1.4.1 What is the moment $\vec{\boldsymbol{M}}$ produced by a 20 N force F acting in the x direction with a lever arm of $\vec{\boldsymbol{r}} = (16\,\mathrm{mm})\,\hat{\boldsymbol{\jmath}}$?

$$\vec{F} = 20\,\mathrm{N}\,\hat{\imath}$$

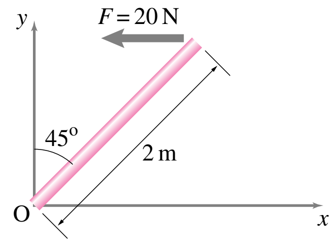
$$\vec{r} = (16\,\mathrm{mm})\,\hat{\jmath}$$

$$\vec{M} = \vec{r} \times \vec{F}$$

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 16\,\mathrm{mm} & 0 \\ 20\,\mathrm{N} & 0 & 0 \end{vmatrix}$$

$$= -32\,\mathrm{N\,cm}$$

1.4.2 Find the moment of the force shown on the rod about point O.



Problem 1.2

$$\vec{r} = (2m \sin 45)\hat{i} + (2m \cos 45)\hat{j}$$

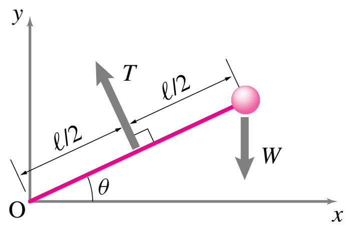
$$= \sqrt{2}\, m\, \hat{i} + \sqrt{2}\, m\, \hat{j}$$

$$\vec{F} = -20N\, \hat{i}$$

$$\vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \sqrt{2}m & \sqrt{2}m & 0 \\ -20N & 0 & 0 \end{vmatrix}$$

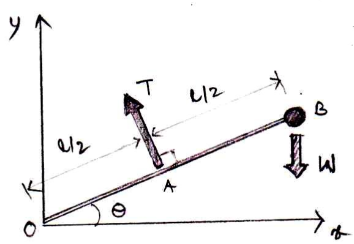
$$= -20\sqrt{2}\, Nm\, \hat{k}$$

1.4.3 Find the sum of moments of forces $\vec{W}$ and $\vec{T}$ about the origin, given that $W = 100\,\text{N}, T = 120\,\text{N}, \ell = 4\,\text{m}$, and $\theta = 30°$.



Problem 1.3

2.4.3



$W = 100\text{N}$
$T = 120\text{N}$
$\theta = 30°$
$\ell = 4\,\text{m}$

Forces $\vec{W}$ and $\vec{T}$ are acting at points B and A respectively.

Moment due to forces W & T about O are given by

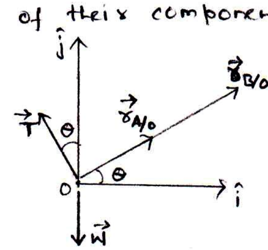
$$\vec{M}_{W/O} = \vec{r}_{B/O} \times \vec{W}$$

$$\vec{M}_{T/O} = \vec{r}_{A/O} \times \vec{T}$$

Writing the forces and vectors in terms of their components:

$$\vec{W} = -100\text{N}\,\hat{j}$$

$$\vec{T} = (-T\sin\theta)\,\hat{i} + (T\cos\theta)\,\hat{j}$$



$$\vec{r}_{A/O} = (\tfrac{L}{2}\cos\theta)\,\hat{i} + (\tfrac{L}{2}\sin\theta)\,\hat{j}$$

$$\vec{r}_{B/O} = (L\cos\theta)\,\hat{i} + (L\sin\theta)\,\hat{j}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

2.4.3 Continued

Now finding the moments

$$\vec{M}_{W/O} = \vec{r}_{B/O} \times \vec{W}$$

$$= [(L\cos\theta)\hat{i} + (L\sin\theta)\hat{j}] \times (-100N)\hat{j}$$

$$= [4\cos 30^\circ m\hat{i} + 4\sin 30^\circ m\hat{j}] \times (-100N)\hat{j}$$

$$= [3.464\, m\hat{i} + 2m\hat{j}] \times [-100N]\hat{j}$$

$$= -346.4\, Nm\, \hat{K}$$

$$\vec{M}_{T/O} = \vec{r}_{A/O} \times \vec{T}$$

$$= [(\tfrac{L}{2}\cos\theta)\hat{i} + (\tfrac{L}{2}\sin\theta)\hat{j}] \times [(-T\sin\theta)\hat{i} + (T\cos\theta)\hat{j}]$$

$$= [\tfrac{L}{2}.T.(\cos\theta)^2\hat{K} + \tfrac{L}{2}.T.(\sin\theta)^2\hat{K}]$$

$$= [\tfrac{4}{2}.120.(\cos 30^\circ)^2 Nm\hat{K} + \tfrac{4}{2}.120.(\sin 30)^2 Nm\hat{K}]$$

$$= 180\, Nm\hat{K} + 60\, Nm\hat{K}$$

$$= 240\, Nm\hat{K}$$

Sum of the moments

$$\vec{M}_{W/O} + \vec{M}_{T/O} = -346.4\, Nm\,\hat{K} + 240\, Nm\hat{K}$$

$$= -106.4\, Nm\hat{K}$$

$|\vec{W}| = 100N$

$|\vec{T}| = 120N$

$\theta = 30^\circ$

$\ell = 4m$

$\vec{W} = -100\,N\,\hat{j}$

$\vec{T} = -120N\cos\theta\,\hat{i} + 120\,N\sin\theta\,\hat{j}$
$= -60\sqrt{3}\,N\hat{i} + 60\,N\,\hat{j}$

$\vec{r}_T = \frac{\ell}{2}\cos\theta\,\hat{i} + \frac{\ell}{2}\sin\theta\,\hat{j}$

$= \sqrt{3}\,m\,\hat{i} + 1\,m\,\hat{j}$

$\vec{r}_W = \ell\cos\theta\,\hat{i} + \ell\sin\theta\,\hat{j}$

$= 2\sqrt{3}\,m\,\hat{i} + 2m\,\hat{j}$

Find Moments,

$$\vec{r}_W \times \vec{W} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2\sqrt{3}m & 2m & 0 \\ 0 & -100\,N & 0 \end{vmatrix}$$

$= -200\sqrt{3}\,Nm\,\hat{k}$

$$\vec{r}_T \times \vec{T} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \sqrt{3}m & 1m & 0 \\ -60\sqrt{3}N & 60N & 0 \end{vmatrix}$$

$= 120\sqrt{3}\,Nm\,\hat{k}$

Sum of Moments $= (-200\sqrt{3} + 120\sqrt{3})\,Nm\,\hat{k}$

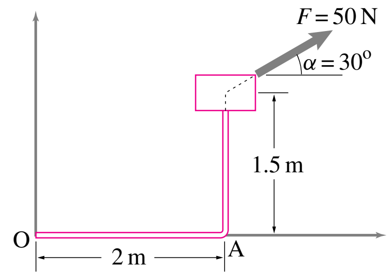
$= -80\sqrt{3}\,Nm\,\hat{k}$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

1.4.4 Find the moment of the force

a) about point A

b) about point O.



Problem 1.4

$\vec{F} = 50\text{N}\cos 30\,\hat{i} + 50\text{ N}\sin 30\,\hat{j}$

$= 25\sqrt{3}\text{ N}\,\hat{i} + 25\text{ N}\hat{j}$

$\vec{r}_{F/A} = 1.5\text{m}\,\hat{j}$

Find Moment Across A and O,

$\vec{M}_{/A} = \vec{r}_{F/A} \times \vec{F}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1.5\text{m} & 0 \\ 25\sqrt{3}\text{N} & 25\text{ N} & 0 \end{vmatrix}$$

$= -37.5\sqrt{3}\text{ Nm}\,\hat{k}$

$\vec{r}_{F/O} = 2\text{m}\,\hat{i} + 1.5\text{m}\,\hat{j}$

$\vec{M}_{/O} = \vec{r}_{F/O} \times \vec{F}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2\text{m} & 1.5\text{m} & 0 \\ 25\sqrt{3}\text{N} & 25\text{N} & 0 \end{vmatrix}$$

$= (50 - 37.5\sqrt{3})\text{ Nm}\,\hat{k}$

$= -14.95\text{ Nm}\,\hat{k}$

1.4.5 The line of action of a force $\vec{F} = 20\,\mathrm{N}\hat{\boldsymbol{j}} - 5\,\mathrm{N}\hat{\boldsymbol{k}}$ passes through a point A with coordinates (200 mm, 300 mm, -100 mm). What is the moment $\vec{M}(= \vec{r} \times \vec{F})$ of the force about the origin?

$$\vec{r} = 20\text{cm}\,\hat{\imath} + 30\text{cm}\,\hat{\jmath} - 10\text{cm}\,\hat{k}$$

$$\vec{F} = 20\text{N}\,\hat{\jmath} - 5\text{N}\,\hat{k}$$

$$\vec{M} = \vec{r} \times \vec{F}$$

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 20\text{cm} & 30\text{cm} & -10\text{cm} \\ 0 & 20\text{ N} & -5\text{N} \end{vmatrix}$$

$$= 50\,\text{Ncm}\,\hat{\imath} + 100\text{Ncm}\,\hat{\jmath} + 400\,\text{Ncm}\,\hat{k}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

1.4.6 Drawing vectors and computing with vectors. In an xyz coordinate system, let point O be the origin. Point A has xyz coordinates $(0m, 5m, 12m)$ and point B has xyz coordinates $(4m, 5m, 12m)$.

a) Make a neat sketch of the vectors OA, OB, and AB.

b) Find a unit vector in the direction of OA, call it $\hat{\boldsymbol{\lambda}}_{OA}$.

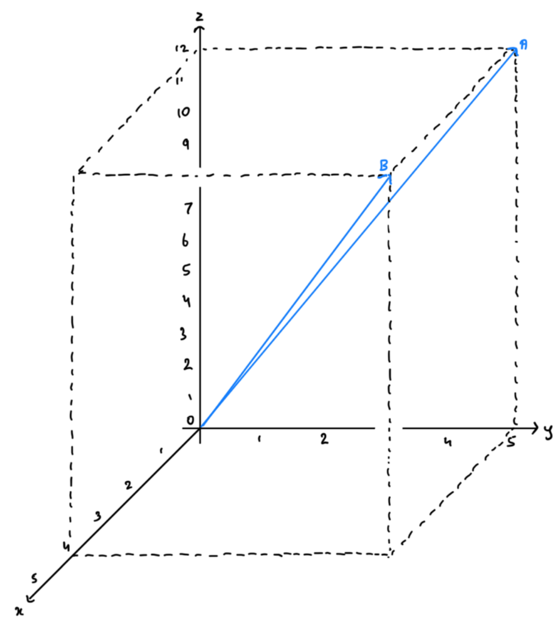
c) Find the force $\vec{\boldsymbol{F}}$ which is 5 N in size and is in the direction of OA.

d) What is the angle between OA and OB?

e) What is $\vec{\boldsymbol{r}}_{BO} \times \vec{\boldsymbol{F}}$?

f) Assume $\vec{\boldsymbol{F}}$ acts at O. What is the moment of $\vec{\boldsymbol{F}}$ about a line parallel to the z axis that goes through point B?

a)



b) $\vec{r}_{OA} = 5m\,\hat{j} + 12\,m\,\hat{k}$

$|\vec{r}_{OA}| = \sqrt{(5m)^2 + (12m)^2}$

$= 13m$

$\hat{\lambda}_{OA} = \frac{5}{13}\hat{j} + \frac{12}{13}\hat{k}$

c) $\vec{F} = 5N\,\hat{\lambda}_{OA}$

$= \frac{25}{13}N\,\hat{j} + \frac{60}{13}N\,\hat{k}$

d) $\vec{r}_{OB} = 4m\hat{i} + 5m\hat{j} + 12m\,\hat{k}$

$|\vec{r}_{OB}| = \sqrt{(4m)^2 + (5m)^2 + (12m)^2}$

$= \sqrt{185}\,m$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

Find unit vector in direction of OB,

$$\hat{\lambda}_{OB} = \frac{4}{\sqrt{185}}\hat{i} + \frac{5}{\sqrt{185}}\hat{j} + \frac{12}{\sqrt{185}}\hat{k}$$

$$\theta = \cos^{-1}\left(\frac{\hat{\lambda}_{OA}\cdot\hat{\lambda}_{OB}}{|\hat{\lambda}_{OA}|\,|\hat{\lambda}_{OB}|}\right)$$

$$= \cos^{-1}(0.955)$$

$$= 17.1^\circ$$

e) $$\vec{r}_{BO}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4m & -5m & -12m \\ 0 & \frac{25}{13}N & \frac{60}{13}N \end{vmatrix}$$

$$= (167.79\hat{i} + 18.46\hat{j} - 7.69\hat{k})\,Nm$$

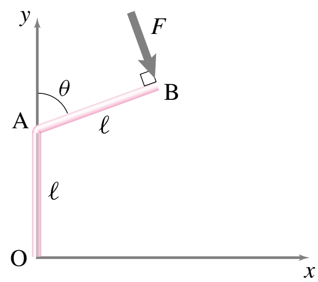
f) Line Parallel to z axis,

$$\vec{r} = 12m\hat{k}$$

$$\vec{r}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 12m \\ 0 & \frac{25}{13}N & \frac{60}{13}N \end{vmatrix}$$

$$= 23.07\,Nm\,\hat{i}$$

1.4.7 In the figure shown, OA = AB = 2 m. The force $F = 40\,\text{N}$ acts perpendicular to the arm AB. Find the moment of $\vec{F}$ about O, given that $\theta = 45°$. If $\vec{F}$ always acts normal to the arm AB, would increasing θ increase the magnitude of the moment? In particular, what value of θ will give the largest moment?



Problem 1.7

$$\vec{F} = 40\text{N} \cos(360-\theta)\hat{\imath} + 40\text{N} \sin(360-\theta)\hat{\jmath}$$

$$= -40\text{N} \cos\theta\, \hat{\imath} - 40\text{N} \sin\theta\, \hat{\jmath}$$

$$= -20\sqrt{2}\ \text{N}\hat{\imath} - 20\sqrt{2}\ \text{N}\hat{\jmath}$$

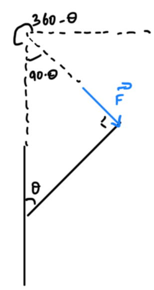


Figure 1:
To determine angle between $\vec{F}$ and horizontal

$$\vec{r}_B = 2\text{m}\,\hat{\jmath} + 2\text{m} \cos(90-\theta)\hat{\imath} + 2\text{m} \sin(90-\theta)\hat{\jmath}$$

$$= 2\text{m}\,\hat{\jmath} + 2\text{m} \sin\theta\,\hat{\imath} + 2\text{m} \cos\theta\,\hat{\jmath}$$

$$= 2\text{m}(1+\cos\theta)\hat{\jmath} + 2\text{m} \sin\theta\,\hat{\imath}$$

$$= 2\text{m}\left(1 + 1/\sqrt{2}\right)\hat{\jmath} + \sqrt{2}\,\text{m}\,\hat{\imath}$$

Find Moment across O,

$$\vec{M}_{/O} = \vec{r}_B \times \vec{F}$$

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 3.41\,\text{m} & 1.41\,\text{m} \\ -20\sqrt{2}\,\text{N} & -20\sqrt{2}\,\text{N} & 0 \end{vmatrix}$$

$$= 40\text{N}\hat{\imath} - 40\text{N}\hat{\jmath} + 96.56\,\text{N}\hat{k}$$

$$\vec{M}_{/O} = \vec{r}_B \times \vec{F}$$

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 2(1+\cos\theta) & 2\sin\theta \\ -40\cos\theta & -40\sin\theta & 0 \end{vmatrix}$$

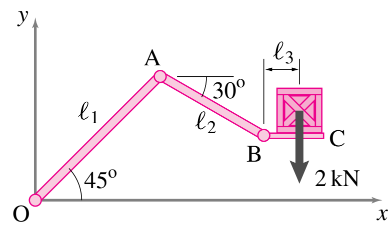
$$= +80\sin^2\theta + 80\cos^2\theta + 80\cos\theta + 80\cos^2\theta$$

$$= 80 + 80\cos^2\theta + 80\cos\theta$$

This is maximised when $\cos\theta = 1$, or $\theta = 0$.

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

1.4.8 Calculate the moment of the 2 kNpayload on the robot arm about (i) joint A, and (ii) joint B, if ℓ_1 = 0.8 m, ℓ_2 = 0.4 m, and ℓ_3 = 0.1 m.



Problem 1.8

(i) $\vec{r}_{AC} = (\ell_2 \cos 30 + \ell_3)\hat{i} + (\ell_2 \sin 30)\hat{j}$

$= (0.2\sqrt{3} + 0.1)m\hat{i} + 0.2m\,\hat{j}$

$\vec{F} = -2\,kN\,\hat{j}$

Moment across A,

$\vec{M}_{/A} = \vec{r}_{AC} \times \vec{F}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ (0.2\sqrt{3}+0.1)m & 0.2m & 0 \\ 0 & -2kN & 0 \end{vmatrix}$$

$= -0.89\ kNm$

$= -890\ kNm$

(ii) $\vec{r}_{BC} = \ell_3 \hat{i}$

$= 0.1m\,\hat{i}$

Moment across B,

$\vec{M}_{/B} = \vec{r}_{BC} \times \vec{F}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.1m & 0 & 0 \\ 0 & -2kN & 0 \end{vmatrix}$$

$= -0.2\ kNm$

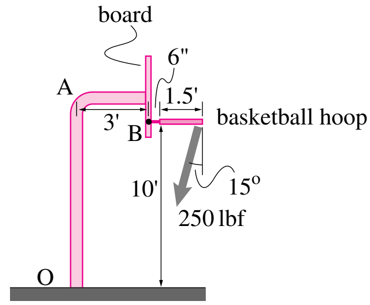
$= -200\ Nm$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

1.4.9 During a slam-dunk, a basketball player pulls on the hoop with a 250 lbf force at point C of the ring as shown in the figure. Find the moment of the force about

a) the point of the ring attachment to the board (point B), and

b) the root of the pole, point O.



Problem 1.9

Let C be the point at which the Force acts.

a) $\vec{r}_{BC} = 1.5\text{in}\ \hat{i}$

$\vec{F} = (250\text{lbf} \times \cos 255)\hat{i} + (250\text{lbf} \times \sin 255)\hat{j}$

$= -64.7\text{lbf}\ \hat{i} - 241.48\ \text{lbf}\ \hat{j}$

Moment across B,

$\vec{M}_{/B} = \vec{r}_{BC} \times \vec{F}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1.5\text{in} & 0 & 0 \\ -64.7\text{lbf} & -241.48\text{lbf} & 0 \end{vmatrix}$

$= -362.22\ \text{lbf in}\ \hat{k}$

b) $\vec{r}_c = (3+1.5)\text{in}\ \hat{i} + 10\ \text{in}\ \hat{j}$

$= 4.5\ \text{in}\ \hat{i} + 10\text{in}\ \hat{j}$

Moment across O,

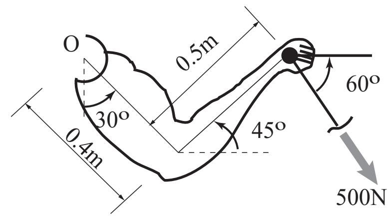
$\vec{M}_{/O} = \vec{r}_c \times \vec{F}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4.5\text{in} & 10\ \text{in} & 0 \\ -64.7\text{lbf} & -241.48\text{lbf} & 0 \end{vmatrix}$

$= -1733.66\ \text{lbf in}$

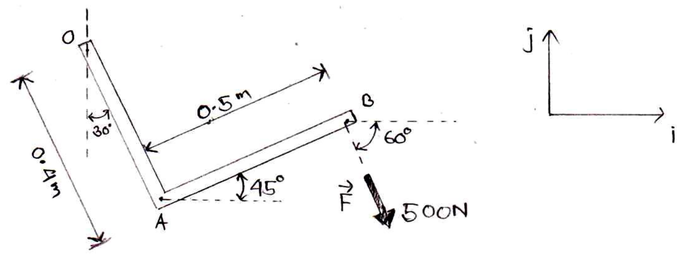
Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

1.4.10 During weight training, an athlete pulls a weight of 500 N with his arms pulling on a handlebar connected to a universal machine by a cable. Find the moment of the force about the shoulder joint O in the configuration shown.



Problem 1.10

2.4.10



The figure above depicts the arm of the athlete

Moment due to force $\vec{F}$ about O is

$$\vec{M}_{/O} = \vec{r}_{B/O} \times \vec{F}$$

$$\vec{r}_{B/O} = \vec{r}_{A/O} + \vec{r}_{B/A}$$

$$\vec{r}_{A/O} = (OA \sin 30^\circ)\,\hat{i} - (OA \cos 30^\circ)\,\hat{j}$$

$$\vec{r}_{B/A} = (AB \cos 45^\circ)\,\hat{i} + (AB \sin 45^\circ)\,\hat{j}$$

Now

$$\begin{aligned}
\vec{r}_{B/O} &= \vec{r}_{A/O} + \vec{r}_{B/A} \\
&= (OA \sin 30^\circ)\,\hat{i} - (OA \cos 30^\circ)\,\hat{j} \\
&\quad + (AB \cos 45^\circ)\,\hat{i} + (AB \sin 45^\circ)\,\hat{j} \\
&= \frac{OA}{2}\,\hat{i} - \frac{\sqrt{3}}{2} OA\,\hat{j} + \frac{AB}{\sqrt{2}}\,\hat{i} + \frac{AB}{\sqrt{2}}\,\hat{j} \\
&= \left(\frac{OA}{2} + \frac{AB}{\sqrt{2}}\right)\hat{i} + \left(-\frac{\sqrt{3}}{2} OA + \frac{AB}{\sqrt{2}}\right)\hat{j}
\end{aligned}$$

Writing force $\vec{F}$ in terms of components along both axes.

$$\begin{aligned}
\vec{F} &= (F \cos 60^\circ)\,\hat{i} - (F \sin 60^\circ)\,\hat{j} \\
&= \left(\frac{F}{2}\right)\hat{i} - \left(\frac{\sqrt{3}}{2} F\right)\hat{j}
\end{aligned}$$

Finding the moment

$$\begin{aligned}
\vec{M}_{/O} &= \vec{r}_{B/O} \times \vec{F} \\
&= \left[\left(\frac{OA}{2} + \frac{AB}{\sqrt{2}}\right)\hat{i} + \left(-\frac{\sqrt{3}}{2} OA + \frac{AB}{\sqrt{2}}\right)\hat{j}\right] \times \left[\frac{F}{2}\,\hat{i} - \frac{\sqrt{3}}{2} F\,\hat{j}\right]
\end{aligned}$$

2.4.10 continued

$$= \left[\left(\frac{OA}{2} + \frac{AB}{\sqrt{2}} \right) \left(-\frac{\sqrt{3}}{2} F \right) \hat{k} + \left(-\frac{\sqrt{3}}{2} OA + \frac{AB}{\sqrt{2}} \right) \left(\frac{F}{2} \right) (-\hat{k}) \right]$$

$$= \left[\left(\frac{0.4}{2} + \frac{0.5}{\sqrt{2}} \right) m \left(-\frac{\sqrt{3}}{2} \times 500 \right) N \right] \hat{k} + \left[\left(-\frac{\sqrt{3}}{2} \times 0.4 + \frac{0.5}{\sqrt{2}} \right) m \left(\frac{500}{2} \right) N \right] (-\hat{k})$$

$$= [0.5535 \times -433.01] \hat{k} + [(-0.3464 + 0.3535) \times 250] (-\hat{k})$$

$$= -239.67 \hat{k} - 1.775 \hat{k}$$

$$= -241.445 \, (Nm) \hat{k}$$

Let A be the point where the athlete holds the weight.

$$\vec{r}_A = (0.4\cos 30 + 0.5\cos 45)\,m\,\hat{i} + (-0.4\sin 30 + 0.5\sin 45)\,m\,\hat{j}$$

$$= 0.699\,m\,\hat{i} + 0.153\,m\,\hat{j}$$

$$\vec{F} = 500\cos 60\,N\,\hat{i} - 500\sin 60\,N\,\hat{j}$$

$$= 250\,N\,\hat{i} - 433.01\,N\,\hat{j}$$

Moment across O,

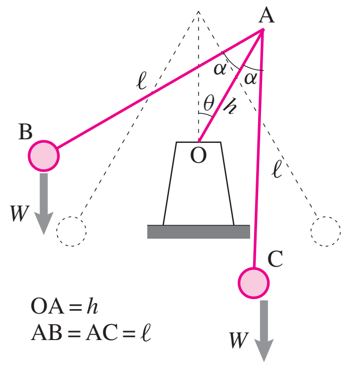
$$\vec{M}_{/O} = \vec{r}_A \times \vec{F}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.699\,m & 0.153\,m & 0 \\ 250\,N & -433.01\,N & 0 \end{vmatrix}$$

$$= -340.92\,Nm\,\hat{k}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

1.4.11 Find the sum of moments due to the two weights of the teeter-totter when the teeter-totter is tipped at an angle θ from its vertical position. Give your answer in terms of the variables shown in the figure. [Note that AC is not vertical.]



OA = h
AB = AC = ℓ

Problem 1.11

$$\vec{r}_A = (h\sin\theta)\hat{i} + (h\cos\theta)\hat{j}$$

Angle of AB with horizontal is determined from Figure 1.

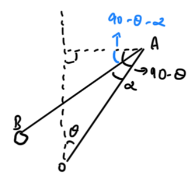


Figure 1:
To find angle of AB to horizontal

$$\vec{r}_B = \vec{r}_A + \vec{r}_{AB}$$

$$= \vec{r}_A + (\ell\cos(270-\theta-\alpha))\hat{i} + (\ell\sin(270-\theta-\alpha))\hat{j}$$

$$= (h\sin\theta + \ell\cos(270-\theta-\alpha))\hat{i} + (h\cos\theta + \ell\sin(270-\theta-\alpha))\hat{j}$$

Angle of AC with horizontal is determined from Figure 2.

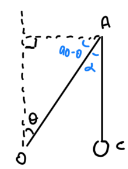


Figure 2:
To find angle of AC to horizontal

$$\vec{r}_C = \vec{r}_A + \vec{r}_{AC}$$

$$= \vec{r}_A + (\ell\cos(270+\alpha-\theta))\hat{i} + (\ell\sin(270+\alpha-\theta))\hat{j}$$

$$= (h\sin\theta + \ell\cos(270+\alpha-\theta))\hat{i} + (h\cos\theta + \ell\sin(270+\alpha-\theta))\hat{j}$$

$$\vec{M}_{B/O} = \vec{r}_B \times \vec{W}_B$$

$$= W(h\sin\theta + \ell\cos(270-\theta-\alpha))\hat{k}$$

$$\vec{M}_{C/O} = \vec{r}_C \times \vec{W}_C$$

$$= W(h\sin\theta + \ell\cos(270-\theta+\alpha))\hat{k}$$

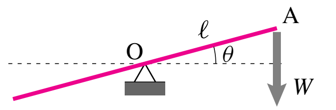
Find Sum of Moments,

$$\vec{M}_{/O} = \vec{M}_{B/O} + \vec{M}_{C/O}$$

$$= W(2h\sin\theta + \ell[\cos(270-\theta-\alpha) + \cos(270-\theta+\alpha)])\hat{k}$$

Disclaimer: *We have lost track of the many people who wrote these solutions. They are not official and have not been reviewed by the authors for correctness.*

1.4.12 Find the percentage error in computing the moment of $\vec{W}$ about the pivot point O as a function of θ, if the weight is assumed to act normal to the arm OA (a good approximation when θ is very small).



Problem 1.12

$$\vec{M}_{/O} = \vec{r}_A \times \vec{W}$$

$$= |\vec{r}_A|\,|\vec{W}| \cos\theta\ \hat{k}$$

$$= \ell W \cos\theta\ \hat{k}$$

For small θ, $\cos\theta \approx 1$

$$= \ell W \hat{k}$$

$$\text{Error} = \frac{\ell W \cos\theta - \ell W}{\ell W \cos\theta} \times 100\,\%$$

$$= (1 - \sec\theta) \times 100\,\%$$

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1.4.13 Vector Calculations and Geometry. The 5 N force $\vec{F}_1$ is along the line OA. The 7 N force $\vec{F}_2$ is along the line OB.

a) Find a unit vector in the direction OB.

b) Find a unit vector in the direction OA.

c) Write both $\vec{F}_1$ and $\vec{F}_2$ as the product of their magnitudes and unit vectors in their directions.

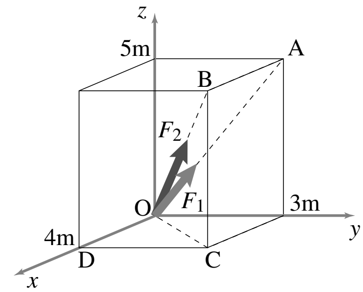
d) What is the angle AOB?

e) What is the component of $\vec{F}_1$ in the x-direction?

f) What is $\vec{r}_{DO} \times \vec{F}_1$? ($\vec{r}_{DO} \equiv \vec{r}_{O/D}$ is the position of O relative to D.)

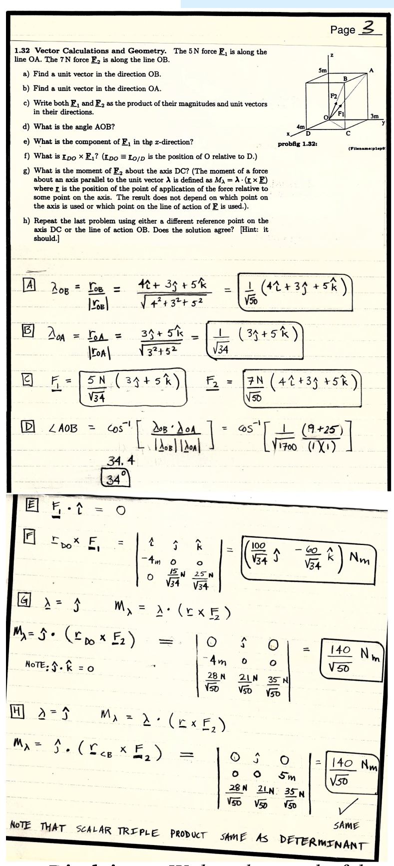
g) What is the moment of $\vec{F}_2$ about the axis DC?

h) Again find the moment of $\vec{F}_2$ about the axis DC. This time use either a different reference point on the axis DC or a different point on the line of action OB. Does the solution agree? [Hint: it should.]



Problem 1.13

Answer:
(a) $\hat{\lambda}_{OB} = \frac{1}{\sqrt{50}}(4\hat{\imath} + 3\hat{\jmath} + 5\hat{k})$.
(b) $\hat{\lambda}_{OA} = \frac{1}{\sqrt{34}}(3\hat{\jmath} + 5\hat{k})$.
(c) $\vec{F}_1 = \frac{5\,\mathrm{N}}{\sqrt{34}}(3\hat{\jmath} + 5\hat{k})$, $\vec{F}_2 = \frac{7\,\mathrm{N}}{\sqrt{50}}(4\hat{\imath} + 3\hat{\jmath} + 5\hat{k})$.
(d) $\angle AOB = 34.45\,\mathrm{deg}$.
(e) $F_{1_x} = 0$
(f) $\vec{r}_{DO} \times \vec{F}_1 = \left(\frac{100}{\sqrt{34}}\hat{\jmath} - \frac{60}{\sqrt{34}}\hat{k}\right)$ N·m.
(g) $M_\lambda = \frac{140}{\sqrt{50}}$ N·m.
(h) $M_\lambda = \frac{140}{\sqrt{50}}$ N·m.(same as (7))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1.32 Vector Calculations and Geometry. The 5 N force $\mathbf{F}_1$ is along the line OA. The 7 N force $\mathbf{F}_2$ is along the line OB.

a) Find a unit vector in the direction OB.

b) Find a unit vector in the direction OA.

c) Write both $\mathbf{F}_1$ and $\mathbf{F}_2$ as the product of their magnitudes and unit vectors in their directions.

d) What is the angle AOB?

e) What is the component of $\mathbf{F}_1$ in the x-direction?

f) What is $\mathbf{r}_{DO} \times \mathbf{F}_1$? ($\mathbf{r}_{DO} \equiv \mathbf{r}_{O/D}$ is the position of O relative to D.)

g) What is the moment of $\mathbf{F}_2$ about the axis DC? (The moment of a force about an axis parallel to the unit vector $\boldsymbol{\lambda}$ is defined as $M_\lambda = \boldsymbol{\lambda}\cdot(\mathbf{r}\times\mathbf{F})$ where $\mathbf{r}$ is the position of the point of application of the force relative to some point on the axis. The result does not depend on which point on the axis is used or which point on the line of action of $\mathbf{F}$ is used.).

h) Repeat the last problem using either a different reference point on the axis DC or the line of action OB. Does the solution agree? [Hint: it should.]

probfig 1.32:

(Filename:pl...

A $\lambda_{OB} = \frac{\underline{r}_{OB}}{|\underline{r}_{OB}|} = \frac{4\hat{\imath} + 3\hat{\jmath} + 5\hat{k}}{\sqrt{4^2+3^2+5^2}} = \boxed{\frac{1}{\sqrt{50}}(4\hat{\imath} + 3\hat{\jmath} + 5\hat{k})}$

B $\lambda_{OA} = \frac{\underline{r}_{OA}}{|\underline{r}_{OA}|} = \frac{3\hat{\jmath} + 5\hat{k}}{\sqrt{3^2+5^2}} = \boxed{\frac{1}{\sqrt{34}}(3\hat{\jmath} + 5\hat{k})}$

C $\underline{F}_1 = \boxed{\frac{5\,N}{\sqrt{34}}(3\hat{\jmath} + 5\hat{k})}$ $\underline{F}_2 = \boxed{\frac{7\,N}{\sqrt{50}}(4\hat{\imath} + 3\hat{\jmath} + 5\hat{k})}$

D $\angle AOB = \cos^{-1}\left[\frac{\lambda_{OB}\cdot\lambda_{OA}}{|\lambda_{OB}||\lambda_{OA}|}\right] = \cos^{-1}\left[\frac{1}{\sqrt{1700}}\frac{(9+25)}{(1)(1)}\right]$

34.4
$\boxed{34^\circ}$

E $\underline{F}_1\cdot\hat{\imath} = 0$

F $\underline{r}_{DO}\times\underline{F}_1 = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -4m & 0 & 0 \\ 0 & \frac{15}{\sqrt{34}}N & \frac{25}{\sqrt{34}}N \end{vmatrix} = \boxed{\left(\frac{100}{\sqrt{34}}\hat{\jmath} - \frac{60}{\sqrt{34}}\hat{k}\right) Nm}$

G $\underline{\lambda} = \hat{\jmath}$ $M_\lambda = \underline{\lambda}\cdot(\underline{r}\times\underline{F}_2)$

$M_\lambda = \hat{\jmath}\cdot(\underline{r}_{DO}\times\underline{F}_2) = \begin{vmatrix} 0 & \hat{\jmath} & 0 \\ -4m & 0 & 0 \\ \frac{28}{\sqrt{50}}N & \frac{21}{\sqrt{50}}N & \frac{35}{\sqrt{50}}N \end{vmatrix} = \boxed{\frac{140}{\sqrt{50}} Nm}$

NOTE: $\hat{\jmath}\cdot\hat{k} = 0$

H $\underline{\lambda} = \hat{\jmath}$ $M_\lambda = \underline{\lambda}\cdot(\underline{r}\times\underline{F}_2)$

$M_\lambda = \hat{\jmath}\cdot(\underline{r}_{CB}\times\underline{F}_2) = \begin{vmatrix} 0 & \hat{\jmath} & 0 \\ 0 & 0 & 5m \\ \frac{28}{\sqrt{50}}N & \frac{21}{\sqrt{50}}N & \frac{35}{\sqrt{50}}N \end{vmatrix} = \boxed{\frac{140}{\sqrt{50}} Nm}$

✓ SAME

NOTE THAT SCALAR TRIPLE PRODUCT SAME AS DETERMINANT

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a) $\vec{r}_B = (4\hat{i} + 3\hat{j} + 5\hat{k})m$

$|\vec{r}_B| = \sqrt{(4m)^2 + (3m)^2 + (5m)^2}$

$= 5\sqrt{2}\ m$

$\hat{\lambda}_{B/O} = \frac{1}{5\sqrt{2}}(4\hat{i} + 5\hat{j} + \hat{k})$

b) $\vec{r}_A = (3\hat{j} + 5\hat{k})\,m$

$|\vec{r}_A| = \sqrt{(3m) + (5m)^2}$

$= \sqrt{34}\ m$

$\hat{\lambda}_{A/O} = \frac{1}{\sqrt{34}}(3\hat{j} + 5\hat{k})$

c) $\vec{F}_1 = 5N\,\hat{\lambda}_{A/O}$

$= \frac{5N}{\sqrt{34}}(3\hat{j} + 5\hat{k})$

$\vec{F}_2 = \frac{7N}{5\sqrt{2}}(4\hat{i} + 3\hat{j} + 5\hat{k})$

d) $\theta_{AOB} = \cos^{-1}\left[\frac{\hat{\lambda}_{A/O} \cdot \hat{\lambda}_{B/O}}{|\hat{\lambda}_{A/O}|\,|\hat{\lambda}_{B/O}|}\right]$

$= \cos^{-1}\left[\frac{4}{\sqrt{17}}\right]$

$= 14.03^\circ$

e) $\vec{F}_1 \cdot \hat{i} = \frac{15}{\sqrt{34}}\ N$

$= 2.57\ N$

f) $\vec{r}_{DO} = 4m\hat{i}$

$$\vec{r}_{DO} \times \vec{F}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4m & 0 & 0 \\ 0 & \frac{15}{\sqrt{34}}N & \frac{25}{\sqrt{34}}N \end{vmatrix}$$

$$= -\frac{100}{\sqrt{34}} Nm\,\hat{j} + \frac{60}{\sqrt{34}} Nm\,\hat{k}$$

$$= \frac{20}{\sqrt{34}} Nm\,(-5\hat{j} + 3\hat{k})$$

g) $$\vec{r}_{DO} \times \vec{F}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4m & 0 & 0 \\ \frac{28N}{5\sqrt{2}} & \frac{21N}{5\sqrt{2}} & \frac{7N}{\sqrt{2}} \end{vmatrix}$$

$$= -14\sqrt{2}\,N\,\hat{j} + \frac{42\sqrt{2}}{5}N\,\hat{k}$$

$\vec{r}_{DC} = 3\hat{j}$

$$M_{/DC} = \vec{r}_{DC} \cdot (\vec{r}_{DO} \times \vec{F}_2)$$

$$= -42\sqrt{2}\,N$$

h) $\vec{r}_C = 4\hat{i} + 3\hat{j}$

$$M_{/DC} = \vec{r}_C \cdot (\vec{r}_{DO} \times \vec{F}_2)$$

$$= -42\sqrt{2}\,N$$

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