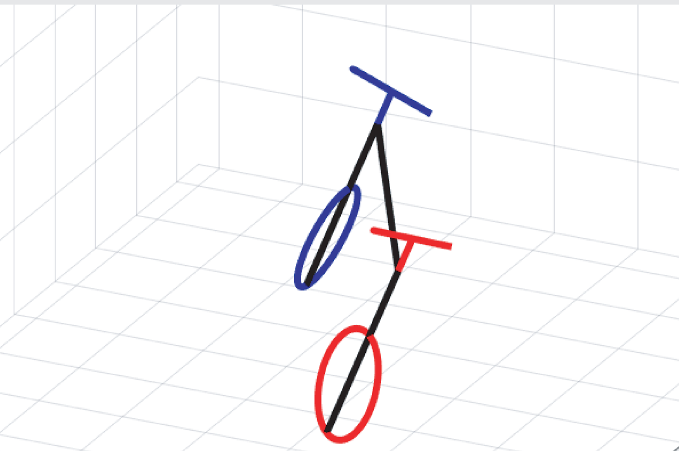
Optimal Control of a Dual-Steering Robotic Bicycle: from Bicycle to Segway and In-Between

Anshuman Das, Andy Ruina
Sibley School of Mechanical and Aerospace Engineering, Cornell University, Ithaca NY

Introduction

We consider a new device, a so called ***bisteercycle***, that is both a bicycle and a Segway, and everything in between; we anticipate that the bisteercycle has the advantage of:

- a Segway: with good balancing and controllability of orientation at zero speed.
- a bicycle: taking little space and little control authority at higher speeds



The bisteercycle is designed to have steering and drive inputs to *both* wheels. Our goal is to develop a controller architecture that can consider a bicycle and a Segway with a single controller, able to vary continuously between, for example:

1.) a co-steering (front and rear wheels steer together) bicycle
2.) a counter-steering (front and rear wheels steer oppositely) bicycle
3.) a bicycle doing a track stand
4.) a bicycle moving forwards at arbitrarily small speeds
5.) a bicycle spinning in place

Dynamics

Overview

Our nonlinear simulation of a bisteercycle has vertical steering axes, infinitessimal wheels, and no trail. The model has a 6-dimensional configuration space:

- Location and heading of the rear wheel (3)
- Steer angles (2)
- Lean angle (3)

As a single-track vehicle, the bisteercycle has 4 velocity DOF. As a Segway, a velocity DOF is added to the front wheel (here, the drive motors are independent); in this configuration, the equations of motion (EOM) are singular. The hybrid nature of the EOM poses an ongoing challenge for the design of a continuously-stable, closed-loop controller. **Our work explores the stability of a controller, designed for a single-track vehicle, that can operate with both bicycle and Segway dynamics.**

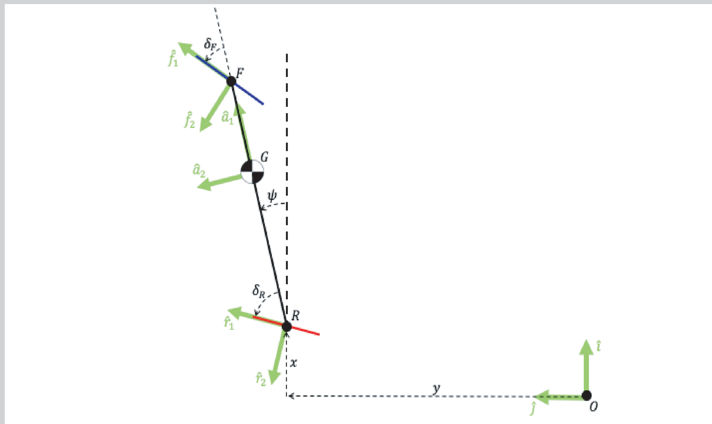
2D Rigid-Body Dynamics

The equations of motion are found through an *angular momentum balance* about the instantaneous center of rotation.

$$\sum \mathbf{M}_{/C} = \mathbf{r}_{G/C} \times m^I \mathbf{a}_{G/O} + I_{33}\ddot{\psi}\hat{\mathbf{k}}$$

Dynamics *(cont.)*

The EOMs use a minimal coordinate set, including the velocity of the rear wheel and front and rear steer angles, as well as the steer angle rates and drive forces as control inputs. Non-cyclic coordinates are solved for through integration.



In the EOMs, u_1 and u_2 are the drive forces. Some example open loop inputs were provided to better understand the behavior of the bisteercycle.

$$\dot{z} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \\ \dot{V}_R \\ \dot{\delta}_F \\ \dot{\delta}_R \end{bmatrix} = \begin{bmatrix} V_R c_{\psi+\delta_R} \\ V_R s_{\psi+\delta_R} \\ \frac{V_R s_{\delta_R - \delta_F}}{(l_R + l_F) c_{\delta_F}} \\ f(V_R, \delta_R, \delta_F, u_3, u_4, u_1, u_2) \\ u_3 \\ u_4 \end{bmatrix}$$

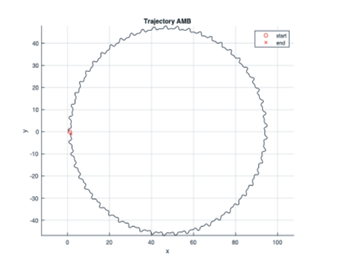


Figure 6: $z_0 = (1, 0, \pi/2, 1, \pi/3, \pi/4)^T$, $u = (0, 0, -2\pi/3 \sin 2t, -\pi/4 \sin t)^T$

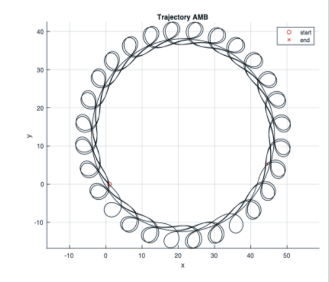


Figure 7: $z_0 = (1, 0, \pi/2, 5, \pi/4, \pi/4)^T$, $u = (0, 0, -\pi/4 \sin t, -\pi/3 \sin t)^T$

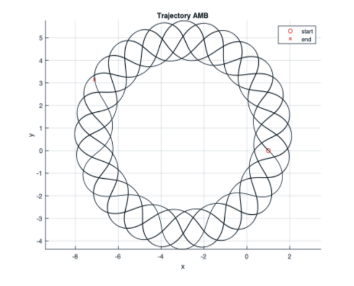


Figure 8: $z_0 = (1, 0, \pi/2, 1, \pi/3, \pi/4)^T$, $u = (0, 0, -\pi/4 \cos t, -\pi/3 \cos t)^T$

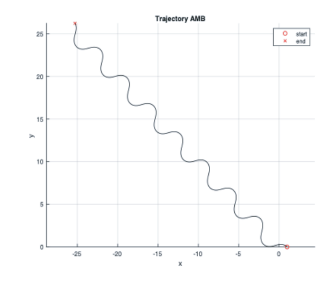


Figure 9: $z_0 = (1, 0, \pi/2, 1, \pi/4, \pi/4)^T$, $u = (0, 0, -\pi/3 \cos t, -\pi/3 \cos t)^T$

For certain sinusoidal inputs, the system exhibits **quasi-periodic behavior**. When the wheels are parallel, the instantaneous center of rotation is generally infinite (straight line motion).

Singularity arises from motion when both wheels are at 90º, perpendicular to the frame.

3D Rigid-Body Dynamics

The rigid frame of the bisteercycle has mass and inertia, defined by a symmetric tensor

$$\mathbf{I} = I_{11}\mathbf{b}_1\mathbf{b}_1 + (I_{11} + I_{33})\mathbf{b}_2\mathbf{b}_2 + I_{33}\mathbf{b}_3\mathbf{b}_3$$

The equations of motion are found through:

1.) an *angular momentum balance* about the axis perpendicular to the ground plane, through the instantaneous center of rotation

$$\sum \mathbf{M}_{/C} \cdot \hat{\mathbf{k}} = (\mathbf{r}_{G/C} \times m^{\mathcal{I}}\mathbf{a}_{G/O} + \mathbf{I} \cdot \boldsymbol{\omega}_{\mathcal{BI}} + \boldsymbol{\omega}_{\mathcal{BI}} \times (\mathbf{I} \cdot \boldsymbol{\omega}_{\mathcal{BI}})) \cdot \hat{\mathbf{k}}$$

2.) an *angular momentum balance* about the axis through the points of contact of both wheels (denoted by R and F).

$$\sum \mathbf{M}_{/E} \cdot \hat{\boldsymbol{\lambda}}_{RF} = (\mathbf{r}_{G/E} \times m^{\mathcal{I}}\mathbf{a}_{G/O} + \mathbf{I} \cdot \boldsymbol{\omega}_{\mathcal{BI}} + \boldsymbol{\omega}_{\mathcal{BI}} \times (\mathbf{I} \cdot \boldsymbol{\omega}_{\mathcal{BI}})) \cdot \hat{\boldsymbol{\lambda}}_{RF}$$

Controller Design

The bisteercycle uses **a linear quadratic regulator at multiple equilibria using gain scheduling (gsLQR)**. For zero-input equilibria, we look for circular orbits of the bisteercycle through root solving. Circular orbits can be characterized by constant lean and steer angles and speed, with 0 lean rate.

Controller Design *(cont.)*

We use LQR to find optimal gain matrices K for step 4. A table of associated gains are tabulated and interpolated for a large subset of the state space.

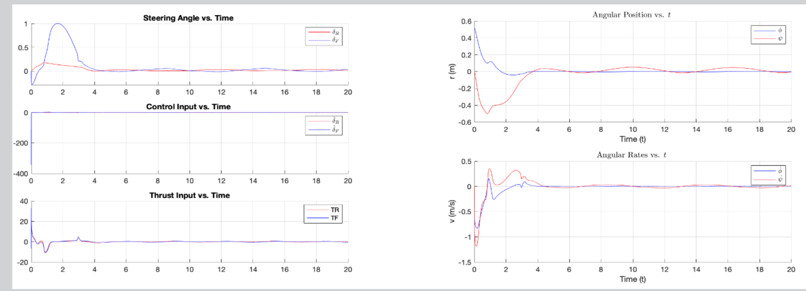
1. $\mathbf{f}(\mathbf{x}^*) = \mathbf{0}$
2. $A = \left(\frac{\partial \mathbf{f}}{\partial \mathbf{x}} + \frac{\partial \mathbf{g}}{\partial \mathbf{x}}\mathbf{u}\right)\Big|_{\mathbf{x}=\mathbf{x}^*, \mathbf{u}=0}$, $B = \mathbf{g}(\mathbf{x})|_{\mathbf{x}=\mathbf{x}^*, \mathbf{u}=0}$
3. Repeat step 2 to linearize about every equilibrium point
4. For each A and B matrix, find the desired gain matrix K that ensures that $A - BK$ is Hurwitz (real part of all eigenvalues are negative)
5. Multi-dimensional interpolation between each K matrix
6. $\mathbf{u} = -\mathbf{K}(\mathbf{x} - \mathbf{x}^{des})$

Results and Conclusion

We attempted four motions with the bisteercycle:

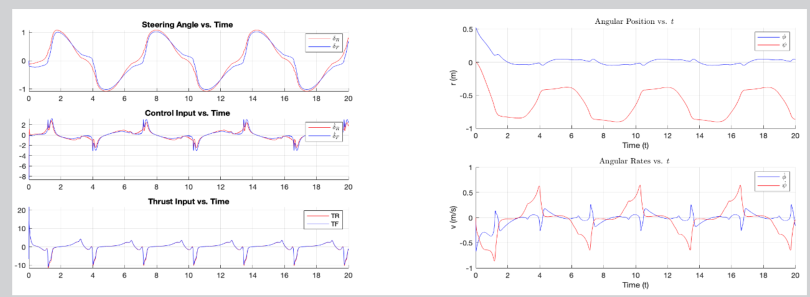
Normal bicycle

- Initial conditions: $(\pi/6, 5, 0, 0, 0)^T$
- Reference inputs: $(0, 2, 0, \pi/3 \sin t, 0)^T$
- LQR weights:
  - $Q = diag([1e3, 1e1, 1e3, 1e2, 1e2])$
  - $R = diag([1e0, 1e0, 1e0, 1e3])$



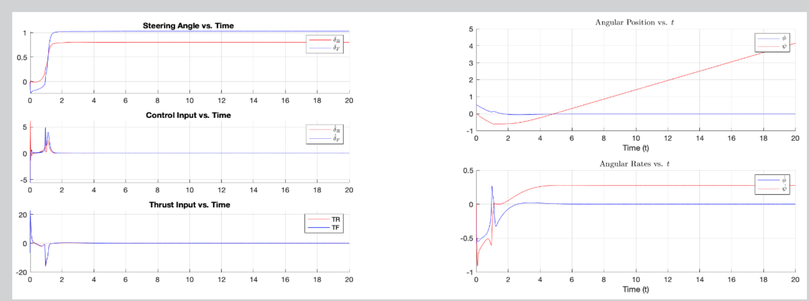
"Segway"

- Initial conditions: $[\pi/6, 5, 0, 0, 0]^T$
- Reference inputs: $[0, 2, 0, \pi/3 \sin t, \pi/3 \sin t]^T$
- LQR weights:
  - $Q = diag([1e3, 1e1, 1e3, 1e2, 1e2])$
  - $R = diag([1e0, 1e0, 1e0, 1e0])$



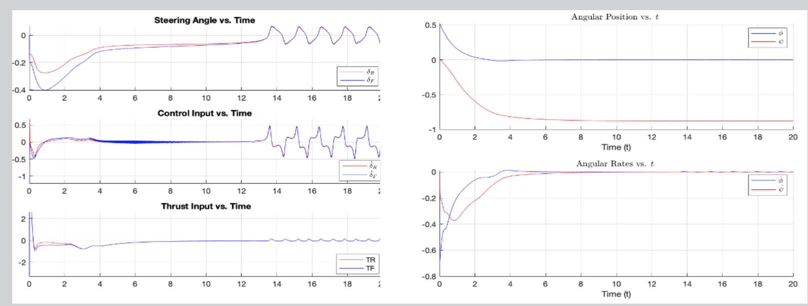
Circular motion

- Initial conditions: $[\pi/6, 5, 0, 0, 0]^T$
- Reference inputs: $[\phi, V_R, 0, \pi/3, \pi/4]^T$
- LQR weights:
  - $Q = diag([1e3, 1e1, 1e3, 1e2, 1e2])$
  - $R = diag([1e0, 1e0, 1e0, 1e0])$



Track stand

- Initial conditions: $[\pi/6, 5, 0, 0, 0]^T$
- Reference inputs: $[0, 0, 0, 0, 0]^T$
- LQR weights:
  - $Q = diag([1e3, 1e1, 1e3, 1e2, 1e2])$
  - $R = diag([1e0, 1e0, 1e0, 1e0])$



Simulation shows that counter-steering helps circular motion, while co-steering helps low speed stability.

References

[1] Chihiro Nakagawa et al. *"Stabilization of a bicycle with two-wheel steering and two-wheeldriving by driving forces at low speed"*. In:Journal of Mechanical Science and Technology23.4 (Apr. 2009), pp. 980–986. ISSN: 1976-3824. DOI: 10.1007/s12206- 009- 0325- 4. URL:https://doi.org/10.1007/s12206-009-0325-4.