1.5 Solving vector equations

1.5.1 Consider the vector equation

$$a\vec{A} + b\vec{B} = \vec{C}$$

with $\vec{A}$, $\vec{B}$, and $\vec{C}$ given. For the cases below find a and b if possible. If there are multiple solutions give at least 2. If there are no solutions explain why.

a) $\vec{A} = \hat{\imath}$, $\vec{B} = \hat{\jmath}$, $\vec{C} = 3\hat{\imath} + 4\hat{\jmath}$

b) $\vec{A} = \hat{\imath}$, $\vec{B} = 2\hat{\imath}$, $\vec{C} = 3\hat{\imath}$

c) $\vec{A} = \hat{\jmath}$, $\vec{B} = 2\hat{\jmath}$, $\vec{C} = 3\hat{\imath}$

d) $\vec{A} = \hat{\imath} + \hat{\jmath}$, $\vec{B} = -\hat{\imath} + \hat{\jmath}$, $\vec{C} = 2\hat{\jmath}$

e) $\vec{A} = \hat{\imath} + 2\hat{\jmath}$, $\vec{B} = 2\hat{\imath} + 3\hat{\jmath}$, $\vec{C} = 3\hat{\imath} + 4\hat{\jmath}$

f) $\vec{A} = \pi\hat{\imath} + e\hat{\jmath}$, $\vec{B} = \sqrt{2}\hat{\imath} + \sqrt{3}\hat{\jmath}$, $\vec{C} = \hat{\imath}$

1.5.2 Consider the vector equation

$$a\vec{A} + b\vec{B} + c\vec{C} = \vec{D}$$

with $\vec{A}$, $\vec{B}$, $\vec{C}$ and $\vec{D}$ given. For the cases below find a if possible, there is no need to find b and c.

a) $\vec{A} = \hat{\imath}$, $\vec{C} = \hat{k}$, $\vec{B} = \hat{\jmath}$, $\vec{D} = 3\hat{\imath} + 4\hat{\jmath} + 19\hat{k}$

b) $\vec{A} = 2\hat{\imath}$, $\vec{C} = 15\hat{\jmath} + 360\hat{k}$, $\vec{B} = 3\hat{\imath} + 4\hat{\jmath}$, $\vec{D} = 2\hat{\imath} + 17\hat{\jmath} + 37\hat{k}$

c) $\vec{A} = \hat{k}$, $\vec{C} = \hat{\imath} + \hat{\jmath} + \hat{k}$, $\vec{B} = \hat{\jmath} + \hat{k}$, $\vec{D} = \hat{\jmath}$

d) $\vec{A} = \hat{\imath} + \hat{\jmath} + \hat{k}$, $\vec{C} = 3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$, $\vec{B} = 2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$, $\vec{D} = 4\hat{\imath} + 5\hat{\jmath} + 7\hat{k}$

e) $\vec{A} = \sqrt{1}\hat{\imath} + \sqrt{2}\hat{\jmath} + \sqrt{3}\hat{k}$, $\vec{C} = \sqrt{7}\hat{\imath} + \sqrt{8}\hat{\jmath} + \sqrt{9}\hat{k}$, $\vec{B} = \sqrt{4}\hat{\imath} + \sqrt{5}\hat{\jmath} + \sqrt{6}\hat{k}$, $\vec{D} = -\hat{\imath} + \pi\hat{\jmath} + e\hat{k}$

1.5.3 In the problems below use matrix algebra on a computer to find a, b and c uniquely if possible. If not possible explain why not. You are given that

$$a\vec{A} + b\vec{B} + c\vec{C} = \vec{D}$$

and that

a) $\vec{A} = \hat{\imath}$, $\vec{B} = \hat{\jmath}$, $\vec{C} = \hat{k}$, $\vec{D} = -2\hat{\imath} + 5\hat{\jmath} + 10\hat{k}$

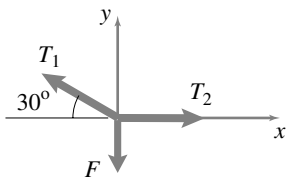
b) $\vec{A} = \hat{\imath} + \hat{\jmath}$, $\vec{B} = -\hat{\imath} + \hat{\jmath}$, $\vec{C} = \hat{k}$, $\vec{D} = 2\hat{\imath}$

c) $\vec{A} = \hat{\imath} + \hat{\jmath} + \hat{k}$, $\vec{B} = 2\hat{\imath} + \hat{\jmath} + \hat{k}$, $\vec{C} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$, $\vec{D} = 2\hat{\imath} + \hat{\jmath} + \hat{k}$

d) $\vec{A} = 1\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, $\vec{B} = 4\hat{\imath} + 5\hat{\jmath} + 6\hat{k}$, $\vec{C} = 7\hat{\imath} + 8\hat{\jmath} + 9\hat{k}$, $\vec{D} = 10\hat{k}$

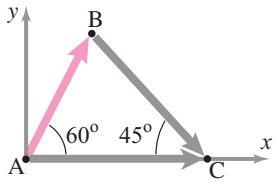
e) $\vec{A} = \sqrt{1}\hat{\imath} + \sqrt{2}\hat{\jmath} + \sqrt{3}\hat{k}$, $\vec{B} = \sqrt{4}\hat{\imath} + \sqrt{5}\hat{\jmath} + \sqrt{6}\hat{k}$, $\vec{C} = \sqrt{7}\hat{\imath} + \sqrt{8}\hat{\jmath} + \sqrt{9}\hat{k}$, $\vec{D} = \sqrt{10}\hat{k}$

1.5.4 The three forces shown in the figure are in equilibrium, *i.e.*, $\vec{T}_1 + \vec{T}_2 + \vec{F} = \vec{0}$. If $|\vec{F}| = 10$ N, find tensions T_1 and T_2 (magnitudes of $\vec{T}_1$ and $\vec{T}_2$).



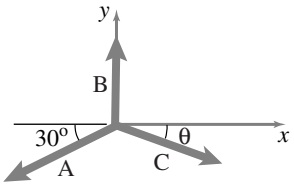
Problem 1.5.4

1.5.5 Points A, B, and C are located in the xy plane as shown in the figure. For position vectors, we can write, $\vec{r}_B + \vec{r}_{C/B} = \vec{r}_C$. Find $|\vec{r}_B|$ and $|\vec{r}_{C/B}|$ if $\vec{r}_C = 10\,\text{m}\,\hat{\imath}$.



Problem 1.5.5

1.5.6 Three vectors, $\vec{A}$, $\vec{B}$, and $\vec{C}$, (shown in the figure) are such that $\vec{A} + \vec{B} + \vec{C} = \vec{0}$. You are given that $A = |\vec{A}| = 5$ and $C = |\vec{C}| = 8$. Find θ.



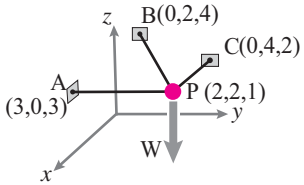
Problem 1.5.6

1.5.7 Let $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{0}$, where $\vec{F}_3 = 10\,\text{N}(\hat{\imath} - \hat{\jmath})$, and $\vec{F}_1/|\vec{F}_1| = 0.250\hat{\imath} + 0.968\hat{\jmath}$ and $\vec{F}_2/|\vec{F}_2| = -0.425\hat{\imath} - 0.905\hat{\jmath}$. Find $|\vec{F}_1|$ from a single scalar equation.

1.5.8 To evaluate the equation $\sum \vec{F} = m\vec{a}$ for some problem, a student writes $\sum \vec{F} = F_x\hat{\imath} - (F_y - 30\,\text{N})\hat{\jmath} + 50\,\text{N}\hat{k}$ in the xyz coordinate system, but $\vec{a} = 2.5\,\text{m/s}^2\hat{\imath}' + 1.8\,\text{m/s}^2\hat{\jmath}' - a_z\hat{k}'$ in a rotated $x'y'z'$ coordinate system. If $\hat{\imath}' = \cos 60^\circ\hat{\imath} + \sin 60^\circ\hat{\jmath}$, $\hat{\jmath}' = -\sin 60^\circ\hat{\imath} + \cos 60^\circ\hat{\jmath}$ and $\hat{k}' = \hat{k}$, find the scalar equations for the x', y', and z' directions.

1.5.9 A car travels straight north-east for a while on a dirt road that leads to a north-south highway. The car travels on the highway due north for a while. When the driver stops, the GPS system indicates that the car is 60 miles north and 30 miles east from the starting point. Find the distance travelled on the dirt road.

1.5.10 A particle is held at point P with the help of three rods PA, PB, and PC. Let the tensions in the three strings be T_A, T_B, and T_C, respectively (so that T_A acts along line PA and so on). The equilibrium of the particle requires that $\vec{T}_A + \vec{T}_B + \vec{T}_C + \vec{W} = \vec{0}$ where $\vec{W} = -10\,\text{N}\,\hat{k}$ is the weight of the particle. Find the magnitudes of tensions in the three rods (note, tension can be negative).



Problem 1.5.10

1.5.11 You are given that $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 5\,\text{kN}\,\hat{\jmath}$ where $\vec{F}_1 = (2\hat{\imath} - 3\hat{\jmath} + 4\hat{k})\,\text{kN}$, $\vec{F}_2 = (\hat{\imath} + 5\hat{k})\,\text{kN}$. Find the direction of $\vec{F}_3$ (An angle measured CCW from the $+x$ axis to the direction of positive $\vec{F}_3$).

1.5.12 A plane intersects the x, y, and z axis at 3,4, and 5 respectively. What point on the plane is in the direction $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

from the point (10,10,10)? (Find the x, y and z components of the point.).

These problems concern the solution of simultaneous equations. These could come from various vector equations.

1.5.13 Write the following equations in matrix form to solve for x, y, and z:

$$\begin{aligned} 2x - 3y + 5 &= 0, \\ y + 2\pi z &= 21, \\ \frac{1}{3}x - 2y + \pi z - 11 &= 0. \end{aligned}$$

1.5.14 Are the following equations linearly independent?

a) $x_1 + 2x_2 + x_3 = 30$

b) $3x_1 + 6x_2 + 9x_3 = 4.5$

c) $2x_1 + 4x_2 + 15x_3 = 7.5$.

1.5.15 Write computer commands (or a program) to solve for x, y and z from the following equations with r as an input variable. Your program should display an error message if, for a particular r, the equations are not linearly independent.

a) $5x + 2r\,y + z = 2$

b) $3x + 6y + (2r - 1)z = 3$

c) $2x + (r - 1)y + 3r\,z = 5$.

Find the solutions for $r = 3$, 4.99, and 5.

1.5.16 An exam problem in statics has three unknown forces. A student writes the following three equations (he knows that he needs three equations for three unknowns!) — one for the force balance in the x-direction and the other two for the moment balance about two different points.

a) $F_1 - \frac{1}{2}F_2 + \frac{1}{\sqrt{2}}F_3 = 0$

b) $2F_1 + \frac{3}{2}F_2 = 0$

c) $\frac{5}{2}F_2 + \sqrt{2}F_3 = 0$.

Can the student solve for F_1, F_2, and F_3 uniquely from these equations? *

1.5.17 What is the solution to the set of equations:

$$\begin{aligned} x + y + z + w &= 0 \\ x - y + z - w &= 0 \\ x + y - z - w &= 0 \\ x + y + z - w &= 2? \end{aligned}$$